Math 2550-04

11/4/24

Today:

- Finish Topic 7
- Review for test

(Topic 7 continued...)

Ex: In $\mathbb{R}^3$, let

$$W = \left\{ \langle x, y, z \rangle \;\middle|\; \begin{array}{l} x + y \qquad = 0 \\ \quad\;\; y - 5z = 0 \end{array} \right\}$$

Last time we had a theorem called "homogeneous subspace theorem" that tells us that W is a subspace of $\mathbb{R}^3$.

Let's find a basis for W.

Suppose $\vec{v} = \langle x, y, z \rangle$ satisfies

$$\boxed{\begin{aligned} x + y \quad\quad &= 0 \quad (1) \\ y - 5z &= 0 \quad (2) \end{aligned}}$$

already reduced

leading: x, y

free: z

Solving:

$z = t$

(2) $y = 5z = 5t$

(1) $x = -y = -5t$

Thus,

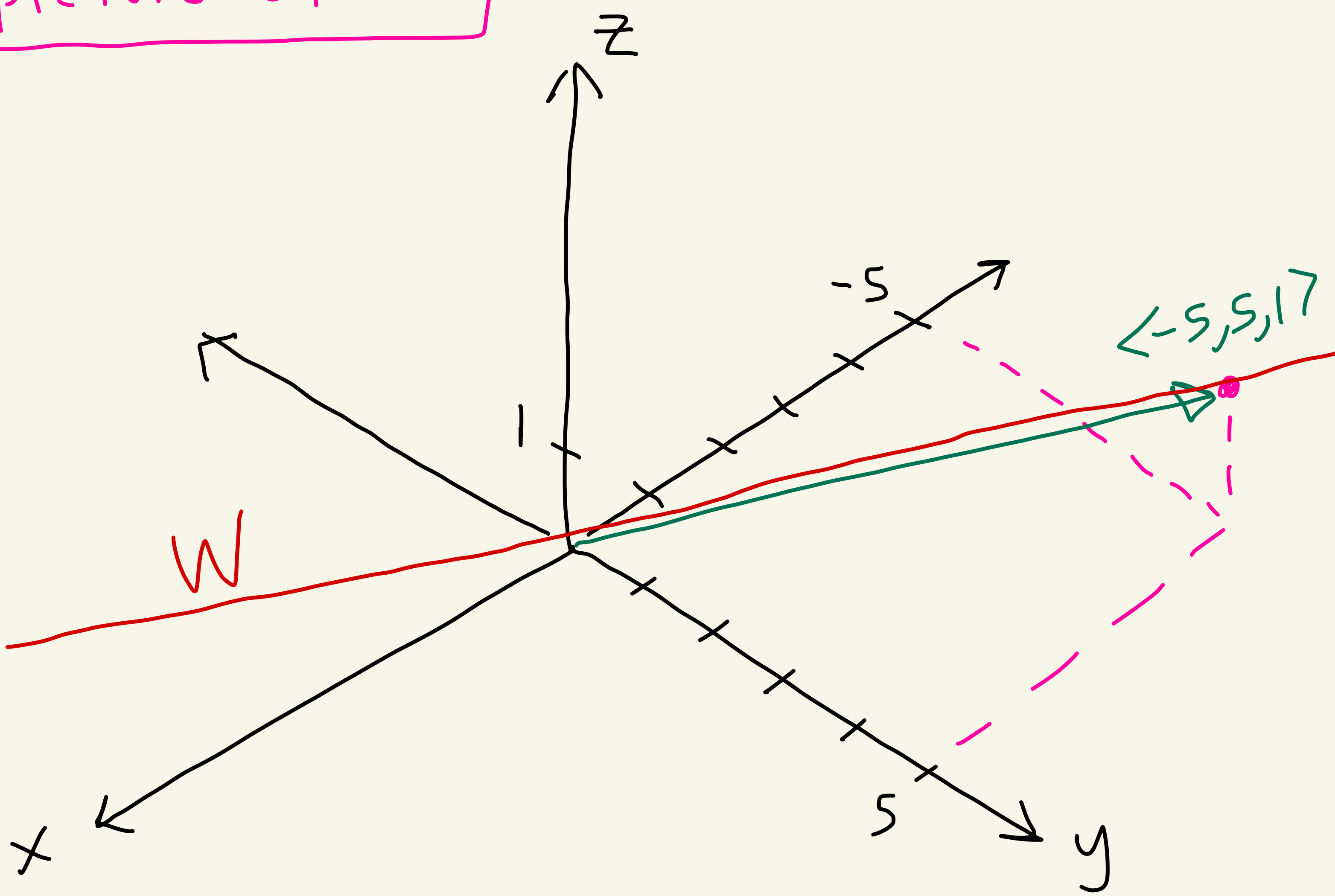
$$\vec{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5t \\ 5t \\ t \end{pmatrix} = t \begin{pmatrix} -5 \\ 5 \\ 1 \end{pmatrix}$$

