Math 2550-04
10/7/24

HW 1 - Part 1

$\vec{v} = \langle 1, -1, 3, 0 \rangle$

$\vec{w} = \langle 2, 0, -2, 4 \rangle$

$$-\vec{v} + 3\vec{w} = -\langle 1, -1, 3, 0 \rangle + 3\langle 2, 0, -2, 4 \rangle$$
$$= \langle -1, 1, -3, 0 \rangle + \langle 6, 0, -6, 12 \rangle$$
$$= \langle 5, 1, -9, 12 \rangle$$

$$\vec{v} \cdot \vec{w} = \langle 1, -1, 3, 0 \rangle \cdot \langle 2, 0, -2, 4 \rangle$$
$$= (1)(2) + (-1)(0) + (3)(-2) + (0)(4)$$
$$= 2 + 0 - 6 + 0 = -4$$

$$\|\vec{v}\| = \sqrt{(1)^2 + (-1)^2 + (3)^2 + (0)^2} = \sqrt{11}$$

Q: List three vectors from S

$$S = \{ c_1 \langle 1, 0, -1 \rangle + c_2 \langle 2, 1, 1 \rangle \mid c_1, c_2 \in \mathbb{R} \}$$

$c_1 = 0, c_2 = 0$

$$0 \cdot \langle 1, 0, -1 \rangle + 0 \cdot \langle 2, 1, 1 \rangle = \boxed{\langle 0, 0, 0 \rangle}$$

$c_1 = 1, c_2 = 0$

$$1 \cdot \langle 1, 0, -1 \rangle + 0 \cdot \langle 2, 1, 1 \rangle = \boxed{\langle 1, 0, -1 \rangle}$$

$c_1 = \pi, c_2 = 1$:

$$\pi \cdot \langle 1, 0, -1 \rangle + 1 \cdot \langle 2, 1, 1 \rangle = \boxed{\langle \pi + 2, 1, -\pi + 1 \rangle}$$

Matrix proof question

Let A, B, C be 2×2 matrices

Prove that $(B+C)A = BA + CA$

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad B = \begin{pmatrix} e & f \\ g & i \end{pmatrix}, \quad C = \begin{pmatrix} j & k \\ l & m \end{pmatrix}$$

Then

$$(B+C)A = \underbrace{\begin{pmatrix} e+j & f+k \\ g+l & i+m \end{pmatrix}}_{B+C} \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{A}$$

$$= \begin{pmatrix} (e+j)a + (f+k)c & (e+j)b + (f+k)d \\ (g+l)a + (i+m)c & (g+l)b + (i+m)d \end{pmatrix}$$

$$= \begin{pmatrix} ea+ja+fc+kc & eb+jb+fd+kd \\ ga+la+ic+mc & gb+lb+id+md \end{pmatrix}$$

And,

$$BA+CA = \underbrace{\begin{pmatrix} e & f \\ g & i \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}} + \underbrace{\begin{pmatrix} j & k \\ l & m \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}}$$

$$= \begin{pmatrix} ea+fc & eb+fd \\ ga+ic & gb+id \end{pmatrix} + \begin{pmatrix} ja+kc & jb+kd \\ la+mc & lb+md \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc+ja+kc & eb+fd+jb+kd \\ ga+ic+la+mc & gb+id+lb+md \end{pmatrix}$$

SAME

So, $(B+C)A = BA+CA$. $\blacksquare$

Hw 2 - Part 1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 & 1 \\ 2 & 1 & 2 \\ 3 & -3 & 5 \end{pmatrix}$$

$$2A - 3B = \underbrace{\begin{pmatrix} 2 & 4 & 6 \\ 0 & 2 & 2 \\ 2 & 4 & 2 \end{pmatrix}}_{2A} + \underbrace{\begin{pmatrix} 3 & 0 & -3 \\ -6 & -3 & -6 \\ -9 & 9 & -15 \end{pmatrix}}_{-3B}$$

$$= \begin{pmatrix} 5 & 4 & 3 \\ -6 & -1 & -4 \\ -7 & 13 & -13 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix} \leftarrow \text{"A transpose"}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 3 \\ 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$\underbrace{A}_{3\times 3}\ \underbrace{C}_{3\times 2} = \begin{pmatrix} (1)(0)+2(1)+3(-1) & (1)(3)+(2)(1)+(3)(2) \\ (0)(0)+(1)(1)+(1)(-1) & (0)(3)+(1)(1)+(1)(2) \\ (1)(0)+(2)(1)+(1)(-1) & (1)(3)+(2)(1)+(1)(2) \end{pmatrix}$$

answer is 3×2

$$= \begin{pmatrix} -1 & 11 \\ 0 & 3 \\ 1 & 7 \end{pmatrix}$$

$\underbrace{C}_{3\times 2}\ \underbrace{A}_{3\times 3}$ is undefined

$2 \neq 3$

$\boxed{\text{Hw 3}}$

① (c)

$$\boxed{\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}}$$

1 ✓

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right)$$

make 0

$$\xrightarrow[\substack{R_1 + R_3 \to R_3 \\ -3R_1 + R_4 \to R_4}]{-2R_1 + R_2 \to R_2} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

make 1

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & \boxed{3} & -6 & 0 & 0 \\ 0 & \boxed{3} & -6 & 0 & 0 \end{array}\right)$$

make 0

$$\xrightarrow[-3R_2+R_4 \to R_4]{-3R_2+R_3 \to R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

in row echelon form

Convert back:

$$\begin{aligned} x - y + 2z - w &= -1 \quad (1) \\ y - 2z &= 0 \quad (2) \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

leading variables: x, y

free variables: z, w

Solve for leading and give free variables new name:

① $x = -1 + y - 2z + w$

② $y = 2z$

③ $z = t$

④ $w = u$

Back-sub:

④ $w = u$

③ $z = t$

② $y = 2z = 2t$

① $x = -1 + y - 2z + w = -1 + 2t - 2t + u$
$= -1 + u$

Answer:

$$\boxed{\begin{aligned} x &= -1+u \\ y &= 2t \\ z &= t \\ w &= u \end{aligned} \quad t, u \text{ can be any numbers}}$$