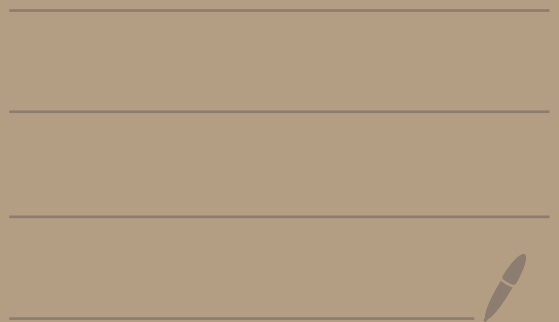


Math 2150-01

4/21/25



HW 8-1(c)

Solve

$$\frac{1}{4}y'' + y' + y = x^2 - 2x$$

Step 1: Solve

$$\frac{1}{4}y'' + y' + y = 0$$

$$\frac{1}{4}r^2 + r + 1 = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \times 4$$

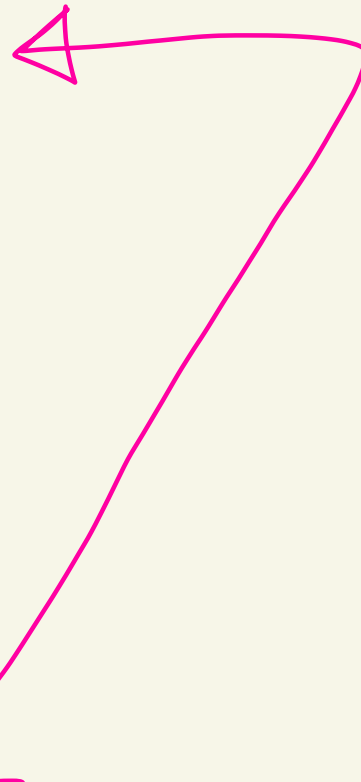
$$r^2 + 4r + 4 = 0 \quad \leftarrow$$

$$(r+2)(r+2) = 0$$

$$r = -2$$

$$y_h = c_1 e^{-2x} + c_2 x e^{-2x}$$

Step 2: Now find y_p for

$$\frac{1}{4}y'' + y' + y = x^2 - 2x$$


Guess:

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

Plug these into the ODE to get

$$\underbrace{\frac{1}{4}(2A) + (2Ax + B) + (Ax^2 + Bx + C)}_{\frac{1}{4}y_p'' + y_p' + y_p} = x^2 - 2x$$

We get

$$\underbrace{Ax^2}_{A=1} + \underbrace{(2A+B)x}_{2A+B=-2} + \underbrace{\left(\frac{1}{2}A+B+C\right)}_{\frac{1}{2}A+B+C=0} = x^2 - 2x$$

So, $A=1$

So, $2(1)+B=-2$, Then $B=-4$

So, $\frac{1}{2}(1)+(-4)+C=0$, Then $C=3.5$

Then,

$$y_p = Ax^2 + Bx + C = x^2 - 4x + 3.5$$

Step 3: The general solution to

$$\frac{1}{4}y'' + y' + y = x^2 - 2x$$

is

$$y = y_h + y_p$$

$$y = c_1 e^{-2x} + c_2 x e^{-2x} + x^2 - 4x + 3.5$$

HW 9 - 1(b)

Suppose you are given that the solution to $y'' + y = 0$ is $y_h = c_1 \cos(x) + c_2 \sin(x)$.

Use variation of parameters to find y_p for

$$y'' + y = \boxed{\sin(x)}$$

$b(x)$

Then state the general solution

Know: $y_1 = \cos(x)$
 $y_2 = \sin(x)$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix}$$

$$\begin{aligned} &= (\cos(x))(\cos(x)) - (\sin(x))(-\sin(x)) \\ &= \cos^2(x) + \sin^2(x) \\ &= 1 \end{aligned}$$

Then,

$$v_1 = \int \frac{-y_2 b(x)}{w(y_1, y_2)} dx$$

$$= \int \frac{-\sin(x) \cdot \sin(x)}{1} dx$$

$$= - \int \sin^2(x) dx$$

$$= - \int \left(\frac{1}{2} - \frac{1}{2} \cos(2x) \right) dx$$

$$= - \left[\frac{1}{2} x - \frac{1}{2} \left(\frac{1}{2} \sin(2x) \right) \right]$$

$$= -\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

And,

$$V_2 = \int \frac{y_1 b(x)}{W(y_1, y_2)} dx = \int \frac{\cos(x) \sin(x)}{1} dx$$

$$= \int \underbrace{\cos(x)}_{u=\sin(x)} \underbrace{\sin(x) dx}_{du=\cos(x) dx}$$

$$= \int u du = \frac{u^2}{2} = \frac{\sin^2(x)}{2}$$

Thus,

$$y_p = V_1 y_1 + V_2 y_2$$

$$= \left(-\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) \cos(x) + \left(\frac{1}{2}\sin^2(x)\right) \sin(x)$$

General solution to $y'' + y = \sin(x)$ is

$$y = y_h + y_p$$

$$= c_1 \cos(x) + c_2 \sin(x)$$

$$+ \left(-\frac{1}{2}x + \frac{1}{4} \sin(2x)\right) \cos(x)$$

$$+ \frac{1}{2} \sin^2(x) \sin(x)$$

HW 10

1(a)

Given $y_1 = x^4$ solves

$$x^2 y'' - 7xy' + 16y = 0$$

on $I = (0, \infty)$.

