2150-02
3/10/25

Practice Test 3

$\boxed{\text{HW 1-1(d)}}$

$$x^2 y''' - 5y'' + \sin(x) y' - 2y = \cos(x) - 2$$

ODE
LINEAR
ORDER 3

$\boxed{\text{HW 3-1(a)}}$

Solve linear equation

$$y' - 2y = 1$$

$I = (-\infty, \infty)$

$$A(x) = \int -2\,dx = -2x$$

Multiply by e^{-2x} to get:

$$\underbrace{e^{-2x}y' - 2e^{-2x}y}_{\left(e^{-2x}y\right)'} = e^{-2x}$$

We get

$$\left(e^{-2x}y\right)' = e^{-2x}$$

Integrate with respect to x:

$$e^{-2x}y = \int e^{-2x}\,dx$$

$$\int e^{-2x}\,dx = \int e^{u}\left(-\tfrac{1}{2}\right)du$$

$$u = -2x$$
$$du = -2\,dx$$
$$-\tfrac{1}{2}\,du = dx$$

$$= -\frac{1}{2}\int e^{u}\,du$$
$$= -\frac{1}{2}e^{u} + C$$
$$= -\frac{1}{2}e^{-2x} + C$$

Thus,

$$e^{-2x}y = -\frac{1}{2}e^{-2x} + C$$

$$y = \frac{1}{e^{-2x}}\left(-\frac{1}{2}e^{-2x} + C\right)$$

$$y = -\frac{1}{2} + \frac{C}{e^{-2x}}$$

$$\boxed{y = -\frac{1}{2} + Ce^{2x}}$$

Practice Test 2

$\boxed{\text{HW 3}}$ 2(b) Solve linear equation:

$$xy' + y = 3x^3 - 1$$

$\boxed{I = (0, \infty)} \leftarrow \boxed{x > 0}$

We have

$$xy' + y = 3x^3 - 1$$

not 1. Divide by x

Divide by x:

$$y' + \frac{1}{x} y = 3x^2 - \frac{1}{x}$$

$$A(x) = \int \frac{1}{x}\, dx = \ln|x| = \ln(x)$$

$\boxed{x > 0}$

Multiply by $e^{A(x)} = e^{\ln(x)} = x$ to get:

$$xy' + y = 3x^3 - 1$$

$$(xy)' = 3x^3 - 1$$

Integrate!

$$xy = 3\frac{x^4}{4} - x + C$$

divide by x

$$\boxed{y = \frac{3}{4}x^3 - 1 + \frac{C}{x}}$$

$\boxed{\text{Practice Test 3}}$

$\boxed{\text{HW 4}}$ 1(e) Solve the separable equation

$$x e^{-y} \sin(x) - y \frac{dy}{dx} = 0$$

We get

$$-y \frac{dy}{dx} = -x e^{-y} \sin(x)$$

We get

$$\frac{y}{e^{-y}} dy = x \sin(x) dx$$

So,

$$ye^y\,dy = x\sin(x)\,dx$$

Thus,

$$\int ye^y\,dy = \int x\sin(x)\,dx$$

LIATE

$$\int ye^y\,dy = ye^y - \int e^y\,dy = ye^y - e^y + C$$

$$\int u\,dv = uv - \int v\,du$$

$u = y$ $\quad du = dy$

$dv = e^y\,dy$ $\quad v = e^y$

$$\int x\sin(x)\,dx = -x\cos(x) + \int \cos(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = x$ $\quad du = dx$

$dv = \sin(x)\,dx$ $\quad v = -\cos(x)$

$$= -x\cos(x) + \sin(x) + D$$

We get

$$ye^y - e^y + C = -x\cos(x) + \sin(x) + D$$

or

$$\boxed{ye^y - e^y = -x\cos(x) + \sin(x) + E}$$

$\boxed{\text{HW 5} \\ \text{1(d)}}$

Consider the equation:

$$\underbrace{\frac{2x}{y}}_{M} - \underbrace{\frac{x^2}{y^2}}_{N} \cdot \frac{dy}{dx} = 0$$

