Math 2150-02
1/27/25

Topic 1 - What is a differential equation?

<u>Ex:</u> $y' = 2y$

is an ordinary differential equation (ODE).

To "solve" this equation we want a function y that satisfies the equation.

Consider $y = e^{2x}$.

Then, $y' = 2e^{2x}$.

So, $y' = 2y$ if $y = e^{2x}$.

Def:

- An equation relating an unknown function and one or more of its derivaties is called a differential equation.

- If a differential equation only has regular derivatives of a single function then its called an ordinary differential equation (ODE)

  If it has partial derivatives its called a partial differential equation (PDE)

- The order of a differential equation is the order of the highest derivative that occurs in the equation.

<u>Ex:</u> $y' = 2y$

ODE of order 1

<u>Ex:</u>

$$\underbrace{\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = 0}_{y'' + y' + 1 = 0}$$

ODE of order 2

y is a function of x

<u>Ex:</u> $4x^2 y'' + y = \cos(x)$

Here y is a function of x where x is a number.

This is an ODE of order 2

<u>Ex:</u> $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is called Laplace's equation

Here u is a function of x and y. $[u = u(x,y)]$

This is a PDE of order 2

Def: An ODE is called linear if it is of the form

$$\underbrace{a_n(x)}\, y^{(n)} + \underbrace{a_{n-1}(x)}\, y^{(n-1)} + \dots + \underbrace{a_1(x)}\, y' + \underbrace{a_0(x)}\, y = \underbrace{b(x)}$$

coefficients only have #'s & x's

Ex:

$$\underbrace{2x}\, y''' - \underbrace{\cos(x^2)}\, y' + \underbrace{2}\, y = \underbrace{e^x}$$

only x's & #'s

is a linear ODE of order 3.

Ex: $2y'' + y' + y = 10x$

is a linear ODE of order 2.

Ex:

