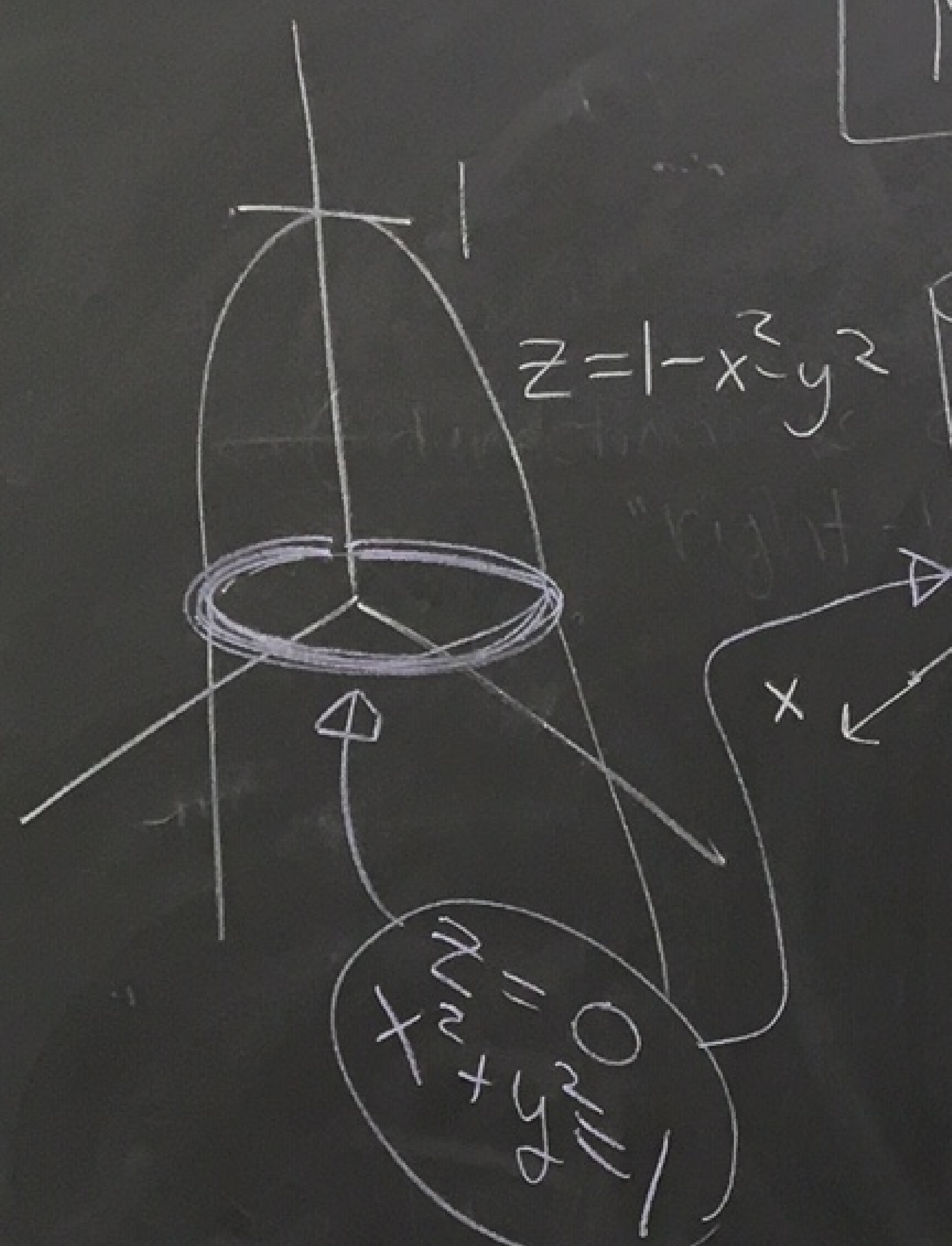
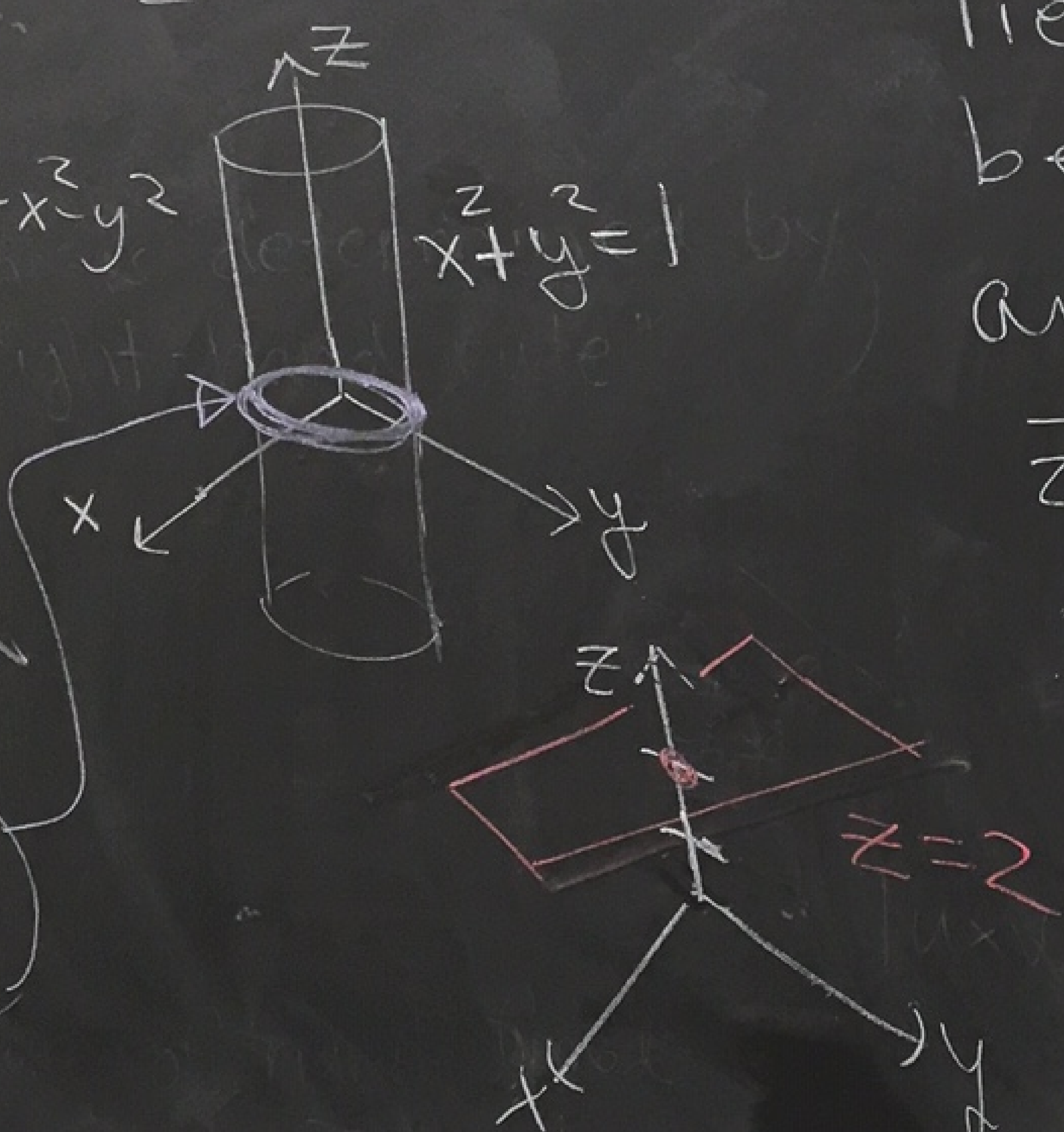
