Math 2150-01
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There was a mistake in HW 1 problem 2(a) solutions in the f_2 calculation. I fixed it and re-uploaded the solutions.

Continue topic 3...

HW 1(d)

Solve

$$\frac{dy}{dx} + 2xy = xe^{-x^2}$$

on $I = (-\infty, \infty)$

We want to solve

$$y' + \underbrace{2x}_{\text{integrate this}}y = xe^{-x^2}$$

$$A(x) = \int 2x\,dx = x^2$$

Multiply the ODE by

$e^{A(x)} = e^{x^2}$ to get:

$$\underbrace{e^{x^2} y' + e^{x^2}(2x)y}_{\text{always } \left(e^{A(x)} \cdot y\right)'} = e^{x^2} \cdot x \cdot e^{-x^2}$$

This becomes:

$$\left(e^{x^2} \cdot y\right)' = \underbrace{e^{x^2} \cdot x \cdot e^{-x^2}}_{e^{x^2} e^{-x^2} = e^{x^2 - x^2} = e^0 = 1}$$

So we get

$$\left(e^{x^2} \cdot y\right)' = x$$

Integrate both sides with respect to x to get

$$e^{x^2} \cdot y = \int x\,dx$$

We get

$$e^{x^2} \cdot y = \frac{1}{2}x^2 + C$$

Divide by e^{x^2} or multiply by e^{-x^2} to get

$$\boxed{y = \frac{1}{2}x^2 e^{-x^2} + Ce^{-x^2}}$$

Answer

Ex: Let's solve

$$y' + \underbrace{\cos(x)}_{\text{integrate this}}\, y = \sin(x)\cos(x)$$

on $I = (-\infty, \infty)$

$$A(x) = \int \cos(x)\,dx = \sin(x)$$

Multiply the ODE by $e^{A(x)} = e^{\sin(x)}$ to get:

$$\underbrace{e^{\sin(x)} y' + e^{\sin(x)} \cos(x)\, y}_{\text{always } \left(e^{A(x)} \cdot y\right)'} = e^{\sin(x)} \sin(x)\cos(x)$$

We get:

$$\left(e^{\sin(x)} \cdot y\right)' = e^{\sin(x)} \sin(x)\cos(x)$$

Integrate both sides with respect to x to get

$$e^{\sin(x)} \cdot y = \int e^{\sin(x)} \sin(x)\cos(x)\,dx$$

$$\int e^{\sin(x)} \sin(x)\cos(x)\,dx$$

$$= \int e^t \cdot t\,dt = \int t e^t\,dt$$

$$= t e^t - \int e^t\,dt$$

$t = \sin(x)$

$dt = \cos(x)\,dx$

$u = t \qquad du = dt$

$dv = e^t\,dt \qquad v = e^t$

LIATE

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$= te^{t} - e^{t} + C$$

$$= \sin(x)e^{\sin(x)} - e^{\sin(x)} + C$$

Thus,

$$e^{\sin(x)} \cdot y = \sin(x)e^{\sin(x)} - e^{\sin(x)} + C$$

Divide by $e^{\sin(x)}$ or multiply by $e^{-\sin(x)}$ to get:

$$\cancel{e^{-\sin(x)}}\,\cancel{e^{\sin(x)}}\,y = \cancel{e^{-\sin(x)}}\sin(x)\cancel{e^{\sin(x)}} - \cancel{e^{-\sin(x)}}\,\cancel{e^{\sin(x)}} + Ce^{-\sin(x)}$$

We get:

$$\boxed{y = \sin(x) - 1 + Ce^{-\sin(x)}}$$

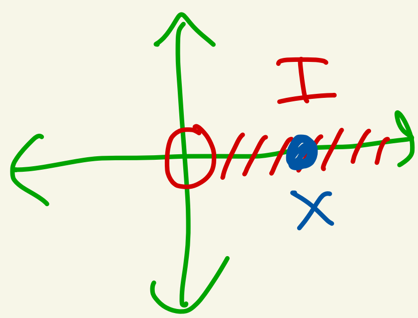
Answer

$I = (-\infty, \infty)$

<u>Ex:</u> Solve

$$xy' + y = 3x^3 + 1$$

on $I = (0, \infty)$



The technique doesn't work with the x in front of y'.

Divide the ODE by x to get

$$y' + \underbrace{\frac{1}{x}}_{\text{integrate this}} y = 3x^2 + \frac{1}{x}$$

Let

$$A(x) = \int \frac{1}{x}\,dx = \ln|x|$$

$$= \ln(x)$$

$I = (0, \infty)$
So, $x > 0$

Multiply the ODE by

$$e^{A(x)} = e^{\ln(x)} = x$$

$e^{\ln(z)} = z$
for any $z > 0$

We get

$$\underbrace{xy' + y}_{\text{always is}} = 3x^3 + 1$$

$$\left(e^{A(x)} \cdot y\right)'$$

This becomes:

$$(xy)' = 3x^3 + 1$$

Integrate with respect to x to get

$$xy = \int (3x^3 + 1)\, dx$$

We get

$$xy = \frac{3}{4}x^4 + x + C$$

Divide by x to get

$$y = \frac{3}{4}x^3 + 1 + \frac{C}{x}$$

So all sols to | A

$$xy' + y = 3x^3 + 1$$

on $I = (0, \infty)$

are of the form

$$y = \frac{3}{4}x^3 + 1 + \frac{c}{x}$$

NSWER

Ex: Solve

$$xy' + y = 3x^3 + 1$$ ODE

$$y(1) = 2$$ Condition on solution

on $I = (0, \infty)$

We already know that

$$y = \frac{3}{4}x^3 + 1 + \frac{c}{x}$$

is the general solution to

$$xy' + y = 3x^3 + 1.$$

Let's make $y(1) = 2$.

We need

$$\underbrace{\frac{3}{4}(1)^3 + 1 + \frac{C}{1}}_{y(1)} = 2$$

We get $\frac{7}{4} + C = 2$

So, $C = 2 - \frac{7}{4} = \frac{1}{4}$

$$\boxed{\text{Answer: } y = \frac{3}{4}x^3 + 1 + \frac{1/4}{x}}$$