

Math 2550-04

8/28/24

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## Topic 1 continued...

Def: Let

$$\vec{v} = \langle a_1, a_2, \dots, a_n \rangle \quad \text{and}$$

$$\vec{w} = \langle b_1, b_2, \dots, b_n \rangle$$

be in  $\mathbb{R}^n$ .

Define the dot product to be

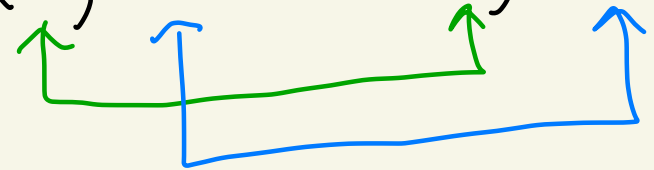
$$\vec{v} \cdot \vec{w} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

Note: The dot product is a number

Ex: In  $\mathbb{R}^2$ , let  $\vec{v} = \langle 1, -1 \rangle$

and  $\vec{w} = \langle 3, -4 \rangle$ .

Then,

$$\vec{v} \cdot \vec{w} = \langle 1, -1 \rangle \cdot \langle 3, -4 \rangle$$


$$= (1)(3) + (-1)(-4)$$

$$= 7$$

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In Calculus in  $\mathbb{R}^2$  or  $\mathbb{R}^3$ ,

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \cdot \|\vec{w}\|}$$

where  $\theta$  is  
the angle  
between  $\vec{v}$   
and  $\vec{w}$

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Ex: In  $\mathbb{R}^5$ , let's calculate

$$\langle 0, 1, -2, 3, \frac{1}{2} \rangle \cdot \langle -1, \frac{1}{3}, 4, 2, 10 \rangle$$


$$\begin{aligned} &= (0)(-1) + (1)\left(\frac{1}{3}\right) + (-2)(4) \\ &\quad + (3)(2) + \left(\frac{1}{2}\right)(10) \\ &= 0 + \frac{1}{3} - 8 + 6 + 5 = 3 + \frac{1}{3} = \frac{10}{3} \end{aligned}$$

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## Properties of dot product

Let  $\vec{u}, \vec{v}, \vec{w}$  be vectors in  $\mathbb{R}^n$ .

Let  $\alpha$  be a scalar in  $\mathbb{R}$ .  
number

Then:

- ①  $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- ②  $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
- ③  $\alpha(\vec{u} \cdot \vec{v}) = (\alpha\vec{u}) \cdot \vec{v} = \vec{u} \cdot (\alpha\vec{v})$

proof of (2) when  $n=3$  :

Let  $\vec{u}, \vec{v}, \vec{w}$  be in  $\mathbb{R}^3$ .

Then,

$$\vec{u} = \langle a, b, c \rangle$$

$$\vec{v} = \langle d, e, f \rangle$$

$$\vec{w} = \langle g, h, i \rangle$$

where  $a, b, c, d, e, f, g, h, i$  are numbers.

Then,

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \langle a, b, c \rangle \cdot (\langle d, e, f \rangle + \langle g, h, i \rangle)$$

$$= \langle a, b, c \rangle \cdot \langle d+g, e+h, f+i \rangle$$

$$= a(d+g) + b(e+h) + c(f+i)$$

$$= ad + ag + be + bh + cf + ci \leftarrow$$

Also, we have

$$\vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

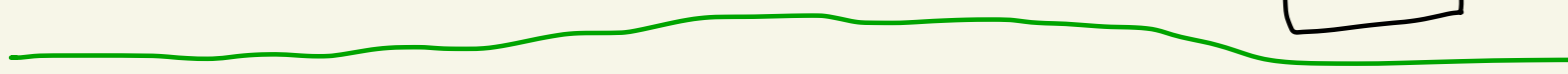
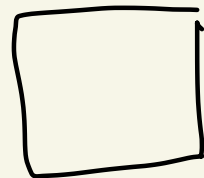
$$= \langle a, b, c \rangle \cdot \langle d, e, f \rangle + \langle a, b, c \rangle \cdot \langle g, h, i \rangle$$

$$= ad + be + cf + ag + bh + ci$$

$$= ad + ag + be + bh + cf + ci$$

L  
A  
V  
Q  
E

$$\text{Thus, } \vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$



# HW 1 - Part 1

10 List 3 elements from the set

$$S = \left\{ c_1 \langle 1, 1, 1 \rangle + c_2 \langle 0, 0, 5 \rangle \mid c_1, c_2 \in \mathbb{R} \right\}$$

means:  $S$  consists of all elements of the form  $c_1 \langle 1, 1, 1 \rangle + c_2 \langle 0, 0, 5 \rangle$  where  $c_1, c_2$  are real #s