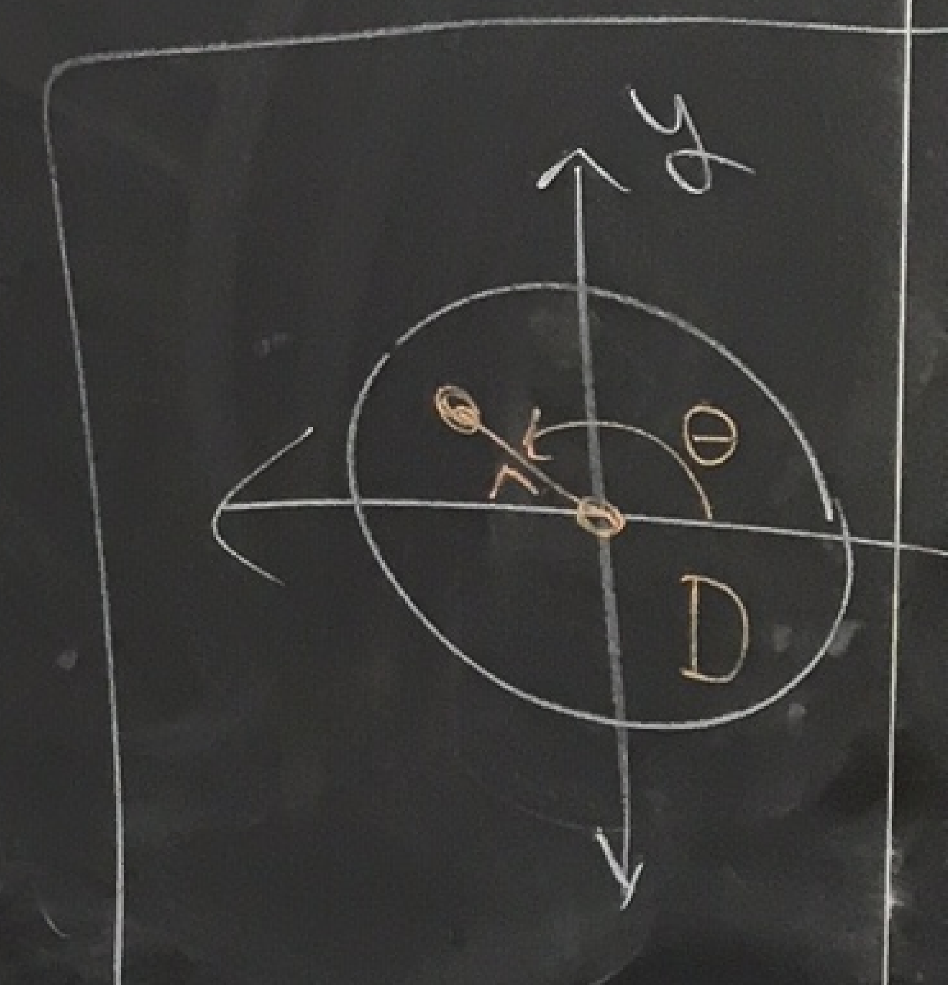
Tuesday
Nov. 5