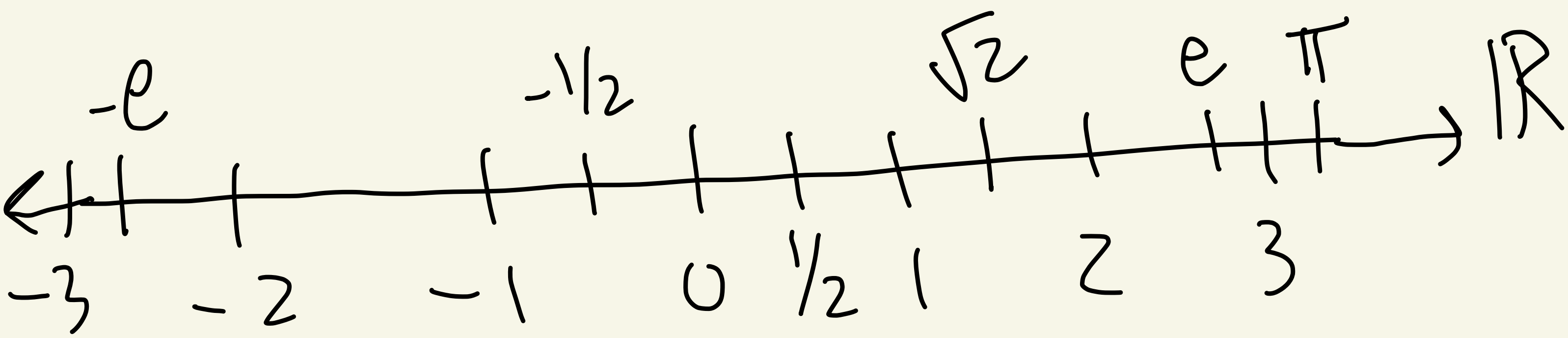
$$y^2 y''' - 2xy + 5x = 0$$

x's & #'s

this term makes the equation non-linear

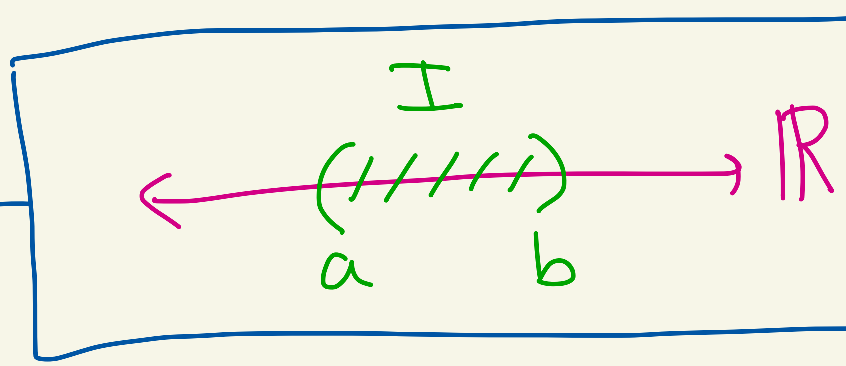
This a non-linear ODE of order 3.

Def: The real number line is denoted by $\mathbb{R}$



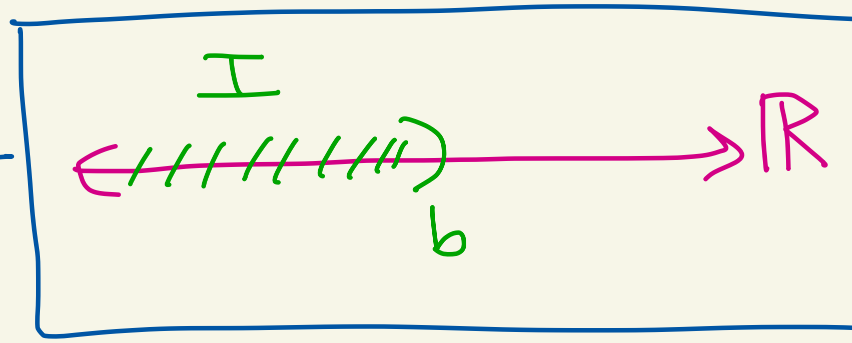
Def: An open interval I is an interval of the form

$I = (a, b)$



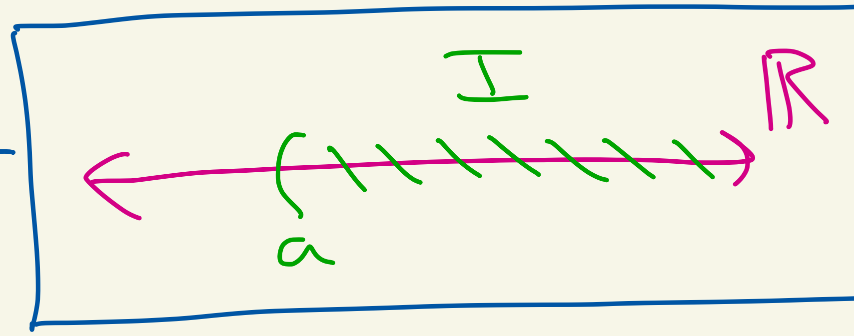
or

$I=(-\infty,b)$



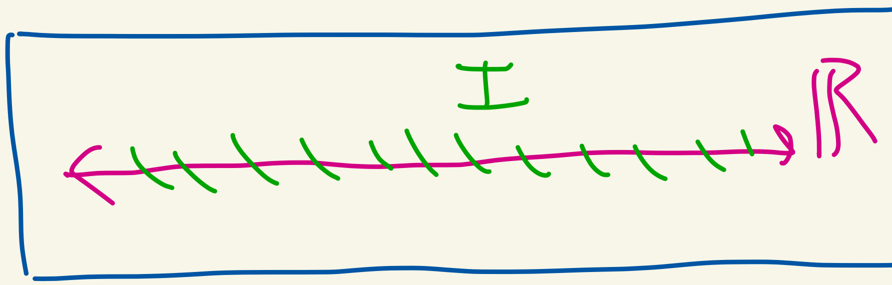
or

$I=(a,\infty)$



or

$I=(-\infty,\infty)$



<u>Def:</u> A function f is a <u>solution</u> to an n-th order ODE on an open interval I if the following is true:

① $f, f', f'', \ldots, f^{(n)}$ exist on I

and

② when you plug f and its derivatives into the ODE it solves the equation for every x in I.

