Math 2150-01
1/27/25

Topic 1 – What is a differential equation?

<u>Ex:</u> $y' = 3y$

To solve this differential equation we want a function y where $y' = 3y$.

Let's try $y = e^{3x}$.

We get $y' = 3e^{3x}$

Notice that here $y' = 3y$.

So, $y = e^{3x}$ solves $y' = 3y$.

Def:

- An equation relating an unknown function and one or more of its derivatives is called a differential equation.
- If a differential equation only has regular derivatives of a single function then its called an ordinary differential equation (ODE).

  If it has partial derivatives then its called a partial differential equation (PDE).
- The order of a differential equation is the order of

the highest derivative that occurs in the equation

Ex: $y' = 3y$

ODE of order 1

Ex: $\underbrace{\frac{dy^2}{d^2x} + \frac{dy}{dx} - 5y = 2}_{y'' + y' - 5y = 2}$

ODE of order 2

Ex: $y'' + 2x^3 y' = \sin(x)$

y is the unknown function

$y = y(x)$ is a function of x

x is a number

ODE of order 2

Ex: (Laplace equation)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Here $u = u(x,y)$ is a function of x and y.

PDE of order 2

Def: An ODE is called <u>linear</u> if it is of the form

$$\underbrace{a_n(x)}\,y^{(n)} + \underbrace{a_{n-1}(x)}\,y^{(n-1)} + \dots + \underbrace{a_1(x)}\,y' + \underbrace{a_0(x)}\,y = \underbrace{b(x)}$$

these terms only have x's and #'s in them

Ex:

$$\underbrace{2x^2}\,y''' \underbrace{-5}\,y' + \underbrace{\tfrac{1}{x}}\,y = \underbrace{\cos(x)}$$