Ex: Let E be the solid that lies within the cylinder $x^2+y^2=1$, below the plane $z=2$, and above the paraboloid $z=1-x^2-y^2$.

Find the volume of E.







x^2+

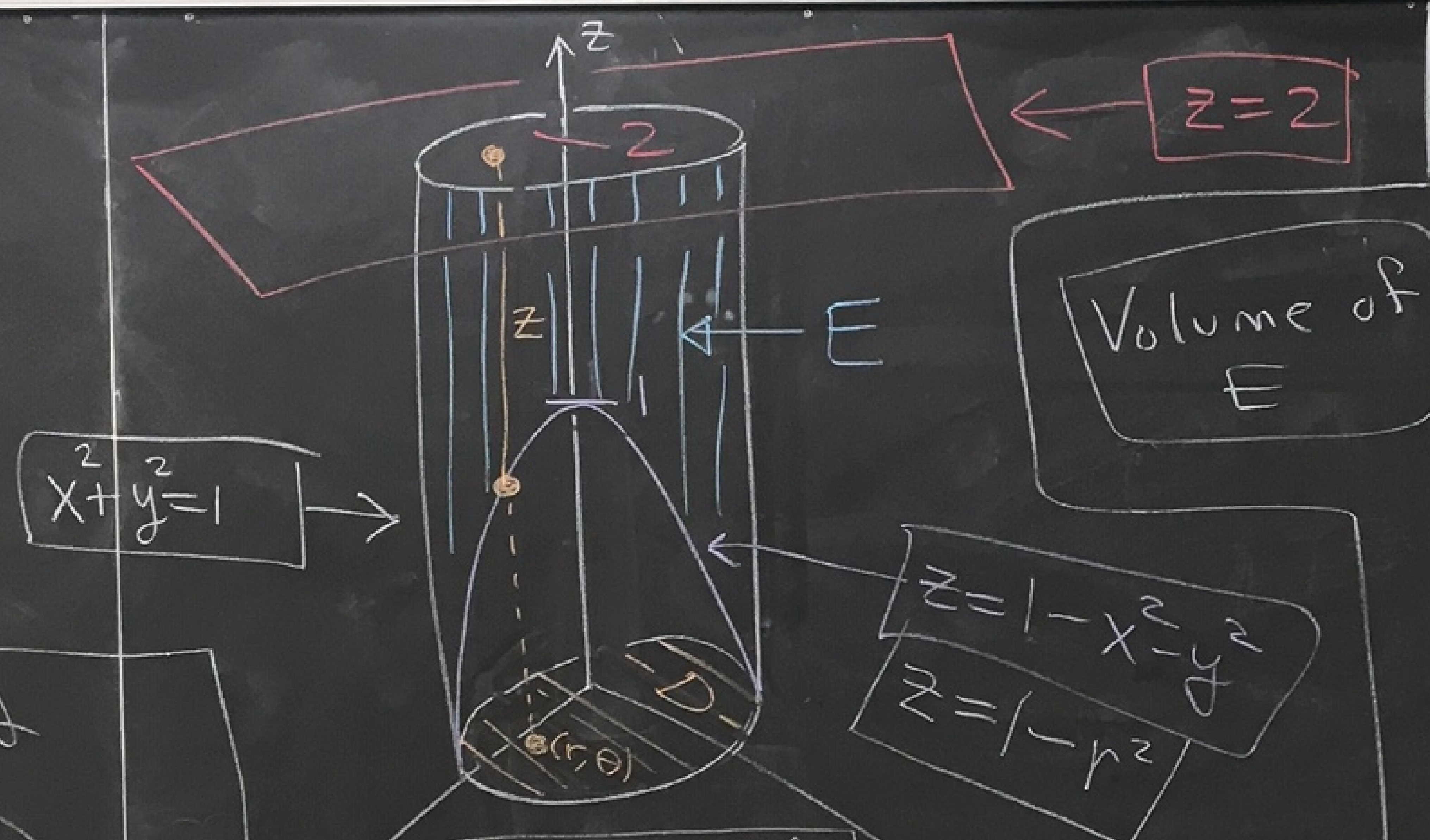
solid that

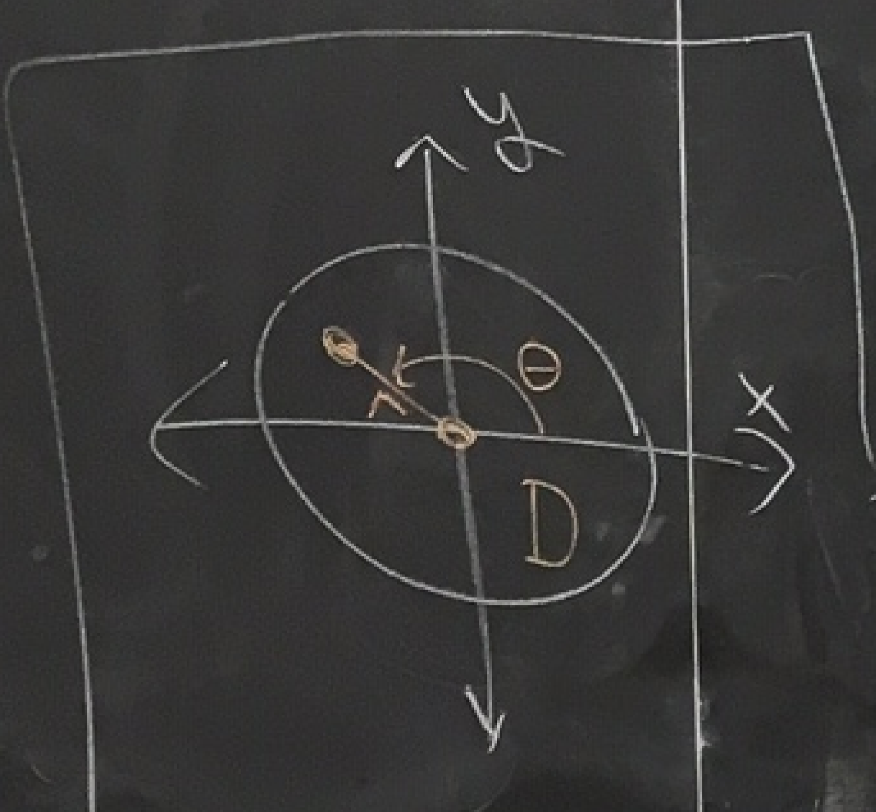
er $x^2+y^2=1$,

2,

aboloid

E.





parameterization of D

$0 \le r \le 1$

$0 \le \theta \le 2\pi$

$$\boxed{\text{Volume of } E} = \iiint_E 1\,dV = \underbrace{\int_0^{2\pi}\int_0^1}_{D} \underbrace{\int_{1-r^2}^{2}}_{z} 1\,\underbrace{dz\,r\,dr\,d\theta}_{dV}$$