In addition, sometimes one is given what $f(x_0), f'(x_0), \ldots, f^{(n-1)}(x_0)$ must equal for some x_0 in I. This extra condition turns the ODE into an <u>initial-value problem</u>.

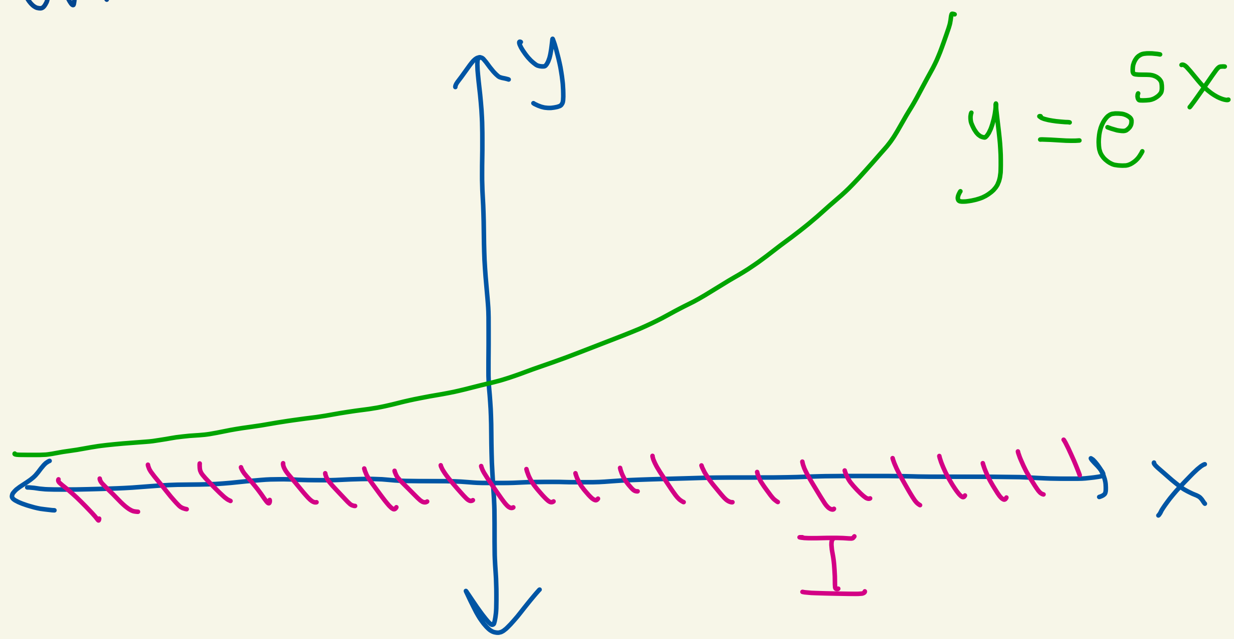
<u>Ex:</u> Let's find a solution to the ODE

$$y'' - 25y = 0$$

on $I = (-\infty, \infty)$.

Consider $y = e^{5x}$.

This function is defined on all of $I = (-\infty, \infty)$.



We have

$$\left.\begin{aligned} y &= e^{5x} \\ y' &= 5e^{5x} \\ y'' &= 25e^{5x} \end{aligned}\right\} \text{these are all defined on } I$$

We get

$$y'' - 25y = 25e^{5x} - 25(e^{5x}) = 0$$

plug y in

So, $y = e^{5x}$ solves

$$y'' - 25y = 0 \quad \text{on} \quad I = (-\infty, \infty)$$