12/5
Thursday

For 13.4, 13.5, 14.1
look at these problems
for sure.

13.4 - 15, 17, 19, 20

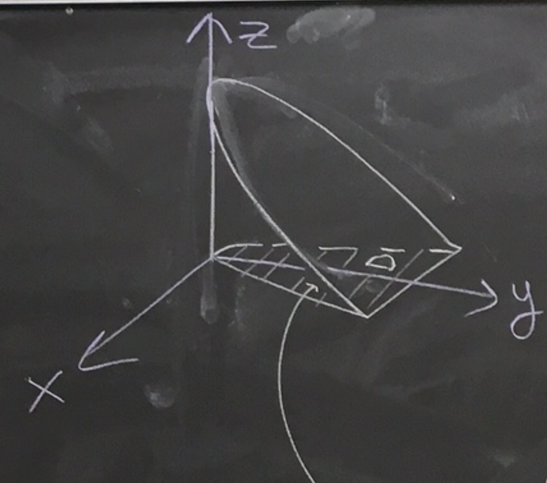
13.5 - 30, 31, 33, 39, 41,
46, 47, 49

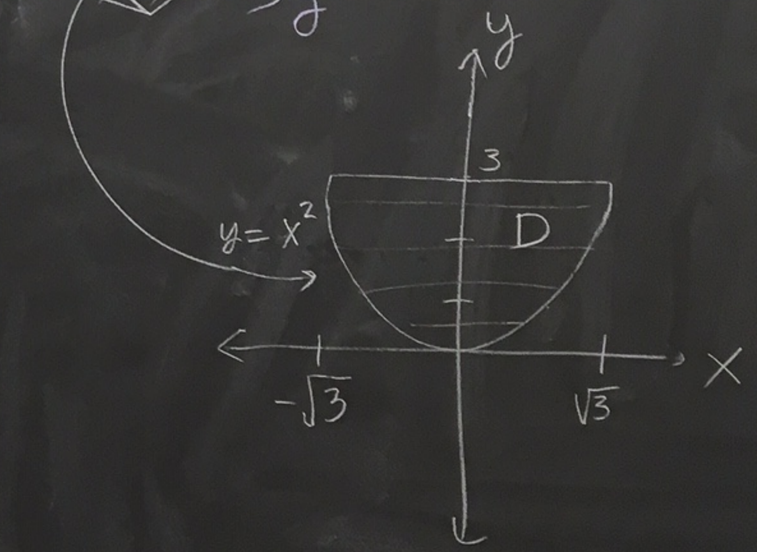
14.1 - 7, 9, 11, 13, 21, 23 (graphing ones)

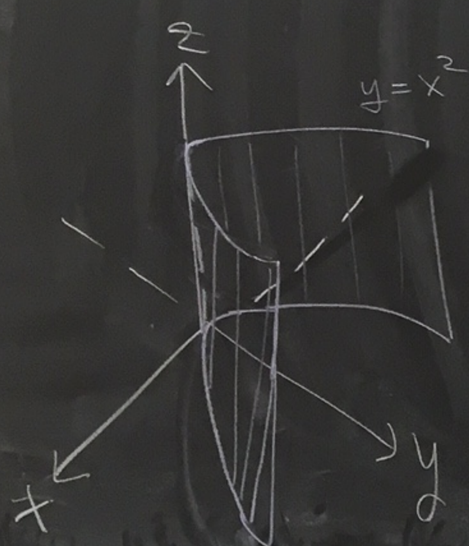
Look at ones from class too.

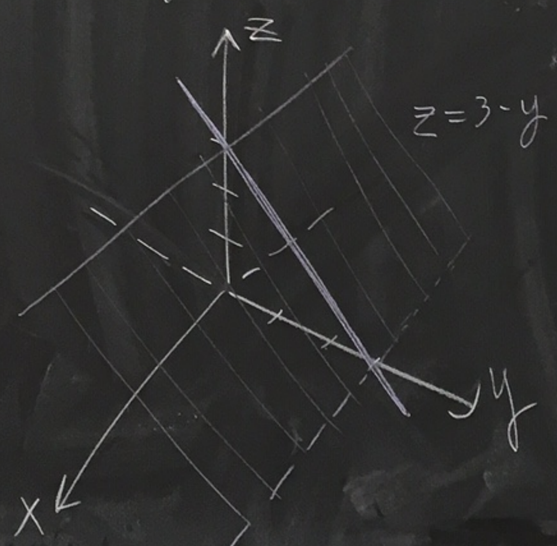
13.4

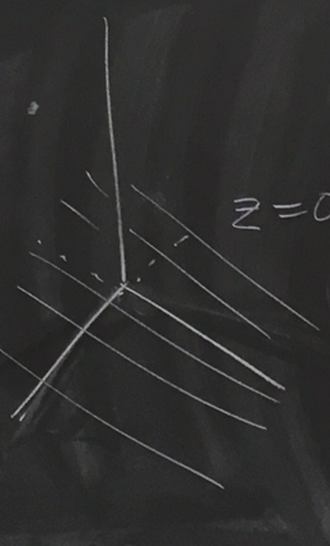
(20) Find the volume of the wedge bounded by the parabolic cylinder $y=x^2$ and the planes $z=3-y$ and $z=0$.