$$= \int_0^{2\pi}\int_0^1\int_{1-r^2}^{2} r\,dz\,dr\,d\theta$$

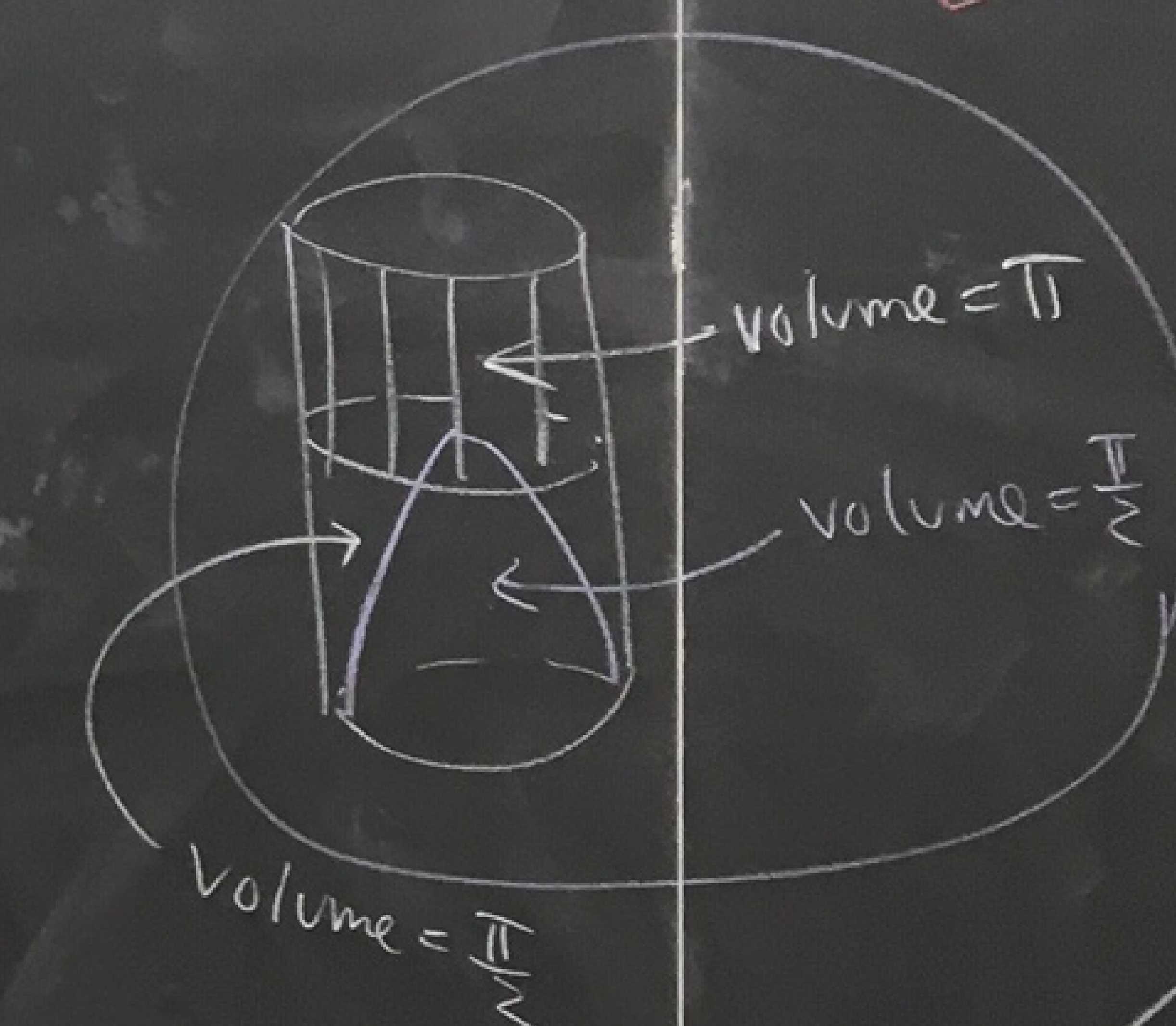
$$= \int_0^{2\pi}\int_0^1 rz\Big|_{z=1-r^2}^{2} dr\,d\theta$$

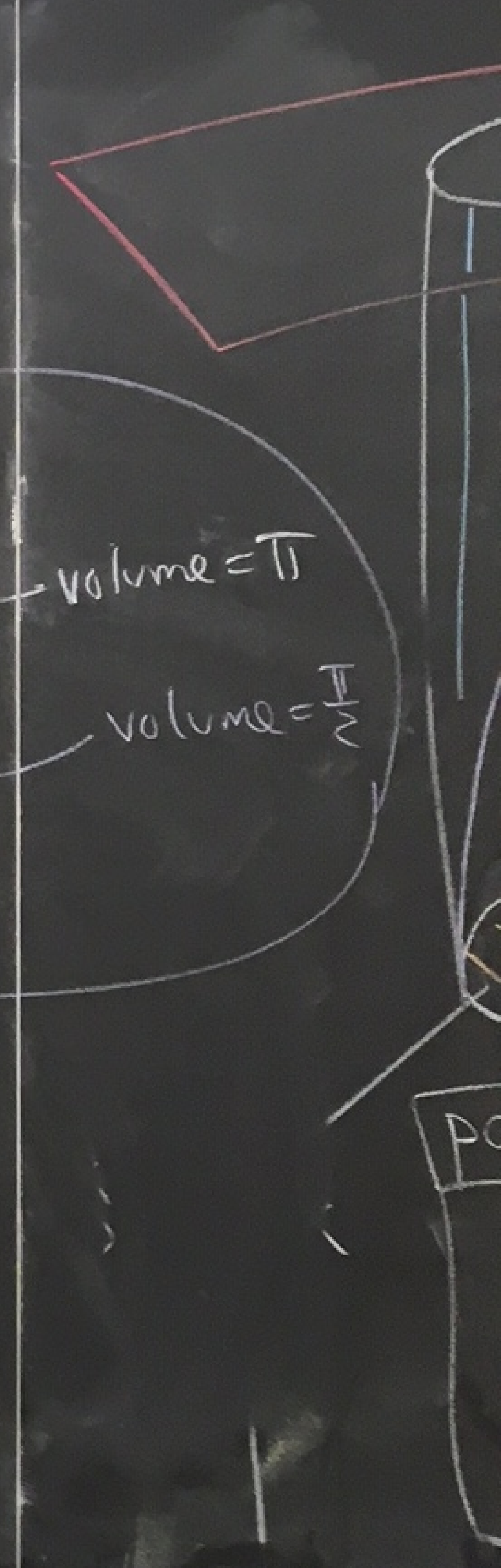
$$= \int_0^{2\pi}\int_0^1 \left(r(2) - r(1-r^2)\right) dr\, d\theta = \int_0^{2\pi}\int_0^1 (r^3 + r)\, dr\, d\theta$$

$$= \int_0^{2\pi}\left[\frac{r^4}{4}+\frac{r^2}{2}\right]_{r=0}^{1} d\theta = \int_0^{2\pi}\left[\frac{1}{4}+\frac{1}{2}\right] d\theta$$

