2150-01
3/12/25

Practice test 3

HW 1

1(a)

$$x^2 y''' - 5y'' + \sin(x) y' - 2y = \cos(x) - 2$$

ODE

order 3

linear

HW 3

1(a)

Solve the linear equation

$$y' - 2y = 1$$

$$A(x) = \int -2\,dx = -2x$$

Multiply $y' - 2y = 1$ by e^{-2x} to get:

$$\underbrace{e^{-2x}y' - 2e^{-2x}y}_{(e^{-2x}y)'} = e^{-2x}$$

always: $(e^{A(x)}y)'$

We get

$$(e^{-2x}y)' = e^{-2x}$$

Integrate:

$$e^{-2x}y = \underbrace{\int e^{-2x}\,dx}$$

$$\int e^{-2x}\,dx = \int -\frac{1}{2}e^{u}\,du$$

$u = -2x$
$du = -2\,dx$
$-\frac{1}{2}du = dx$

$$= -\frac{1}{2}e^{u} + C = -\frac{1}{2}e^{-2x} + C$$

$$e^{-2x}y = -\frac{1}{2}e^{-2x} + C$$

$$y = \frac{-\frac{1}{2}\cancel{e^{-2x}}}{\cancel{e^{-2x}}} + \frac{C}{e^{-2x}}$$

$$\boxed{y = -\frac{1}{2} + \frac{C}{e^{-2x}}}$$

$$\boxed{y = -\frac{1}{2} + Ce^{2x}}$$

$\boxed{\text{HW 4}}$

$\boxed{1(e)}$ Solve the separable eqn

$$xe^{-y}\sin(x) - y\frac{dy}{dx} = 0$$

$$-y\frac{dy}{dx} = -xe^{-y}\sin(x)$$

$$y\frac{dy}{dx} = xe^{-y}\sin(x)$$

$$\frac{y\,dy}{e^{-y}} = x\sin(x)\,dx$$

$$ye^{y}\,dy = x\sin(x)\,dx$$

$$\underbrace{\int ye^{y}\,dy} = \underbrace{\int x\sin(x)\,dx}$$

LIATE

$$\int ye^{y}\,dy = ye^{y} - \int e^{y}\,dy = ye^{y} - e^{y}$$

$$\int u\,dv = uv - \int v\,du$$

$u = y$ $\quad du = dy$

$dv = e^{y}\,dy$ $\quad v = e^{y}$

$$\int x\sin(x)\,dx = -x\cos(x) + \int \cos(x)\,dx$$

$u = x$ $\quad du = dx$

$dv = \sin(x)\,dx$ $\quad v = -\cos(x)$

$$= -x\cos(x) + \sin(x)$$

Answer:

$$ye^y - e^y = -x\cos(x) + \sin(x) + C$$

$\boxed{\text{HW 5}}$

$\boxed{1(d)}$

$$\frac{2x}{y} - \frac{x^2}{y^2} \cdot \frac{dy}{dx} = 0$$

$$\underbrace{2xy^{-1}}_{M} - \underbrace{x^2 y^{-2}}_{N} \cdot y' = 0$$

Check if exact

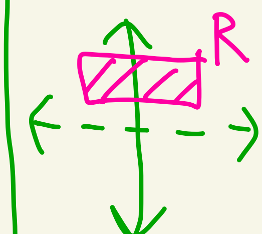
$M = 2xy^{-1}$

$N = -x^2 y^{-2}$

$$\frac{\partial M}{\partial x} = 2y^{-1} \qquad \frac{\partial N}{\partial x} = -2xy^{-2}$$

$$\frac{\partial M}{\partial y} = -2xy^{-2} \qquad \frac{\partial N}{\partial y} = 2x^2 y^{-3}$$

cts when $y \neq 0$



We have

$$\frac{\partial M}{\partial y} = -2xy^{-2} = \frac{\partial N}{\partial x}$$

So its exact.

Let's find the solution.

Need:

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= 2xy^{-1} \quad (1) \\ \frac{\partial f}{\partial y} &= -x^2y^{-2} \quad (2) \end{aligned}} \longleftarrow \boxed{\begin{aligned} \frac{\partial f}{\partial x} &= M \\ \frac{\partial f}{\partial y} &= N \end{aligned}}$$

Integrate (1) with respect to x:

$$f(x,y) = x^2y^{-1} + \underbrace{A(y)}_{\text{constant with respect to } x}$$

Integrate (2) with respect to y:

$$f(x,y) = x^2 y^{-1} + \underbrace{B(x)}_{\text{constant with respect to } y}$$

Set equal:

$$\cancel{x^2 y^{-1}} + A(y) = \cancel{x^2 y^{-1}} + B(x)$$

So,

$$A(y) = B(x)$$

Set $A(y) = 0$ and $B(x) = 0$

Thus,

$$f(x,y) = x^2 y^{-1} + A(y)$$

$$f(x,y) = x^2 y^{-1} + 0$$

$$f(x,y) = x^2 y^{-1}$$

Answer: $f(x,y) = c$

$$x^2 y^{-1} = c$$

or

$$\boxed{y = \frac{1}{c} x^2}$$

where c is any constant

$\boxed{\text{HW 6} \\ 2(c)}$

Suppose you know that

$y_h = c_1 e^{2x} + c_2 x e^{2x}$ is the gen. sol. to

$$y'' - 4y' + 4y = 0$$

and

$y_p = x^2 e^{2x} + x - 2$ solves

$$y'' - 4y' + 4y = 2e^{2x} + 4x - 12$$

whats the gen. sol. to

$$y'' - 4y' + 4y = 2e^{2x} + 4x - 12 \quad ?$$

Answer:

$$y = y_h + y_p = c_1 e^{2x} + c_2 x e^{2x} + x^2 e^{2x} + x - 2$$

HW 7

1(e) Solve

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$$

$$y'' - 10y' + 25y = 0$$

$$r^2 - 10r + 25 = 0$$

$$r = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{0}}{2} = \frac{10}{2} = 5$$

repeated real root

Answer:

$$\boxed{y_h = c_1 e^{5x} + c_2 x e^{5x}}$$