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HW 1 - Part 1

Solutions

(1) (a) $\|\langle 4,-3\rangle\| = \sqrt{4^2+(-3)^2} = \sqrt{25} = 5$

(b) $\|\langle 2,3\rangle\| = \sqrt{2^2+3^2} = \sqrt{13}$

(c) $\|\langle -5,0\rangle\| = \sqrt{(-5)^2+0^2} = 5$

(d) $\|\langle 2,2,2\rangle\| = \sqrt{2^2+2^2+2^2} = \sqrt{12}$

(e) $\|\langle -7,2,-1\rangle\| = \sqrt{(-7)^2+2^2+(-1)^2} = \sqrt{54}$

(f) $\|\langle 0,6,0\rangle\| = \sqrt{0^2+6^2+0^2} = 6$

(2) (a) $\vec{u}+\vec{v} = \langle 2-1, -3+5\rangle = \langle 1,2\rangle$

$\vec{u}-\vec{v} = \langle 2-(-1), -3-5\rangle = \langle 3,-8\rangle$

$\alpha\vec{v} = \langle -\frac{1}{2}, \frac{5}{2}\rangle$ $\alpha\vec{u} = \langle 1, -3/2\rangle$

(b) $\vec{u}+\vec{v} = \langle 1+0, 1+5, -\frac{1}{2}+2\rangle = \langle 1,6,\frac{3}{2}\rangle$

$\vec{u}-v = \langle 1-0, 1-5, -\frac{1}{2}-2\rangle = \langle 1,-4,-\frac{5}{2}\rangle$

$\alpha\vec{u} = \langle -2,-2,1\rangle$ $\alpha\vec{v} = \langle 0,-10,-4\rangle$

(3) (a) $\langle 2,3\rangle\cdot\langle 5,-7\rangle = 2\cdot 5+3(-7) = -11$

(b) $\langle -6,-2\rangle\cdot\langle 4,0\rangle = (-6)(4)+(-2)(0) = -24$

(c) $\langle 1,-5,4\rangle\cdot\langle 3,3,3\rangle = (1)(3)+(-5)(3)+(4)(3) = 0$

(d) $\langle -2,2,3\rangle\cdot\langle 1,7,-4\rangle = (-2)(1)+(2)(7)+(3)(-4) = 0$

4 (a) $\|\vec{v}\| = \sqrt{1^2 + 5^2 + (-1)^2 + 0^2 + \pi^2}$

$= \sqrt{27 + \pi^2}$

(b) $\|\vec{w}\| = \sqrt{0^2 + 2^2 + \left(\frac{1}{2}\right)^2 + (-1)^2}$

$= \sqrt{5 + \frac{1}{4}} = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$

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$\vec{u} + 2\vec{v} = \langle 2, 0, 8, -4, 10 \rangle + 2\langle 0, \frac{1}{2}, 3, 10, -1 \rangle$

$= \langle 2, 0, 8, -4, 10 \rangle + \langle 0, 1, 6, 20, -2 \rangle$

$= \langle 2, 1, 14, 16, 8 \rangle$

$\frac{1}{2}\vec{u} - \vec{v} = \frac{1}{2}\langle 2, 0, 8, -4, 10 \rangle - \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$

$= \langle 1, 0, 4, -2, 5 \rangle - \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$

$= \langle 1, -\frac{1}{2}, 1, -12, 6 \rangle$

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(a)

$\vec{u} \cdot \vec{v} = \langle 1, 0, 2, -1, 5 \rangle \cdot \langle -1, \pi, \sqrt{2}, 13, -2 \rangle$

$= (1)(-1) + (0)(\pi) + (2)(\sqrt{2}) + (-1)(13) + (5)(-2)$

$= -1 + 0 + 2\sqrt{2} - 13 - 10$

$= -24 + 2\sqrt{2}$

(b)

$\vec{u} \cdot \vec{v} = \langle 1, 2, 3, 4, 5, 6, 7 \rangle \cdot \langle -7, -6, -5, -4, -3, -2, -1 \rangle$

$= (1)(-7) + (2)(-6) + (3)(-5) + (4)(-4) + (5)(-3) + (6)(-2) + (7)(-1)$

$= -7 - 12 - 15 - 16 - 15 - 12 - 7$

$= -84$

⑦

$$S = \{ t\langle 1,-1\rangle \mid t \in \mathbb{R} \}$$

5 randomly selected elements from S:

$$3\langle 1,-1\rangle = \langle 3,-3\rangle$$
$$\pi\langle 1,-1\rangle = \langle \pi,-\pi\rangle$$
$$0\langle 1,-1\rangle = \langle 0,0\rangle$$
$$1\langle 1,-1\rangle = \langle 1,-1\rangle$$
$$-\tfrac{1}{2}\langle 1,-1\rangle = \langle -\tfrac{1}{2},\tfrac{1}{2}\rangle$$

you can let t be any real number. I just picked these randomly

So,

$$S = \{ \langle 3,-3\rangle, \langle \pi,-\pi\rangle, \langle 0,0\rangle, \langle 1,-1\rangle, \langle -\tfrac{1}{2},\tfrac{1}{2}\rangle, \ldots \}$$

infinitely many more elements that we didn't list

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$$S = \left\{ t\langle 3,1\rangle + s\langle -1,5\rangle \mid s,t \in \mathbb{R} \right\}$$

