2150-01
3/10/25

Practice Test 2

$\boxed{\text{HW 1 - 1(e)}}$

$$y'' + yx^3 y' + x^2 y = 0$$

ODE
not linear
order 2

$\boxed{\text{HW 3 - 2(c)}}$

Solve the linear equation

$$x^2 y' + x(x+2)y = e^x$$

on $I = (0, \infty) \leftarrow \boxed{x > 0}$

Divide by x^2

$$y' + \boxed{\frac{x(x+2)}{x^2}}\, y = \frac{1}{x^2} e^x$$

$$\frac{x+2}{x} = 1 + \frac{2}{x}$$

$$A(x) = \int \left(1 + \frac{2}{x}\right) dx = x + 2\ln|x|$$
$$= x + 2\ln(x)$$

$\boxed{x > 0}$

We have:

$$e^{x+2\ln(x)} = e^x e^{2\ln(x)} = e^x e^{\ln(x^2)}$$

$\boxed{A\ln(B) = \ln(B^A)}$

$$= e^x \cdot x^2 = x^2 e^x$$

Multiply

$$y' + \left(1 + \frac{2}{x}\right) y = \frac{e^x}{x^2}$$

by $x^2 e^x$ to get

$$\underbrace{x^2 e^x y' + x^2 e^x \left(1 + \frac{2}{x}\right) y}_{\left(x^2 e^x \cdot y\right)'} = \underbrace{x^2 e^x \left(\frac{e^x}{x^2}\right)}_{e^{2x}}$$

$$\left(x^2 e^x \cdot y\right)' = e^{2x}$$

So,

$$x^2 e^x y = \underbrace{\int e^{2x}\,dx}$$

$$\int e^{2x}\,dx = \int \frac{1}{2} e^u\,du$$

$$u = 2x$$
$$du = 2\,dx$$
$$\frac{1}{2}\,du = dx$$

$$= \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{2x} + C$$

$$x^2 e^x y = \frac{1}{2} e^{2x} + C$$

$$y = \frac{1}{x^2 e^x} \cdot \frac{1}{2} e^{2x} + \frac{C}{x^2 e^x}$$

$$y = \frac{e^x}{2x^2} + \frac{c}{x^2 e^x}$$

$$\frac{e^{2x}}{e^x} = e^{2x} e^{-x} = e^{2x-x} = e^x$$

HW 4

1(c,d) Solve the separable problem

$$\frac{dy}{dx} = -\frac{x}{y}, \quad \boxed{y(4) = 3}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$y\,dy = -x\,dx$$

$$\int y\,dy = -\int x\,dx$$

$$\boxed{\frac{y^2}{2} = -\frac{x^2}{2} + C}$$

Plug in $\underbrace{y(4)=3}_{x=4,\ y=3}$:

$$\frac{3^2}{2} = \frac{-(4^2)}{2} + C$$

$$\frac{9}{2} = \frac{-16}{2} + C$$

$$25/2 = C$$

Thus, $\frac{y^2}{2} = \frac{-x^2}{2} + \frac{25}{2}$

So, $\boxed{y^2 = -x^2 + 25}$

$\boxed{y(4) = 3}$

Solve for y:

$$y = \pm\sqrt{-x^2 + 25}$$

Need + to make $y(4) = 3$.

$$y(4) = \pm\sqrt{-(4^2) + 25} = \pm\sqrt{9} = \pm 3$$

Need $\boxed{y = \sqrt{-x^2 + 25}}$

$\boxed{\text{HW 5}}$

$\boxed{1(b)}$ Consider

$$\underbrace{(5x+4y)}_{M} + \underbrace{(4x-8y^3)}_{N} y' = 0$$

Check if exact:

$M = 5x + 4y$
$N = 4x - 8y^3$

$\frac{\partial M}{\partial x} = 5$ $\quad$ $\frac{\partial N}{\partial x} = 4$

$\frac{\partial M}{\partial y} = 4$ $\quad$ $\frac{\partial N}{\partial y} = -24y^2$

continuous every-where

Yes it's exact since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Solve it!

Find $f(x,y)$ where

$$\boxed{\frac{\partial f}{\partial x} = M \qquad \frac{\partial f}{\partial y} = N} \longrightarrow \boxed{\frac{\partial f}{\partial x} = 5x + 4y \quad (1) \qquad \frac{\partial f}{\partial y} = 4x - 8y^3 \quad (2)}$$

Integrate (1) with respect to x:

$$f(x,y) = \frac{5x^2}{2} + 4xy + \underbrace{C(y)}_{\text{constant with respect to } x}$$

Integrate (2) with respect to y:

$$f(x,y) = 4xy - 2y^4 + \underbrace{D(x)}_{\text{constant with respect to } y}$$

Set equal:

$$\frac{5}{2}x^2 + \cancel{4xy} + C(y) = \cancel{4xy} - 2y^4 + D(x)$$

$$\underbrace{\frac{5}{2}x^2} + \underbrace{C(y)} = \underbrace{-2y^4} + \underbrace{D(x)}$$

Set $C(y) = -2y^4$, $D(x) = \frac{5}{2}x^2$

So,

$$f(x,y) = \frac{5}{2}x^2 + 4xy + C(y)$$

$$f(x,y) = \frac{5}{2}x^2 + 4xy - 2y^4$$

<u>Answer:</u>

$$\frac{5}{2}x^2 + 4xy - 2y^4 = b$$

where b is any constant

HW 6

2(c)

Given that $y_h = c_1 x^2 + c_2 x^4$ is the general solution to

$$x^2 y'' - 5xy' + 8y = 0$$

and $y_p = 3$ is a particular solution to

$$x^2 y'' - 5xy' + 8y = 24$$

what is the general solution to $x^2 y - 5xy' + 8y = 24$?

<u>Answer:</u>

$$y = y_h + y_p = c_1 x^2 + c_2 x^4 + 3$$

Hw 7

1(c) Solve $y'' + 9y = 0$

$$r^2 + 9 = 0$$

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{\pm\sqrt{-36}}{2} = \frac{\pm\sqrt{-1}\sqrt{36}}{2}$$

$$= \frac{\pm i(6)}{2} = \pm 3i$$

$\alpha = 0$
$\beta = 3$

$$= \boxed{0 \pm 3i} \leftarrow \boxed{\alpha \pm \beta i}$$

$$y_h = \underbrace{c_1 e^{0x}\cos(3x)}_{c_1 e^{\alpha x}\cos(\beta x)} + \underbrace{c_2 e^{0x}\sin(3x)}_{c_2 e^{\alpha x}\sin(3x)}$$

$$y_h = c_1 \cos(3x) + c_2 \sin(3x)$$

$$\boxed{e^{0x} = e^0 = 1}$$

$\boxed{\text{Hw 3}}$

1(d) $\quad y' + \underbrace{2x}y = xe^{-x^2}$

$$A(x) = \int 2x\,dx = x^2$$

multiply by $e^{A(x)} = e^{x^2}$ to get

$$e^{x^2}y' + e^{x^2}(2x)y = e^{x^2}xe^{-x^2}$$

$$\left(e^{x^2}y\right)' = x$$

Integrate

$$e^{x^2} y = \int x \, dx$$

$$e^{x^2} y = \frac{x^2}{2} + C$$

$$\boxed{y = \frac{x^2}{2} \frac{1}{e^{x^2}} + \frac{C}{e^{x^2}}}$$

$$\boxed{y = \frac{1}{2} x^2 e^{-x^2} + C e^{-x^2}}$$

$$\boxed{\text{LHS}: \left(e^{A(x)} \cdot y\right)'}$$