(a) Find another linearly independent solution y_2 .

(b) State the general solution.

Divide by x^2 to get

$$y'' - \frac{7}{x} y' + \frac{16}{x^2} y = 0$$

$$\boxed{a_1(x) = -\frac{7}{x}}$$

Then,

$$y_2 = y_1 \int \frac{e^{-\int a_1(x) dx}}{y_1^2} dx$$

$$= x^4 \int \frac{e^{-\int -\frac{7}{x} dx}}{(x^4)^2} dx$$

$$= x^4 \int \frac{e^{7 \int \frac{1}{x} dx}}{x^8} dx$$

$$= x^4 \int \frac{e^{7 \ln(x)}}{x^8} dx$$

$$\int \frac{1}{x} dx = \ln|x|$$
$$= \ln(x)$$

$$I = (0, \infty), x > 0$$

$$= x^4 \int \frac{e^{\ln(x^7)}}{x^8} dx$$

$$A \ln(B) = \ln(B^A)$$

$$= x^4 \int \frac{x^7}{x^8} dx$$

$$e^{\ln(A)} = A$$

$$= x^4 \int \frac{1}{x} dx = x^4 \ln(x)$$

So, $y_2 = x^4 \ln(x)$

The general solution is

$$\begin{aligned} y_h &= c_1 y_1 + c_2 y_2 \\ &= c_1 x^4 + c_2 x^4 \ln(x) \end{aligned}$$

HW 11
1(a)

Find a power series
for $f(x) = x^3 + x$
centered at $x_0 = 1$.

$f(x) = x^3 + x$	←	$f(1) = 1^3 + 1 = 2$
$f'(x) = 3x^2 + 1$	←	$f'(1) = 3(1)^2 + 1 = 4$
$f''(x) = 6x$	←	$f''(1) = 6(1) = 6$
$f'''(x) = 6$	←	$f'''(1) = 6$
$f^{(4)}(x) = 0$		$f^{(4)}(1) = 0$
$\vdots$		$\vdots$

0 from this point on

all 0

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4 + \dots$$

all 0

$$\underbrace{x^3 + x}_{f(x)} = 2 + 4(x-1) + \frac{6}{2!}(x-1)^2 + \frac{6}{3!}(x-1)^3$$

$$x^3 + x = 2 + 4(x-1) + 3(x-1)^2 + (x-1)^3$$

HW 12 #1

Find the first 4 non-zero terms of a power series solution to

$$y'' - (x+1)y' + x^2y = 0$$

$$y'(0) = 1, \quad y(0) = 1$$

What's the radius of convergence?

$$x_0 = 0$$

coefficients

$$\begin{aligned} -(x+1) &= -1 - x + 0x^2 + 0x^3 + \dots \\ x^2 &= 0 + 0x + 1 \cdot x^2 + 0x^3 + \dots \\ 0 &= 0 + 0x + 0x^2 + 0x^3 + \dots \end{aligned}$$

these are
all
polynomials
so they
all have
 $r = \infty$

Our solution will have
radius of convergence $r = \infty$

Let's find the solution.

$$\begin{aligned} y(x) &= y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 \\ &\quad + \frac{y'''(0)}{3!}x^3 + \dots \end{aligned}$$

Given: $y(0)=1$ $y'(0)=1$

Know $\rightarrow y'' - (x+1)y' + x^2y = 0$

$$y'' = (x+1)y' - x^2y$$

$$y''(0) = (0+1)[y'(0)] - 0^2[y(0)]$$

$$= 1 \cdot 1 - 0 \cdot 1 = 1$$

$$y''(0) = 1$$

Differentiate to get

$$y''' = (1)y' + (x+1)y'' - 2xy - x^2y'$$

$$y'''(0) = [y'(0)] + (0+1)[y''(0)]$$

$$- 2(0)[y(0)] - 0^2[y'(0)]$$

$$= 1 + 1 \cdot 1 - 0 - 0 = 2$$

$$y'''(0) = 2$$

So,

$$y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2$$

$$+ \frac{y'''(0)}{3!}x^3 + \dots$$

$$y(x) = 1 + 1 \cdot x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \dots$$

$$y(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$$