$$= \int_0^{2\pi}\frac{3}{4}\, d\theta = \frac{3}{4}\theta\Big|_0^{2\pi}$$

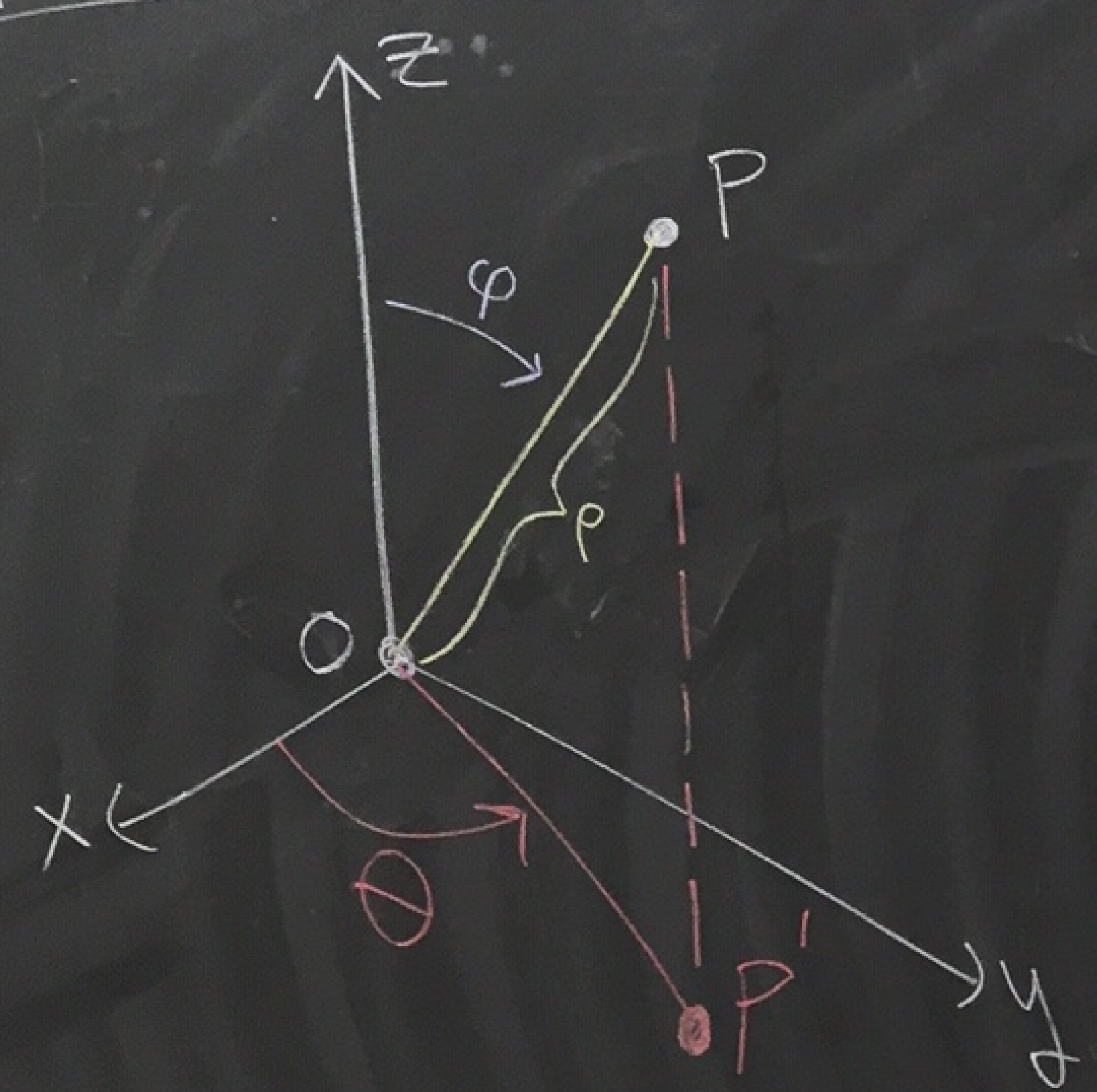
$$= \frac{3}{4}(2\pi) = \boxed{\frac{3}{2}\pi}$$





13.5 continued...

Spherical coordinates



Let P be a point in $\mathbb{R}^3$ ← this means real 3-dimensional space

Let O be the origin.

- ρ is the distance between O and P.
- φ is the angle between the positive z-axis and the line segment $\overline{OP}$.
- Project P into the xy-plane, call this point P'. θ is angle part of the polar coordinates of P'.