<u>Check if exact:</u>

$$M(x,y) = 2xy^{-1}$$

$$N(x,y) = -x^2y^{-2}$$

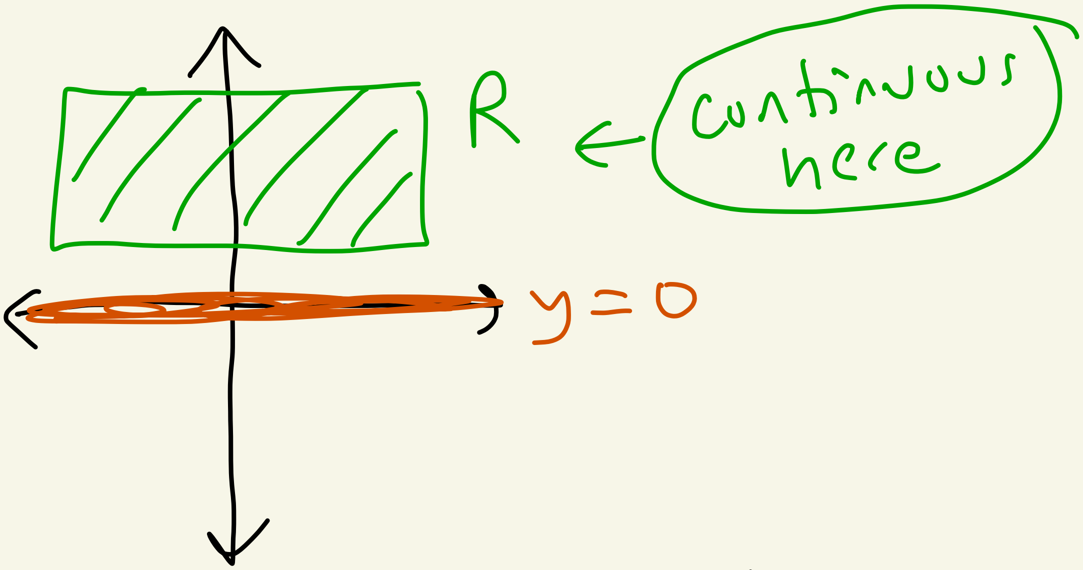
$$\frac{\partial M}{\partial x} = 2y^{-1}$$

$$\frac{\partial M}{\partial y} = -2xy^{-2}$$

$$\frac{\partial N}{\partial x} = -2xy^{-2}$$

$$\frac{\partial N}{\partial y} = 2x^2y^{-3}$$

These are continuous when $y \neq 0$



Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ it is exact.

Let's find a solution.

Need $f(x,y)$ where

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= M \\ \frac{\partial f}{\partial y} &= N \end{aligned}} \longrightarrow \boxed{\begin{aligned} \frac{\partial f}{\partial x} &= 2xy^{-1} \quad (1) \\ \frac{\partial f}{\partial y} &= -x^2y^{-2} \quad (2) \end{aligned}}$$

Integrate (1) with respect to x:

$$f(x,y) = 2\left(\frac{x^2}{2}\right)y^{-1} + \underbrace{C(y)}_{\text{constant w/ respect to } x}$$

$$\boxed{f(x,y) = x^2y^{-1} + c(y)}$$

Integrate ② with respect to y:

$$f(x,y) = -x^2\left(\frac{y^{-1}}{-1}\right) + \underbrace{D(x)}_{\text{constant w/ respect to } y}$$

$$\boxed{f(x,y) = x^2y^{-1} + D(x)}$$

So,

$$\cancel{x^2y^{-1}} + \underbrace{c(y)}_{0} = \cancel{x^2y^{-1}} + \underbrace{D(x)}_{0}$$

So,

$$f(x,y) = x^2y^{-1}$$

Answer:

$$\boxed{x^2y^{-1} = c}$$

$$y = \frac{1}{c}x^2$$

$$y = Ex^2$$

Hw 7
1(c)

Solve $y'' + 8y' + 16y = 0$

$$r^2 + 8r + 16 = 0$$

$$r = \frac{-8 \pm \sqrt{8^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{0}}{2} = -4$$

$$\boxed{y_h = c_1 e^{-4x} + c_2 x e^{-4x}}$$