5 randomly selected elements from S:

$$1\cdot\langle 3,1\rangle - 5\cdot\langle -1,5\rangle = \langle 3,1\rangle + \langle 5,-25\rangle$$
$$= \boxed{\langle 8,-24\rangle}$$

$$-1\cdot\langle 3,1\rangle + 0\cdot\langle -1,5\rangle = \boxed{\langle -3,-1\rangle}$$

$$\frac{1}{2}\cdot\langle 3,1\rangle + \frac{1}{2}\langle -1,5\rangle = \left\langle \frac{3}{2},\frac{1}{2}\right\rangle + \left\langle -\frac{1}{2} + \frac{5}{2}\right\rangle$$
$$= \boxed{\langle 1,3\rangle}$$

$$\pi\langle 3,1\rangle + 2\langle -1,5\rangle = \langle 3\pi,\pi\rangle + \langle -2,10\rangle$$
$$= \boxed{\langle 3\pi-2,\pi+10\rangle}$$

$$0\langle 3,1\rangle + 0\langle -1,5\rangle = \langle 0,0\rangle + \langle 0,0\rangle = \boxed{\langle 0,0\rangle}$$

So, $S = \{ \langle 8,-24\rangle, \langle -3,-1\rangle, \langle 1,3\rangle,$
$\langle 3\pi-2,\pi+10\rangle, \langle 0,0\rangle, \ldots \}$

infinitely many more elements that we didn't list

⑨

$$S = \left\{ c_1\langle 1,1\rangle + c_2\langle 0,-1\rangle + c_3\langle -2,1\rangle \;\middle|\; c_1, c_2, c_3 \in \mathbb{R} \right\}$$

5 randomly selected elements from S:

$$1\cdot\langle 1,1\rangle + 0\cdot\langle 0,-1\rangle - 3\cdot\langle -2,1\rangle = \langle 1,1\rangle + \langle 0,0\rangle + \langle 6,-3\rangle$$
$$= \boxed{\langle 7,-2\rangle}$$

$$0\cdot\langle 1,1\rangle + 1\cdot\langle 0,-1\rangle + 1\cdot\langle -2,1\rangle = \langle 0,0\rangle + \langle 0,-1\rangle + \langle -2,1\rangle$$
$$= \boxed{\langle -2,0\rangle}$$

$$1\cdot\langle 1,1\rangle + 1\cdot\langle 0,-1\rangle + \tfrac{1}{2}\langle -2,1\rangle = \langle 1,1\rangle + \langle 0,-1\rangle + \langle -1,\tfrac{1}{2}\rangle$$
$$= \boxed{\langle 0,\tfrac{1}{2}\rangle}$$

$$0\cdot\langle 1,1\rangle + 0\langle 0,-1\rangle + 0\langle -2,1\rangle = \langle 0,0\rangle + \langle 0,0\rangle + \langle 0,0\rangle$$
$$= \boxed{\langle 0,0\rangle}$$

$$\tfrac{1}{2}\langle 1,1\rangle + \pi\langle 0,-1\rangle - \tfrac{1}{2}\langle -2,1\rangle = \langle \tfrac{1}{2},\tfrac{1}{2}\rangle + \langle 0,-\pi\rangle + \langle 1,\tfrac{-1}{2}\rangle$$
$$= \boxed{\langle \tfrac{3}{2},-\pi\rangle}$$

So, ↓

$$S = \left\{ (7,-2),\ \langle -2,0\rangle,\ \langle 0,\tfrac{1}{2}\rangle,\ \langle 0,0\rangle,\ \left(\tfrac{3}{2},-\pi\right),\ \ldots \right\}$$

$\boxed{\text{infinitely many more elements that we didn't list}}$

<u>Note:</u> You can let c_1, c_2, c_3 be any real numbers.

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$$S = \{ a\langle 1,1,1\rangle + b\langle 0,0,5\rangle \mid a,b \in \mathbb{R} \}$$

5 randomly selected elements from S:

$$0\cdot\langle 1,1,1\rangle + 0\langle 0,0,5\rangle = \langle 0,0,0\rangle + \langle 0,0,0\rangle = \boxed{\langle 0,0,0\rangle}$$

$$1\cdot\langle 1,1,1\rangle + 0\langle 0,0,5\rangle = \langle 1,1,1\rangle + \langle 0,0,0\rangle = \boxed{\langle 1,1,1\rangle}$$

$$-1\cdot\langle 1,1,1\rangle + 1\cdot\langle 0,0,5\rangle = \langle -1,-1,-1\rangle + \langle 0,0,5\rangle = \boxed{\langle -1,-1,4\rangle}$$

$$0\langle 1,1,1\rangle + 1\cdot\langle 0,0,5\rangle = \langle 0,0,0\rangle + \langle 0,0,5\rangle = \boxed{\langle 0,0,5\rangle}$$

$$\tfrac{1}{2}\langle 1,1,1\rangle - \tfrac{1}{2}\langle 0,0,5\rangle = \langle \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\rangle + \langle 0,0,-\tfrac{5}{2}\rangle = \boxed{\langle \tfrac{1}{2},\tfrac{1}{2},2\rangle}$$

So,

$$S = \{ \langle 0,0,0 \rangle, \langle 1,1,1 \rangle, \langle -1,-1,4 \rangle, \langle 0,0,5 \rangle, \langle \tfrac{1}{2}, \tfrac{1}{2}, 2 \rangle, \ldots \}$$

infinitely many more elements that we didn't list