So, $W = \text{span}(\langle -5, 5, 1 \rangle)$

Since $\langle -5,5,1\rangle \neq \vec{0}$ we get that $\beta = [\langle -5,5,1\rangle]$ is a basis for W. Thus,

$\dim(W) = \underline{1}$ since β has 1 vector in it.

$\boxed{\text{picture of W}}$



Theorem (Dimension is well-defined)

Suppose W is a subspace of $\mathbb{R}^n$ and you have two bases $\beta_1 = [\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_a]$ and $\beta_2 = [\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_b]$.

Then, $a = b$.

Idea:

$\dim(W) = a$

$\dim(W) = b$

so you better have $a = b$

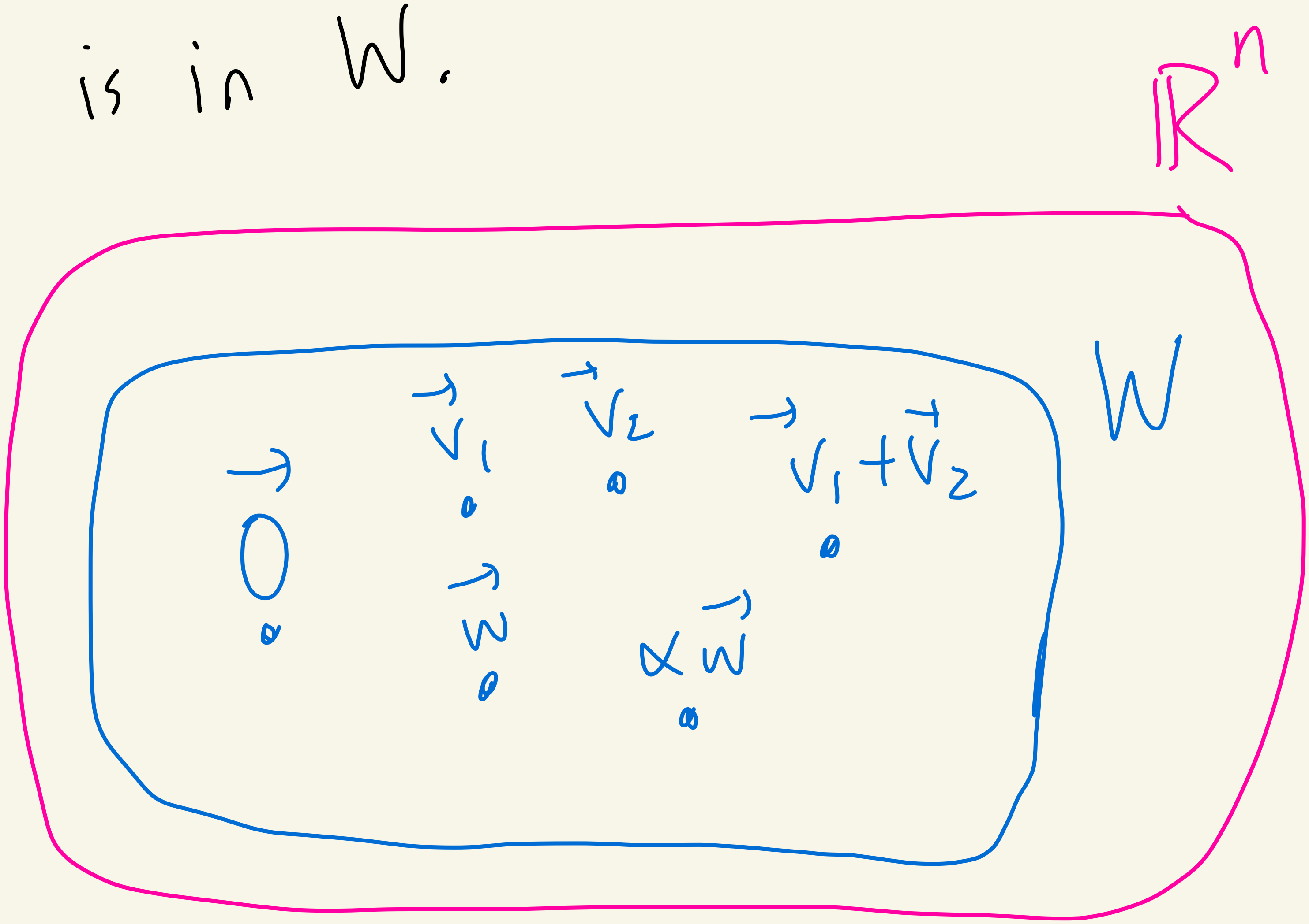
Below is the usual way to define subspace.