#'s and x's

linear ODE of order 3

<u>Ex:</u>

$$5y^{(7)} - xy^{(4)} - y' + 5 = 0$$

#'s & x's

linear ODE of order 7

<u>Ex:</u>

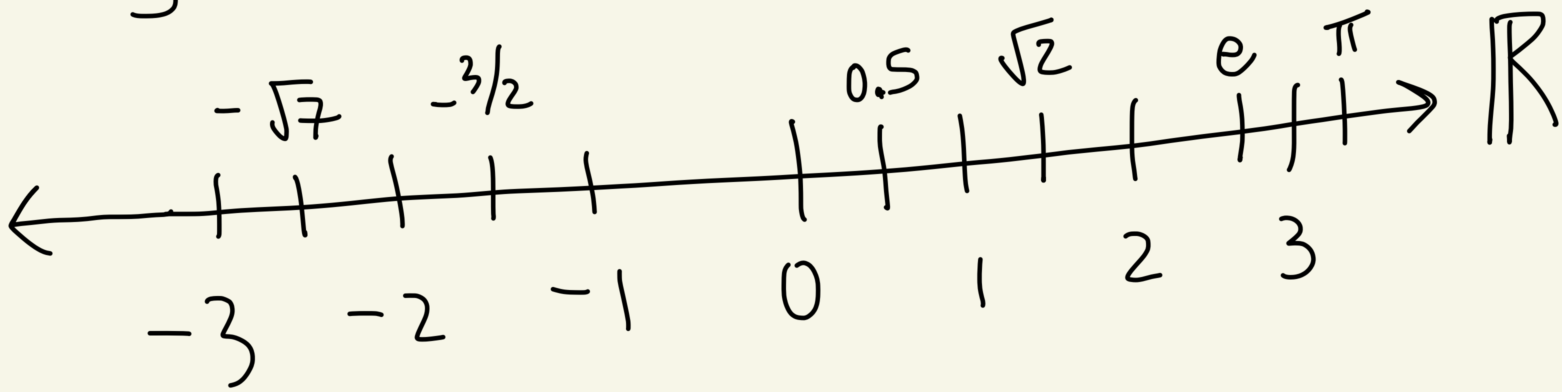
$$y^2 y' - 25y = x$$

not #'s & x's

#'s & x's

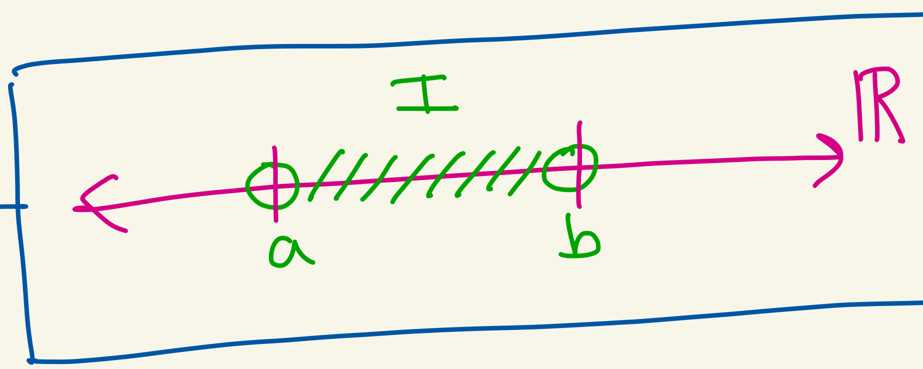
non-linear ODE of order 1

<u>Def:</u> The real number is denoted by $\mathbb{R}$.



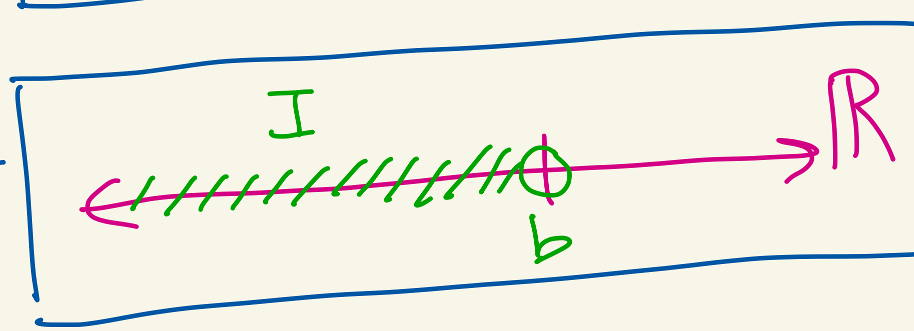
<u>Def:</u> An <u>open interval</u> I is an interval of the form:

$I = (a, b)$



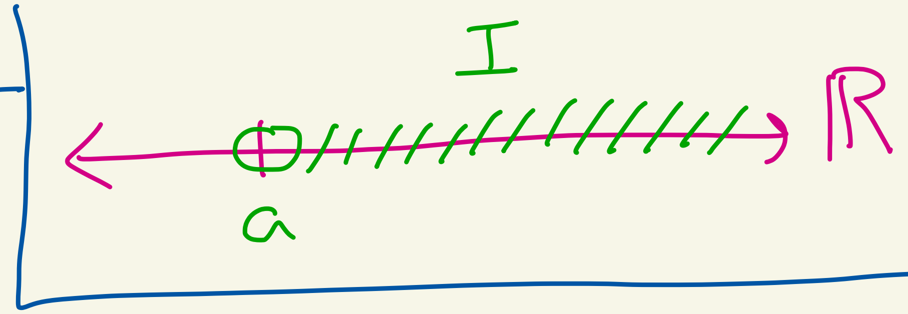
or

$I = (-\infty, b)$



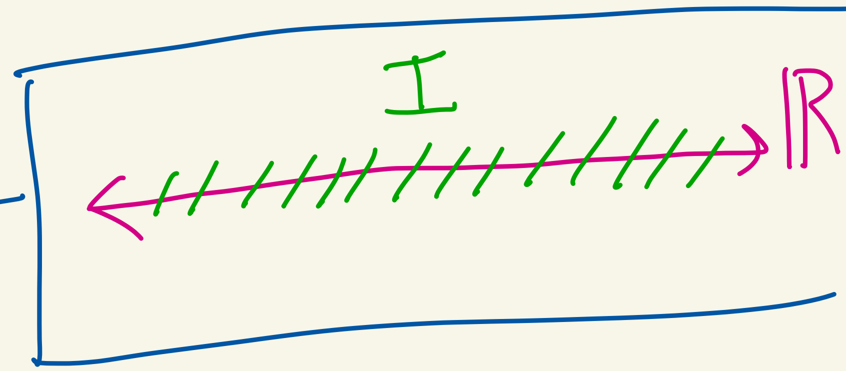
or

$I = (a, \infty)$



or

$I = (-\infty, \infty)$



Def: A function f is a solution to an n-th order ODE on an open interval I if:

1. $f, f', f'', \ldots, f^{(n)}$ exist on I

and

2. when you plug f and it's derivatives into the ODE it solves it for all x in I

In addition, sometimes one is given what

$$f(x_0), f'(x_0), \ldots, f^{(n-1)}(x_0)$$

must equal for some x_0 in I.