When  $c_1 = 1$  and  $c_2 = 2$  we get

$$\begin{aligned} & c_1 \langle 1, 1, 1 \rangle + c_2 \langle 0, 0, 5 \rangle \\ &= 1 \cdot \langle 1, 1, 1 \rangle + 2 \langle 0, 0, 5 \rangle \\ &= \langle 1, 1, 1 \rangle + \langle 0, 0, 10 \rangle \end{aligned}$$

$$= \langle 1, 1, 1 \rangle$$

So,  $\langle 1, 1, 1 \rangle$  is in  $S$

When  $c_1 = 0$  and  $c_2 = 0$  we get

$$0 \cdot \langle 1, 1, 1 \rangle + 0 \cdot \langle 0, 0, 5 \rangle$$

$$= \langle 0, 0, 0 \rangle + \langle 0, 0, 0 \rangle$$

$$= \langle 0, 0, 0 \rangle$$

So,  $\langle 0, 0, 0 \rangle$  is in  $S$

When  $c_1 = 1$  and  $c_2 = 0$

$$1 \cdot \langle 1, 1, 1 \rangle + 0 \cdot \langle 0, 0, 5 \rangle$$

$$= \langle 1, 1, 1 \rangle + \langle 0, 0, 0 \rangle$$

$$= \langle 1, 1, 1 \rangle$$

So,  $\langle 1, 1, 1 \rangle$  is in  $S$



$S_0,$

$$S = \{ \langle 1, 1, 1 \rangle, \langle 0, 0, 0 \rangle, \langle 1, 1, 1 \rangle, \dots \}$$

infinite  
many  
more

## Topic 2 - Matrices

Def: A matrix is a rectangular array of numbers. If  $M$  is a matrix and it has  $m$  rows and  $n$  columns then  $M$  is an  $m \times n$  matrix.  
read: "m by n"

Abstractly we can write an  $m \times n$  matrix like this:

$$M = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

Where  $a_{ij}$  is in row  $i$  and column  $j$ .

Ex:

$$M = \begin{pmatrix} 0 & 5 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

M is 2 x 2  
↑                    ↑  
2 rows            2 columns

$$\begin{aligned} a_{11} &= 0 \\ a_{12} &= 5 \\ a_{21} &= 3 \\ a_{22} &= -1 \end{aligned}$$

Ex:

$$A = (1 \ 5 \ 3) = (a_{11} \ a_{12} \ a_{13})$$

A is 1 x 3  
↑                    ↑  
1 row                3 columns

$$\begin{aligned} a_{11} &= 1 \\ a_{12} &= 5 \\ a_{13} &= 3 \end{aligned}$$

Ex:

$$B = \begin{pmatrix} 1 & 3 & -1 \\ 7 & 2 & 3 \\ 10 & 5 & 1 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}$$

$a_{33} = 1$

$a_{42} = 0$

B is  $4 \times 3$

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You can think of a vector as a matrix, either as a row or column.

For example,  $\vec{v} = \langle 5, 2, -3 \rangle$

You can think of  $\vec{v}$  as:

$$(5 \ 2 \ -3)$$

$1 \times 3$  matrix

Or

$$\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$$

$3 \times 1$  matrix

Def: Let  $A$  and  $B$  be  $m \times n$  matrices [so,  $A$  and  $B$  have the same dimensions.]

$$\text{Let } A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$$

Define  $A+B$  to be

$$A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \dots & a_{2n}+b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots & a_{mn}+b_{mn} \end{pmatrix}$$

Define  $A - B$  to be

$$A - B = \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \dots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \dots & a_{2n} - b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \dots & a_{mn} - b_{mn} \end{pmatrix}$$

If  $\alpha$  is a scalar, define

$$\alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \dots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \dots & \alpha a_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha a_{m1} & \alpha a_{m2} & \dots & \alpha a_{mn} \end{pmatrix}$$