2550

HW 1 - Part 2

Solutions

1 (a)

Let $\vec{u} \in \mathbb{R}^2$.
Then $\vec{u} = \langle a, b \rangle$ where a & b are in $\mathbb{R}$.

And $\vec{0} = \langle 0, 0 \rangle$.

Then,

$$\vec{u} + \vec{0} = \langle a, b \rangle + \langle 0, 0 \rangle$$
$$= \langle a+0,\ b+0 \rangle$$
$$= \langle a, b \rangle$$
$$= \vec{u}$$

1 (b)

Let $\vec{v} \in \mathbb{R}^2$ and $\alpha, \beta \in \mathbb{R}$.

Then $\vec{v} = \langle a, b \rangle$ where a, b are real numbers.

We have that

$$(\alpha+\beta)\vec{v} = (\alpha+\beta)\langle a, b\rangle$$
$$= \langle (\alpha+\beta)a, (\alpha+\beta)b \rangle$$
$$= \langle \alpha a + \beta a, \alpha b + \beta b \rangle$$

and

$$\alpha\vec{v} + \beta\vec{v} = \alpha\langle a, b\rangle + \beta\langle a, b\rangle$$
$$= \langle \alpha a, \alpha b\rangle + \langle \beta a, \beta b\rangle$$
$$= \langle \alpha a + \beta a, \alpha b + \beta b \rangle$$

We can see from above that

$$(\alpha+\beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}.$$

1 (c)

Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^2$.

Then, $\vec{u} = \langle a, b \rangle$, $\vec{v} = \langle c, d \rangle$, and $\vec{w} = \langle e, f \rangle$ for some real numbers a, b, c, d, e, f.

Thus,

$$(\vec{u} + \vec{v}) + \vec{w} = \left(\langle a, b \rangle + \langle c, d \rangle \right) + \langle e, f \rangle$$

$$= \langle a+c, b+d \rangle + \langle e, f \rangle$$

$$= \langle (a+c) + e, (b+d) + f \rangle$$

a, b, c, d, e, f are real #s so $(a+c) + e = a + (c+e)$ and $(b+d) + f = b + (d+f)$

$$= \langle a + (c+e), b + (d+f) \rangle$$

$$= \langle a, b \rangle + \langle c+e, d+f \rangle$$

$$= \langle a, b \rangle + \left(\langle c, d \rangle + \langle e, f \rangle \right)$$

$$= \vec{u} + (\vec{v} + \vec{w})$$

① (d)

Let $\vec{u}, \vec{v} \in \mathbb{R}^2$ and $\alpha \in \mathbb{R}$.

Then $\vec{u} = \langle a, b \rangle$ and $\vec{v} = \langle c, d \rangle$

for some real numbers a, b, c, d.

So,

$$\alpha(\vec{u} \cdot \vec{v}) = \alpha\left(\langle a, b \rangle \cdot \langle c, d \rangle\right)$$
$$= \alpha(ac + bd) = \alpha ac + \alpha bd$$

And

$$(\alpha \vec{u}) \cdot \vec{v} = \left(\alpha \langle a, b \rangle\right) \cdot \langle c, d \rangle$$
$$= \langle \alpha a, \alpha b \rangle \cdot \langle c, d \rangle$$
$$= (\alpha a)c + (\alpha b)d$$
$$= \alpha(ac) + \alpha(bd)$$

We see from above that

$$\alpha(\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}.$$

2 (a)

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$.

Then $\vec{u} = \langle a, b, c \rangle$ and $\vec{v} = \langle d, e, f \rangle$

where a, b, c, d, e, f are real numbers.

Then

$$\vec{u} + \vec{v} = \langle a, b, c \rangle + \langle d, e, f \rangle$$

$$= \langle a+d, b+e, c+f \rangle$$

$$= \langle d+a, e+b, f+c \rangle$$

$$= \langle d, e, f \rangle + \langle a, b, c \rangle$$

$$= \vec{v} + \vec{u}$$

because a, b, c, d, e, f are real #s we know that

$a+d = d+a$

$b+e = e+b$

$c+f = f+c$

② (b)

Let $\vec{v} \in \mathbb{R}^3$ and $\alpha, \beta \in \mathbb{R}$.

Then $\vec{v} = \langle a, b, c \rangle$ where $a, b, c \in \mathbb{R}$.

So,

$$
\begin{aligned}
(\alpha+\beta)\vec{v} &= (\alpha+\beta)\langle a, b, c\rangle \\
&= \langle (\alpha+\beta)a, (\alpha+\beta)b, (\alpha+\beta)c \rangle \\
&= \langle \alpha a + \beta a, \alpha b + \beta b, \alpha c + \beta c \rangle \\
&= \langle \alpha a, \alpha b, \alpha c \rangle + \langle \beta a, \beta b, \beta c \rangle \\
&= \alpha \langle a, b, c \rangle + \beta \langle a, b, c \rangle \\
&= \alpha \vec{v} + \beta \vec{v}
\end{aligned}
$$

(2) (c)

Let $\vec{v} \in \mathbb{R}^3$ and $\alpha, \beta \in \mathbb{R}$.

Then $\vec{v} = \langle a, b, c \rangle$ where $a, b, c \in \mathbb{R}$.

So,

$$\alpha(\beta\vec{v}) = \alpha\left(\beta\langle a, b, c\rangle\right)$$

$$= \alpha\langle \beta a, \beta b, \beta c\rangle$$

$$= \left\langle \alpha(\beta a), \alpha(\beta b), \alpha(\beta c)\right\rangle$$

Since α, β, a, b, c are all real #s we know that
$\alpha(\beta a) = (\alpha\beta)a$
$\alpha(\beta b) = (\alpha\beta)b$
$\alpha(\beta c) = (\alpha\beta)c$

$$= \left\langle (\alpha\beta)a, (\alpha\beta)b, (\alpha\beta)c \right\rangle$$

$$= (\alpha\beta)\langle a, b, c\rangle$$

$$= (\alpha\beta)\vec{v}$$

2 (d)

Let $\vec{u} \in \mathbb{R}^3$ and $\vec{0}$ be the zero vector in $\mathbb{R}^3$.

Then $\vec{u} = \langle a, b, c \rangle$ where $a, b, c \in \mathbb{R}$ and $\vec{0} = \langle 0, 0, 0 \rangle$.

So,

$$\vec{0} \cdot \vec{u} = \langle 0, 0, 0 \rangle \cdot \langle a, b, c \rangle$$

$$= 0a + 0b + 0c$$

$$= 0 + 0 + 0$$

$$= 0$$

② (e) Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$.

Then $\vec{u} = \langle a, b, c \rangle$, $\vec{v} = \langle d, e, f \rangle$,

$\vec{w} = \langle g, h, i \rangle$ where $a, b, c, d, e, f, g, h, i$

are real numbers.

We have that

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \langle a, b, c \rangle \cdot \left(\langle d, e, f \rangle + \langle g, h, i \rangle \right)$$

$$= \langle a, b, c \rangle \cdot \langle d+g, e+h, f+i \rangle$$

$$= a(d+g) + b(e+h) + c(f+i)$$

$$= ad + ag + be + bh + cf + ci$$

and

$$\vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w} = \langle a, b, c \rangle \cdot \langle d, e, f \rangle + \langle a, b, c \rangle \cdot \langle g, h, i \rangle$$

$$= ad + be + cf + ag + bh + ci$$

$$= ad + ag + be + bh + cf + ci$$

Thus, we see from the above calculations that $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$