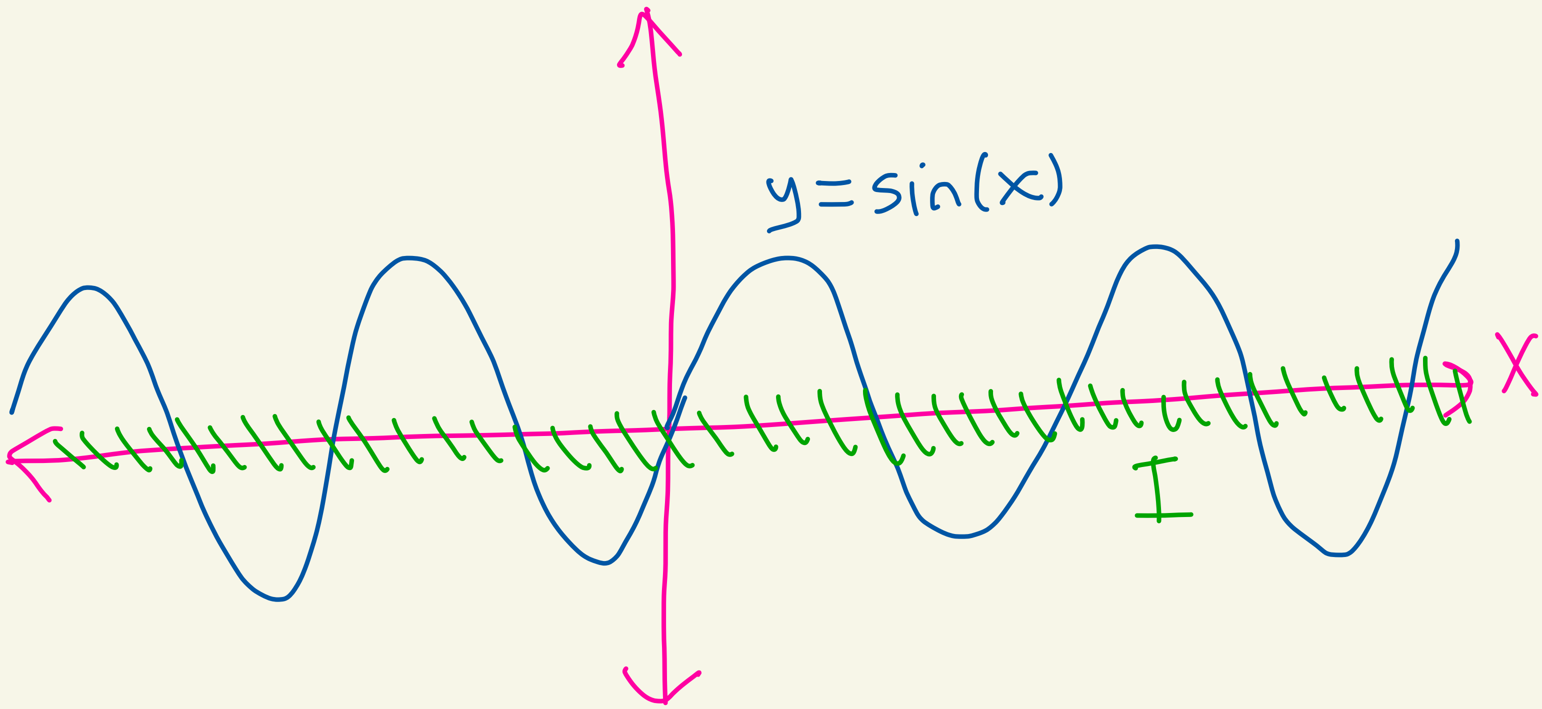
This turns the ODE into an *initial value problem*.

Ex: Let's find a solution to

$$y'' = -y$$

on $I = (-\infty, \infty)$.

Let $y = \sin(x)$



$y = \sin(x)$ is defined for all $\underbrace{x \text{ in } I}_{-\infty < x < \infty}$.

We have

$$y = \sin(x)$$
$$y' = \cos(x)$$
$$y'' = -\sin(x)$$

So, $y'' = -y$

Thus, $y = \sin(x)$ solves

$y'' = -y$ on $I = (-\infty, \infty)$.