2150-01
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What can go wrong with the guessing method for y_p?

If your y_p guess appears as a term in y_h then you need to multiply your guess by powers of x until your guess doesn't appear as a term in y_h

Ex: Solve

$$y'' - 5y' + 4y = 8e^x$$

Step 1: Solve

$$y'' - 5y' + 4y = 0$$

The characteristic equation is

$$r^2 - 5r + 4 = 0$$

$$(r-4)(r-1) = 0$$
$$r = 4, 1$$

The roots are

$$r = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{9}}{2} = \frac{5 \pm 3}{2}$$

$$= \frac{5+3}{2}, \frac{5-3}{2} = \boxed{4, 1}$$

The solution to $y'' - 5y' + 4y = 0$

is $\boxed{y_h = c_1 e^{4x} + c_2 e^{x}}$

Step 2: Guess a solution y_p to

$$y'' - 5y' + 4y = \underbrace{8e^{x}}_{b(x)}$$

The table says to guess

$$y_p = Ae^{x}$$

This won't work, $y_p = Ae^{x}$ appears as a term in y_h.

Try plugging it into

We have $y_p = Ae^{x}$, $y_p' = Ae^{x}$, $y_p'' = Ae^{x}$

Plugging into the equation gives:

$$\underbrace{(Ae^x) - 5(Ae^x) + 4(Ae^x)}_{y'' - 5y' + 4y} = 8e^x$$

This gives

$$0 = 8e^x$$

This isn't solvable.

Since $y_p = Ae^x$ appears as a term in $y_h = c_1 e^{4x} + c_2 e^x$

we need to multiply our guess by an x.

Instead guess: $\boxed{y_p = Axe^x}$

We have $y_p' = Ae^x + Axe^x$

$$y_p'' = Ae^x + (Ae^x + Axe^x)$$
$$= 2Ae^x + Axe^x$$

Now plug into $y'' - 5y' + 4y = 8e^x$ to get:

$$\underbrace{(2Ae^x + Axe^x)}_{y_p''} - 5\underbrace{(Ae^x + Axe^x)}_{y_p'} + 4\underbrace{(Axe^x)}_{y_p} = 8e^x$$

This gives:

$$2Ae^x + Axe^x - 5Ae^x - 5Axe^x + 4Axe^x = 8e^x$$

$$\underbrace{Axe^x - 5Axe^x + 4Axe^x}_{0}$$

We get:

$$-3Ae^x = 8e^x$$

Need

$$-3A = 8$$

So, $A = -8/3$

$$\boxed{\text{Thus, } y_p = \frac{-8}{3}xe^x \text{ solves } y'' - 5y' + 4y = 8e^x}$$

Step 3: The general solution to

$$y'' - 5y' + 4y = 8e^{x}$$

is

$$\boxed{\begin{aligned} y &= y_h + y_p \\ &= c_1 e^{4x} + c_2 e^{x} - \frac{8}{3} x e^{x} \end{aligned}}$$

Ex: Solve

$$y'' - 2y' + y = e^{x}$$

Step 1: Solve

$$y'' - 2y' + y = 0$$

The characteristic equation is

$$r^2 - 2r + 1 = 0$$

We get

$$(r-1)(r-1) = 0$$

So we get a repeated real root $r = 1$

Then,

$$\boxed{y_h = c_1 e^x + c_2 x e^x}$$

is the general solution to

$$y'' - 2y' + y = 0$$

Step 2: Now we guess y_p for

$$y'' - 2y' + y = \underbrace{e^x}_{b(x)}$$

The table says to guess $y_p = Ae^x$

But this appears in $y_h = c_1 e^x + c_2 x e^x$

So multiply by x and guess $y_p = Axe^x$

our guess

But this appears in $y_h = c_1 e^x + c_2 x e^x$

So multiply our guess by x again

to get $y_p = Ax^2 e^x$ ← doesn't appear in y_h

Now we plug it in.

$$y_p = Ax^2 e^x$$

$$y_p' = 2Axe^x + Ax^2 e^x$$

$$y_p'' = (2Ae^x + 2Axe^x) + (2Axe^x + Ax^2 e^x)$$

$$= 2Ae^x + 4Axe^x + Ax^2 e^x$$

Plug these into

$$y'' - 2y' + y = e^x$$

to get:

$$(2Ae^x + 4Axe^x + Ax^2e^x) - 2(2Axe^x + Ax^2e^x) + Ax^2e^x = e^x$$

This gives:

$$2Ae^x + 4Axe^x + Ax^2e^x - 4Axe^x - 2Ax^2e^x + Ax^2e^x = e^x$$

$$0 \qquad 0$$

We get:

$$2Ae^x = e^x$$

Need

$$2A = 1$$

So,

$$A = 1/2$$

Thus, $y_p = \frac{1}{2}x^2e^x$ solves

$$y'' - 2y' + y = e^x$$

Step 3: The general solution to

$$y'' - 2y' + y = e^x$$

is

$$y = y_h + y_p = \boxed{c_1 e^x + c_2 x e^x + \frac{1}{2} x^2 e^x}$$

Ex: What would you guess for y_p for

$$y'' + 2y' = \underbrace{2x + 5}_{} - e^x \quad ?$$

table says guess $Ax + B$

table says guess Ce^x

You'd guess: $y_p = Ax + B + Ce^x$

Ex: What would you guess for

$$y'' + y = \underbrace{2\sin(x)}_{\text{table says}} + \underbrace{x^2}_{\text{table says}}$$

$A\cos(x) + B\sin(x)$ $\quad$ $Cx^2 + Dx + E$

So guess

$$y_p = A\cos(x) + B\sin(x) + Cx^2 + Dx + E$$