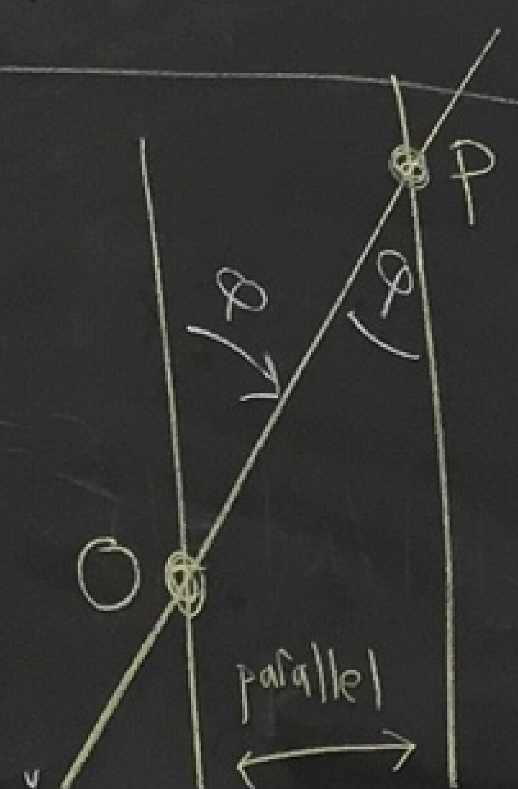
Formulas

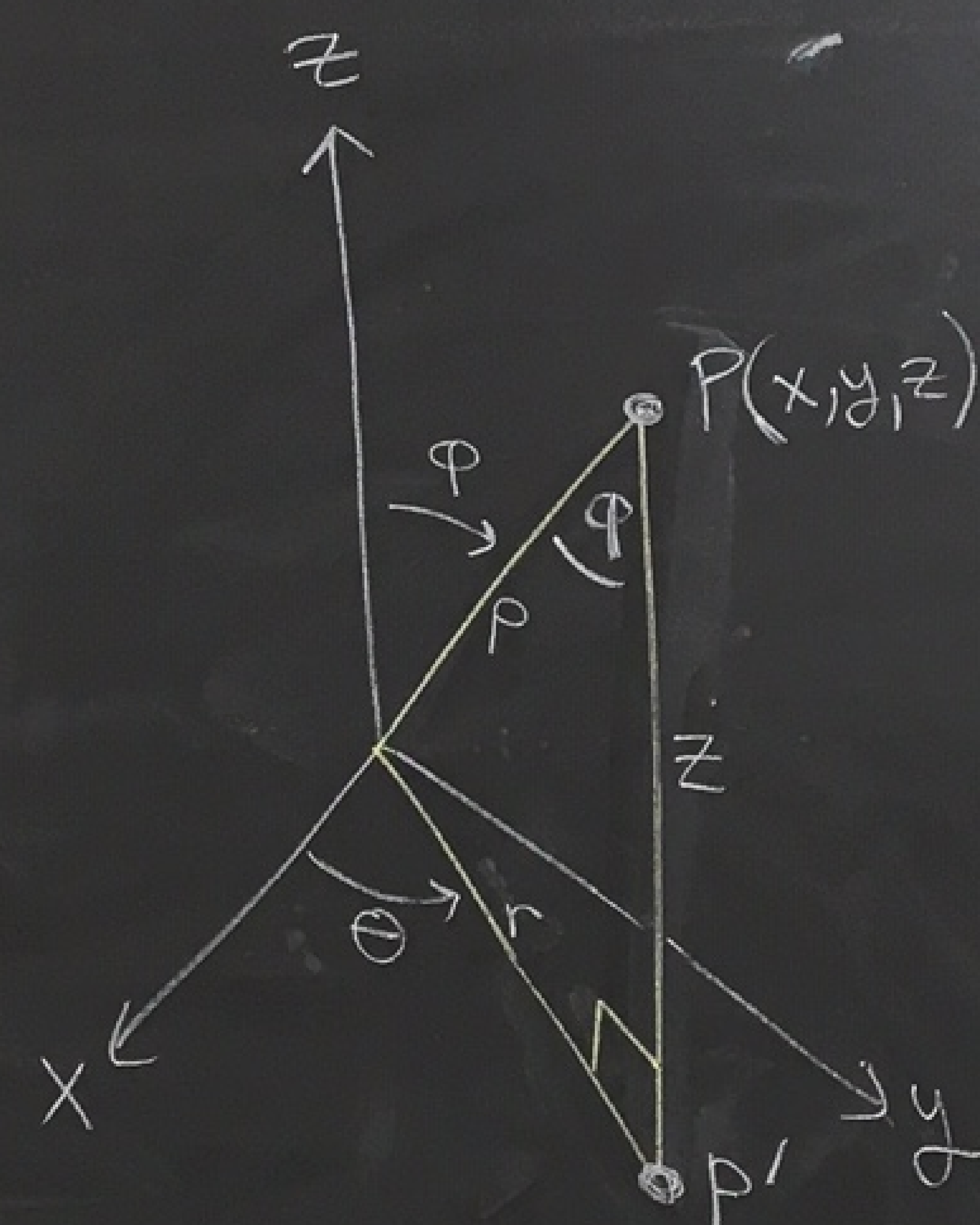
$$\rho^2 = x^2 + y^2 + z^2$$

$$x = \rho \sin(\varphi)\cos(\theta)$$

$$y = \rho \sin(\varphi)\sin(\theta)$$

$$z = \rho \cos(\varphi)$$





ρ = rho
φ = phi
θ = theta

$$\sin(\varphi) = \frac{\text{opp}}{\text{hyp}} = \frac{r}{\rho}$$

$$r = \rho\sin(\varphi)$$

$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{z}{\rho}$$