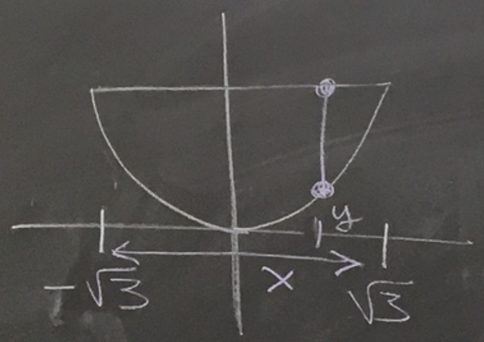






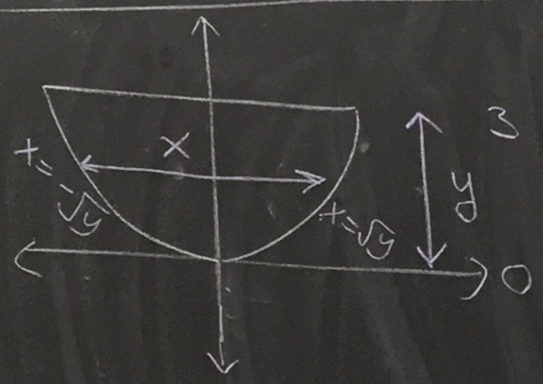


parameterize D



$$-\sqrt{3} \le x \le \sqrt{3}$$
$$x^2 \le y \le 3$$

$$\Rightarrow \text{Volume} = \int_{-\sqrt{3}}^{\sqrt{3}} \int_{x^2}^{3} \int_{0}^{3-y} 1 \, dz \, dy \, dx$$



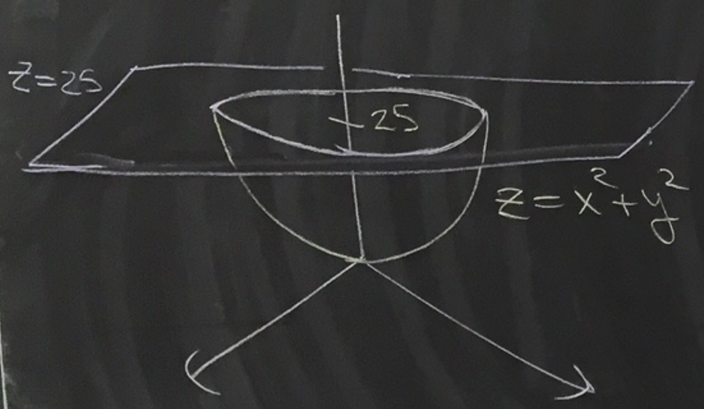
$$0 \le y \le 3$$
$$-\sqrt{y} \le x \le \sqrt{y}$$

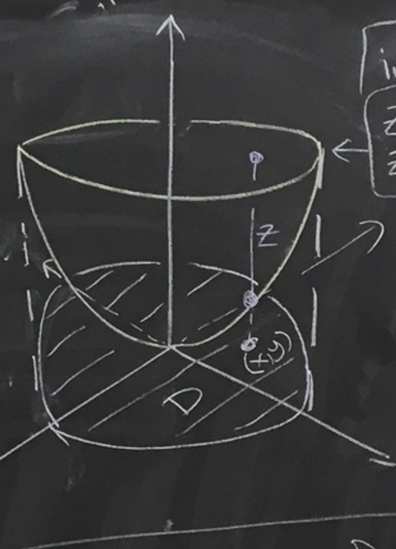
$$\Rightarrow \text{Volume} = \int_{0}^{3} \int_{-\sqrt{y}}^{\sqrt{y}} \int_{0}^{3-y} 1 \, dz \, dx \, dy$$

$$y = x^2$$
$$x = \pm\sqrt{y}$$

13.5

(30) Find the volume of the solid bounded by $z = 25$ and $z = x^2 + y^2$





intersection of

$z = 25$

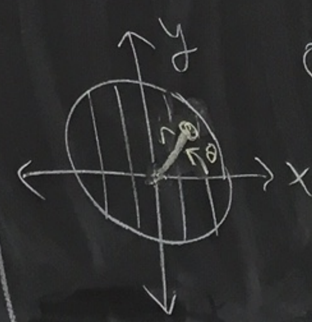
$z = x^2 + y^2$

$\rightarrow 25 = x^2 + y^2$

$5^2 = x^2 + y^2$

$$\iiint_D \left[\int_{x^2+y^2}^{25} 1 \, dz \right] dA$$

parameterize D



$0 \le r \le 5$

$0 \le \theta \le 2\pi$

$$= \int_0^{2\pi} \int_0^5 \int_{r^2}^{25} 1 \, dz \; r \, dr \, d\theta$$

$x^2 + y^2 = r^2$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$dA = r\,dr\,d\theta$

(41) $\iiint_D \frac{dV}{(x^2+y^2+z^2)^{3/2}}$

D is the solid between the spheres of radius 1 and radius 2 centered at the origin.

