2150-02
3/12/25

Practice Test 2

HW 1 1(e)

$$\frac{d^2y}{dx^2} + \boxed{y}x^3\boxed{\frac{dy}{dx}} + x^2y = 0$$

ODE
order 2
not linear

HW 3 (practice Test 1)

1(b) Solve linear equation

$$y' + 2xy = x$$

on $I=(-\infty,\infty)$

$$A(x)=\int 2x\,dx = 2\frac{x^2}{2} = x^2$$

multiply $y'+2xy=x$

by $e^{A(x)} = e^{x^2}$ to get

$$\underbrace{e^{x^2}y' + e^{x^2}2xy}_{(e^{x^2}y)'} = xe^{x^2}$$

$$\boxed{\text{always } (e^{A(x)}y)'}$$

$$(e^{x^2}y)' = xe^{x^2}$$

Integrate:

$$e^{x^2} y = \underbrace{\int x e^{x^2} dx}$$

$$\int x e^{x^2} dx = \frac{1}{2}\int e^u du$$

$$\boxed{\begin{aligned} u &= x^2 \\ du &= 2x\,dx \\ \tfrac{1}{2} du &= x\,dx \end{aligned}}$$

$$= \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{x^2} + C$$

We get

$$e^{x^2} y = \frac{1}{2} e^{x^2} + C$$

So,

$$y = \frac{\frac{1}{2}\cancel{e^{x^2}}}{\cancel{e^{x^2}}} + \frac{c}{e^{x^2}}$$

Thus,

$$\boxed{y = \frac{1}{2} + Ce^{-x^2}}$$

HW 4

1(h) Solve the separable problem:

$$xy' = 4y, \quad y(1) = 5$$

$$xy' = 4y$$

$$x\frac{dy}{dx} = 4y$$

$$\frac{1}{y}dy = \frac{4}{x}dx$$

$$\int \frac{1}{y}dy = \int \frac{4}{x}dx$$

$$\ln|y| = 4\ln|x| + C$$

$$e^{\ln|y|} = e^{4\ln|x| + C}$$

$$|y| = e^{4\ln|x|} \cdot e^{C}$$

$$|y| = \left(e^{\ln|x|}\right)^4 \cdot e^{C}$$

$$|y| = |x|^4 \cdot e^{C}$$

$|x|^4 =$

$$|y| = e^c x^4$$

$= |x^4|$
$= x^4$

$$y = \pm e^c x^4$$

$$\boxed{y = Ax^4 \text{ where } A \text{ is a constant}}$$

Let's make our solution satisfy $\underbrace{y(1) = 5}_{\substack{x=1 \\ y=5}}$. We need:

$$5 = A(1)^4$$

$y = Ax^4$
$x = 1$
$y = 5$

So, $A = 5$

$$\boxed{\text{Thus, } y = 5x^4}$$

$\boxed{\text{HW 5}}$

$\boxed{1(b)}$ Consider

$$\underbrace{(5x+4y)}_{M}+\underbrace{(4x-8y^3)}_{N}y'=0$$

Check that it's exact

$M = 5x+4y$

$N = 4x-8y^3$

$\frac{\partial M}{\partial x} = 5$ $\qquad$ $\frac{\partial N}{\partial x} = 4$

$\frac{\partial M}{\partial y} = 4$ $\qquad$ $\frac{\partial N}{\partial y} = -24y^2$

continuous everywhere

$$\boxed{\text{Check: } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}\ \checkmark$$

So it's exact.

Need $f(x,y)$ where

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= 5x+4y \quad (1) \\ \frac{\partial f}{\partial y} &= 4x-8y^3 \quad (2) \end{aligned}}$$

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= M \\ \frac{\partial f}{\partial y} &= N \end{aligned}}$$

Integrate (1) with respect to x:

$$f(x,y) = \frac{5}{2}x^2 + 4xy + \underbrace{C(y)}_{\text{constant w/ respect to } x}$$

Integrate (2) with respect to y:

$$f(x,y) = 4xy - 2y^4 + \underbrace{D(x)}_{\text{constant with respect to } y}$$

Set equal:

$$\frac{5}{2}x^2 + \cancel{4xy} + C(y) = \cancel{4xy} - 2y^4 + D(x)$$

$$\frac{5}{2}x^2 + C(y) = -2y^4 + D(x)$$

Set $C(y) = -2y^4$, $D(x) = \frac{5}{2}x^2$

Plug $C(y)$ into above:

$$f(x,y) = \frac{5}{2}x^2 + 4xy + C(y)$$

$$f(x,y) = \frac{5}{2}x^2 + 4xy - 2y^4$$

Answer

$$\boxed{\frac{5}{2}x^2 + 4xy - 2y^4 = c}$$

where c is any constant

$f(x,y) = c$

$\boxed{\text{HW 7} \\ 1(c)}$ Solve

$$y'' + 9y = 0$$

$$r^2 + 9 = 0$$

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{\pm\sqrt{-36}}{2} = \frac{\pm\sqrt{36}\sqrt{-1}}{2}$$

$$= \pm \frac{6i}{2} = \pm 3i$$

$$= \underbrace{0 \pm 3i}_{\alpha \pm \beta i}$$

$$\alpha = 0$$

$$\beta = 3$$

Answer:

$$y_h = c_1 \underbrace{e^{0x}\cos(3x)}_{e^{\alpha x}\cos(\beta x)} + c_2 \underbrace{e^{0x}\sin(3x)}_{e^{\alpha x}\sin(\beta x)}$$

$\boxed{\text{Use } e^{0x} = e^0 = 1}$

$$\boxed{y_h = c_1\cos(3x) + c_2\sin(3x)}$$

HW 6

2(c) Suppose you know that $y_h = c_1 x^2 + c_2 x^4$ for $x^2y'' - 5xy' + 8y = 0$ and $y_p = 3$ for

$$x^2y'' - 5xy' + 8y = 24.$$

What's the general solution to

$$x^2y'' - 5xy' + 8y = 24\ ?$$

Answer:

$$y = y_h + y_p = c_1x^2 + c_2x^4 + 3$$