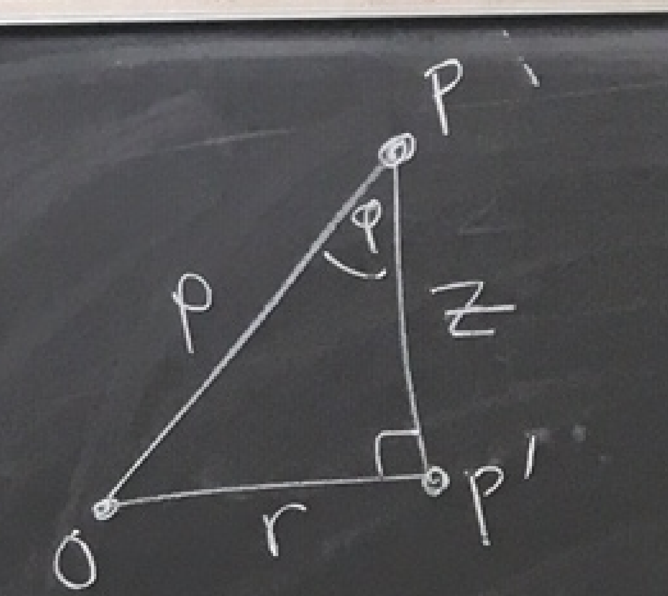
$$z = \rho\cos(\varphi)$$

$$x = r\cos(\theta)$$

$$x = \rho\sin(\varphi)\cos(\theta)$$

$$y = r\sin(\theta)$$

$$y = \rho\sin(\varphi)\sin(\theta)$$



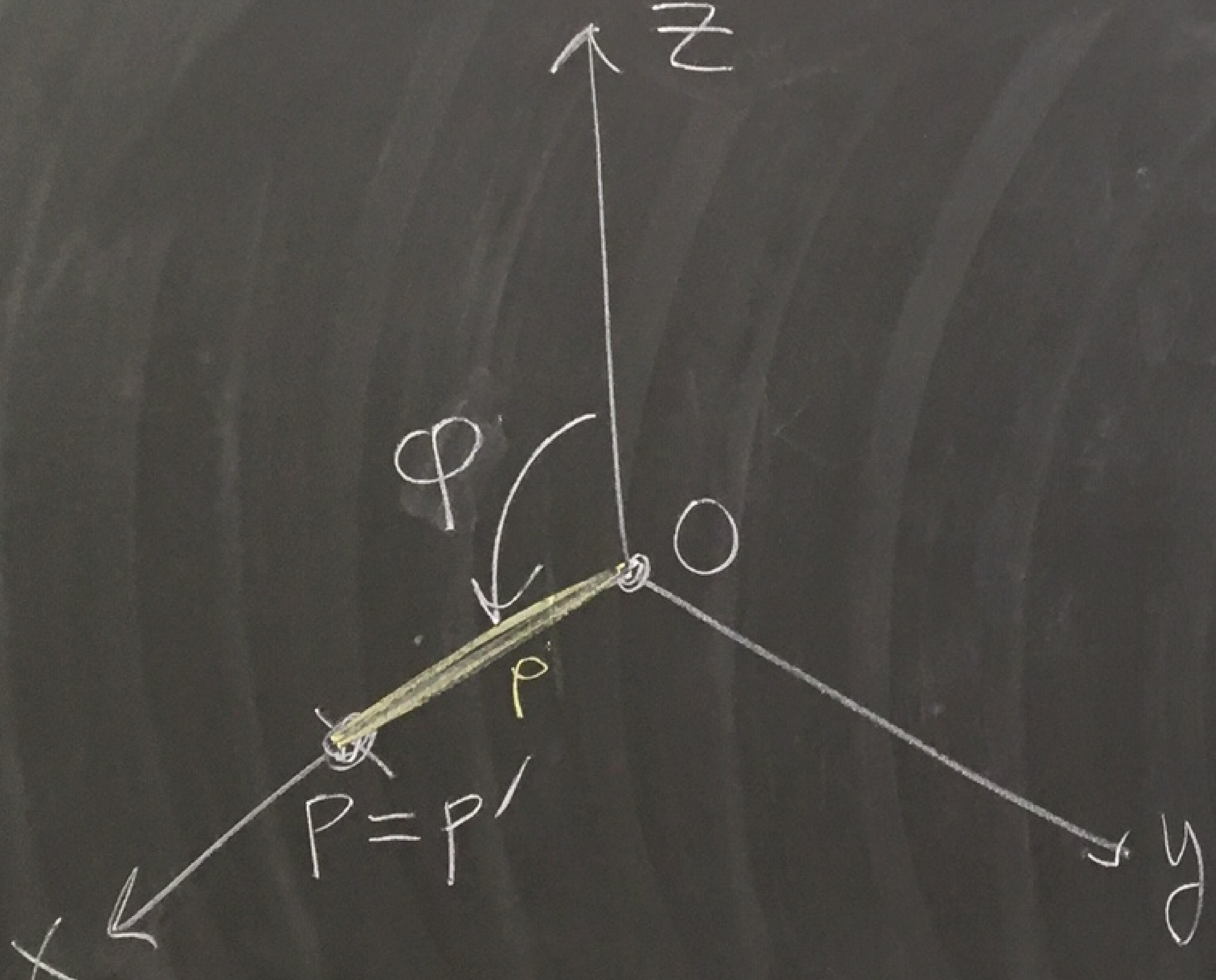
$$x^2 + y^2 = r^2$$

$$\rho^2 = r^2 + z^2$$

$$\rho^2 = x^2 + y^2 + z^2$$

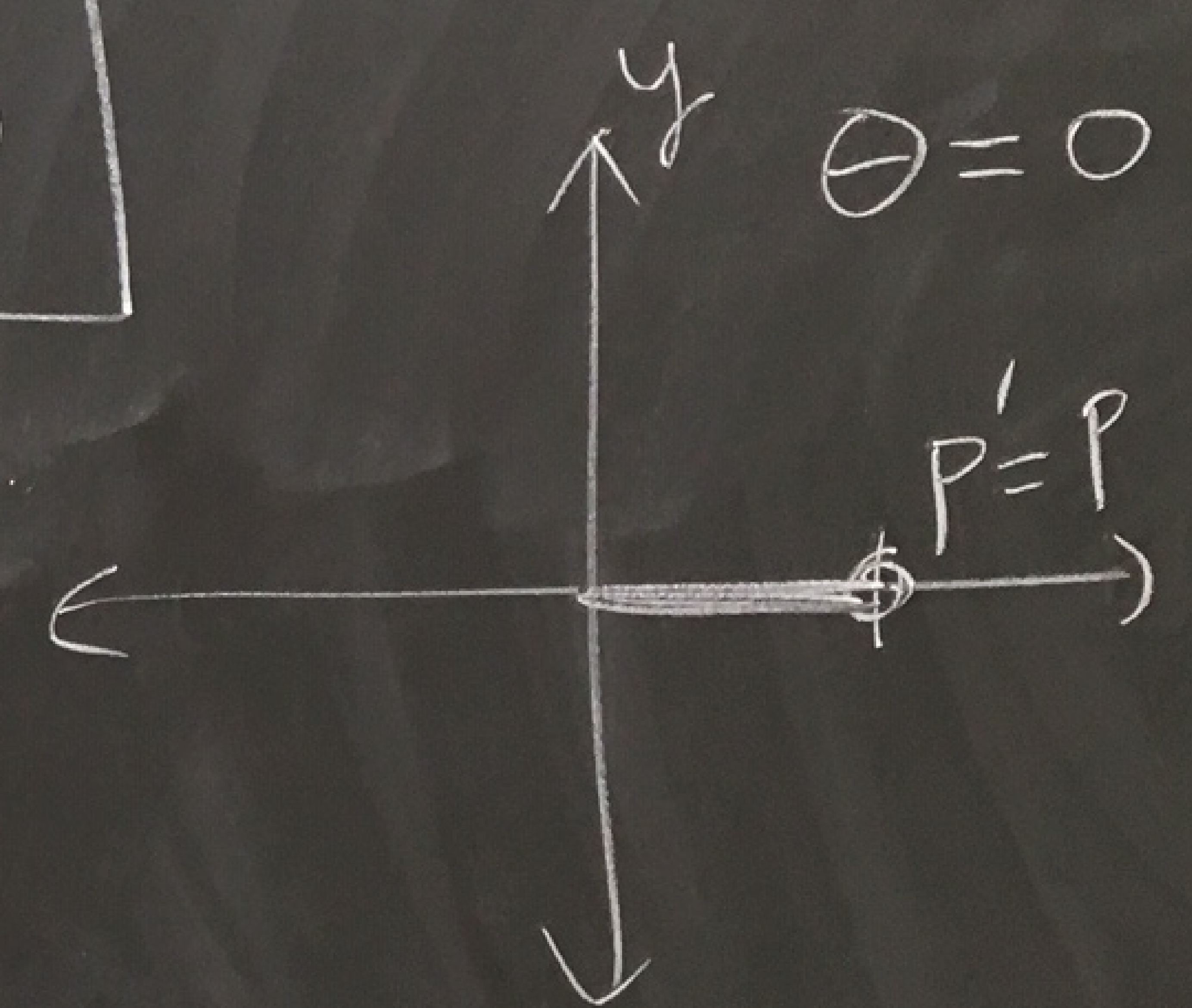
Ex: Convert $P(x,y,z) = P(1,0,0)$

into spherical coordinates.