I'll put this on final

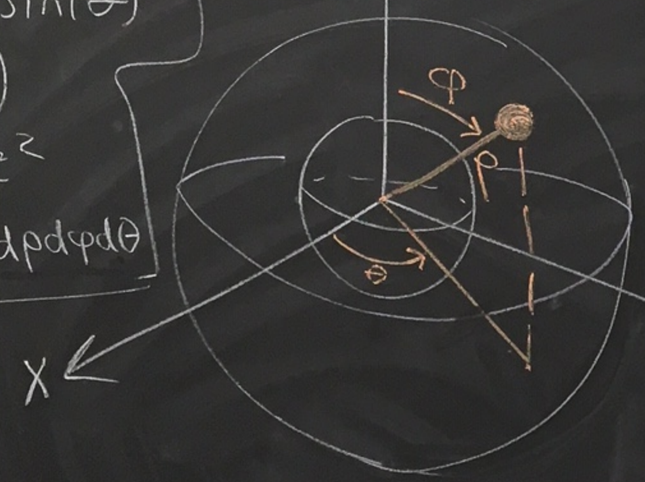
$x = \rho\sin(\varphi)\cos(\theta)$

$y = \rho\sin(\varphi)\sin(\theta)$

$z = \rho\cos(\varphi)$

$\rho^2 = x^2+y^2+z^2$

$dV = \rho^2\sin(\varphi)\,d\rho d\varphi d\theta$



parameterize D

$1 \le \rho \le 2$

$0 \le \varphi \le \pi$

$0 \le \theta \le 2\pi$

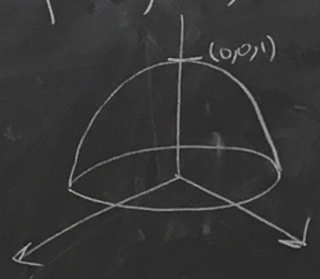
$$\iiint_D \frac{dV}{(x^2+y^2+z^2)^{3/2}}$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_1^2 \frac{\rho^2\sin(\varphi)}{(\rho^2)^{3/2}}\,d\rho d\varphi d\theta$$

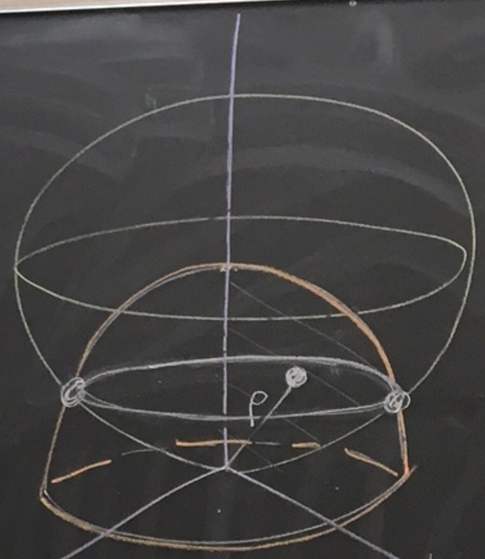
$$= \int_0^{2\pi}\int_0^{\pi}\int_1^2 \frac{\sin(\varphi)}{\rho}\,d\rho d\varphi d\theta$$

(47) Find the volume of the solid bounded by the sphere $\rho = 2\cos(\varphi)$ and the hemisphere given by $\rho = 1, z \geq 0$.

$\rho = 1, z \geq 0$



top half of sphere of radius 1 centered at origin



intersect
$\rho = 1$
$\rho = 2\cos(\varphi)$

$1 = 2\cos(\varphi)$

$\frac{1}{2} = \cos(\varphi)$

$\varphi = \frac{\pi}{3}$

$\rho = 2\cos(\varphi)$

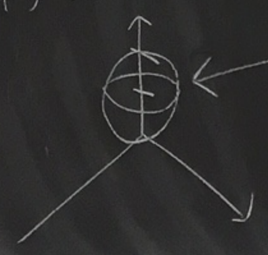
$\rho^2 = 2\rho\cos(\varphi)$

$x^2 + y^2 + z^2 = 2z$

$x^2 + y^2 + z^2 - 2z = 0$

$x^2 + y^2 + z^2 - 2z + 1 = 1$

$x^2 + y^2 + (z-1)^2 = 1$



Sphere centered at $(0,0,1)$ with radius 1.

$0 \leq \theta \leq 2\pi$	$0 \leq \theta \leq 2\pi$
$0 \leq \varphi \leq \pi/3$	$\pi/3 \leq \varphi \leq \frac{\pi}{2}$
$0 \leq \rho \leq 1$	$0 \leq \rho \leq 2\cos(\varphi)$

$$\text{Volume} = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \underline{1}\,\rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta$$

$$+\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\int_0^{2\cos(\varphi)} 1\;\rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta$$