Theorem: Let W be a subset of $\mathbb{R}^n$. Then W is a subspace if and only if the following 3 conditions hold:

① $\vec{0}$ is in W

② if $\vec{v}_1, \vec{v}_2$ are in W, then $\vec{v}_1 + \vec{v}_2$ is in W.

③ if $\vec{w}$ is in W and α is a real number, then $\alpha\vec{w}$ is in W.



Test review starts now...

HW 4 – Part 1

① (a) Determine if A and B are inverses where

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1/2 \\ 0 & 1/2 \end{pmatrix}$$

We need to check if $AB = I$.

We get

$$AB = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1/2 \\ 0 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} (1)(1)+(-1)(0) & (1)\left(\frac{1}{2}\right)+(-1)\left(\frac{1}{2}\right) \\ (0)(1)+(2)(0) & (0)\left(\frac{1}{2}\right)+(2)\left(\frac{1}{2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

Since $AB = I_2$ we know that A and B are inverses.

$\boxed{\text{HW 4 - Part 1}}$

$\boxed{3(a)}$ Find A^{-1} if it exists where

$$A = \begin{pmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{pmatrix}$$

We get

$$\left(\begin{array}{ccc|ccc} 3 & 4 & -1 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 2 & 5 & -4 & 0 & 0 & 1 \end{array}\right)$$

$\boxed{\text{Goal: Turn left side into identity}}$

A identity

$$\xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 3 & 4 & -1 & 1 & 0 & 0 \\ 2 & 5 & -4 & 0 & 0 & 1 \end{array}\right)$$

$\boxed{\text{make these } 0}$

$$\xrightarrow[-2R_1 + R_3 \to R_3]{-3R_1 + R_2 \to R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 4 & -10 & 1 & -3 & 0 \\ 0 & 5 & -10 & 0 & -2 & 1 \end{array}\right)$$

make this a 1

$$\xrightarrow{\frac{1}{4}R_2 \to R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -5/2 & 1/4 & -3/4 & 0 \\ 0 & 5 & -10 & 0 & -2 & 1 \end{array}\right)$$

$\boxed{\text{make these } 0}$

$$\xrightarrow{-5R_2+R_3\to R_3}\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -5/2 & 1/4 & -3/4 & 0 \\ 0 & 0 & 5/2 & -5/4 & \frac{7}{4} & 1 \end{array}\right)$$

make this: $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\xrightarrow{R_3+R_2\to R_2}\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 5/2 & -5/4 & \frac{7}{4} & 1 \end{array}\right)$$

make 1

$$\xrightarrow{\frac{2}{5}R_3\to R_3}\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{7}{10} & \frac{2}{5} \end{array}\right)$$

make 0

$$\xrightarrow{-3R_3+R_1\to R_1}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -11/10 & -6/5 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1/2 & \frac{7}{10} & 2/5 \end{array}\right)$$

$$\underbrace{}_{I_3} \quad \underbrace{}_{A^{-1}}$$

So,

$$A^{-1} = \begin{pmatrix} 3/2 & -11/10 & -6/5 \\ -1 & 1 & 1 \\ -1/2 & 7/10 & 2/5 \end{pmatrix}$$

HW 4 - Part 1

$\boxed{5(c)}$ Solve

$$x_1 + 3x_2 + x_3 = 4$$
$$2x_1 + 2x_2 + x_3 = -1$$
$$2x_1 + 3x_2 + x_3 = 3$$

by inverting the coefficient matrix.

The above is equivalent to

$$\underbrace{\begin{pmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \qquad (*)$$

One can find that $A^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{pmatrix}$

Multiply $(*)$ on the left by A^{-1} to get:

$$\underbrace{\underbrace{\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{pmatrix}}_{A^{-1}} \underbrace{\begin{pmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}}_{A}}_{I_3} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{pmatrix}}_{A^{-1}} \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

So we get

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{pmatrix} \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

(pick up on Weds...)