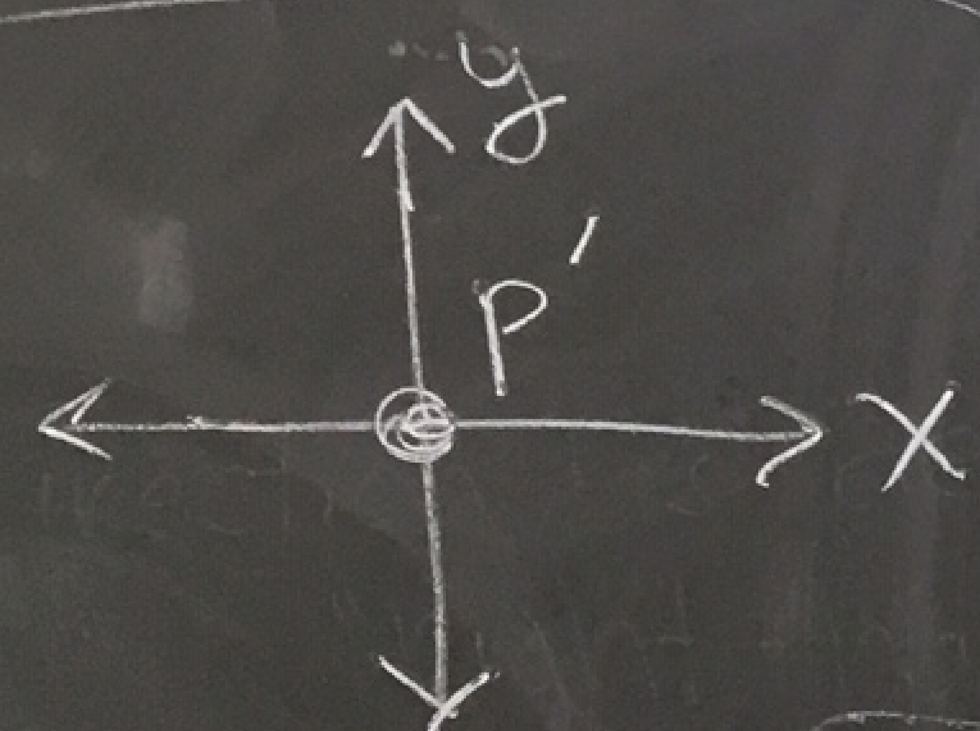


$$\rho^2 = x^2 + y^2 + z^2 = 1^2 + 0^2 + 0^2 = 1$$

$$\boxed{\begin{aligned} \rho &= 1 \\ \varphi &= \frac{\pi}{2} \\ \theta &= 0 \end{aligned}}$$



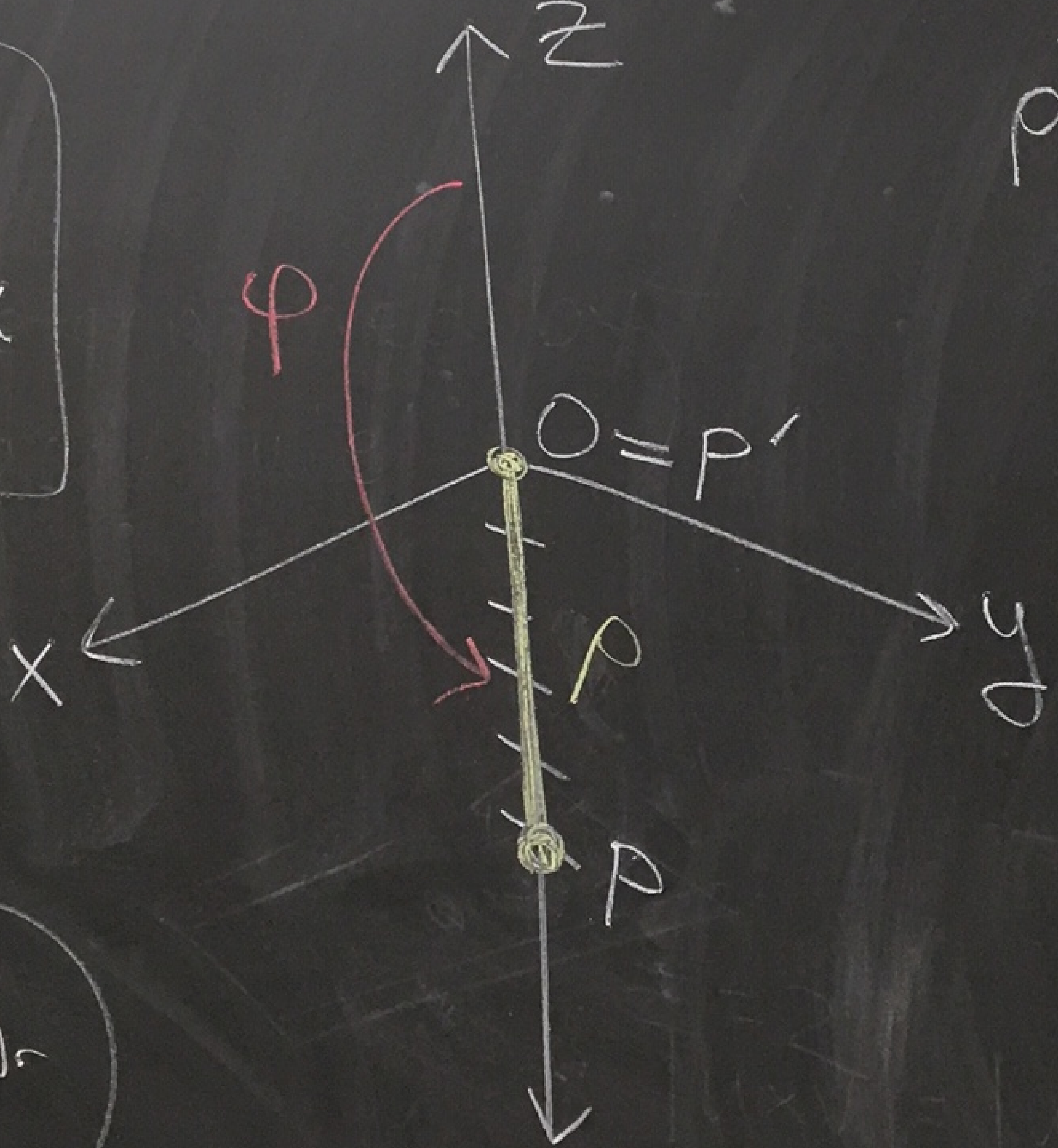
Ex: Convert $P(x,y,z) = P(0,0,-5)$ into spherical coordinates.



$r = 0$

$\theta = 0$

you could pick any angle for the origin



$$\rho^2 = x^2 + y^2 + z^2$$
$$= 0^2 + 0^2 + (-5)^2 = 25$$

$$\rho = 5$$
$$\varphi = \pi$$
$$\theta = 0$$

Ex: Convert $(x,y,z) = (1, \sqrt{3}, 2)$ into spherical coordinates.

Next time