

Math 2550-01

10/7/24



HW 1-Part 1

$$\vec{a} = \langle -1, 2, 4, 6 \rangle$$

$$\vec{b} = \langle 3, 1, 0, -2 \rangle$$

$$2\vec{a} - 3\vec{b}$$

$$= \langle -2, 4, 8, 12 \rangle + \langle -9, -3, 0, 6 \rangle$$

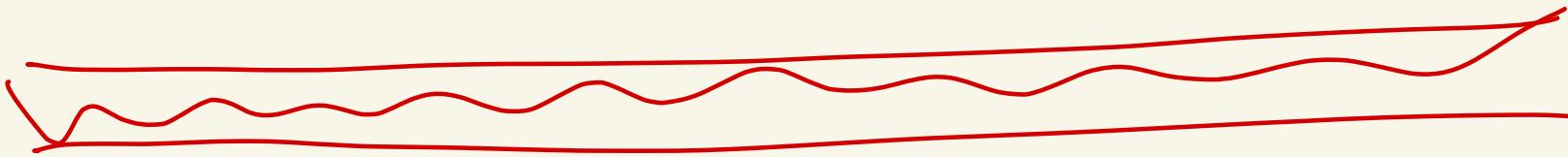
$$= \boxed{\langle -11, 1, 8, 18 \rangle}$$

$$\vec{a} \cdot \vec{b} = \langle -1, 2, 4, 6 \rangle \cdot \langle 3, 1, 0, -2 \rangle$$

$$= (-1)(3) + (2)(1) + (4)(0) + (6)(-2)$$

$$= -3 + 2 + 0 - 12 = \boxed{-13}$$

$$\|\vec{b}\| = \sqrt{3^2 + 1^2 + 0^2 + (-2)^2} \\ = \sqrt{14}$$



Q: Find three vectors in S .

$$S = \left\{ c_1 \langle 1, 0 \rangle + c_2 \langle 2, -1 \rangle + c_3 \langle 5, 1 \rangle \mid c_1, c_2, c_3 \in \mathbb{R} \right\}$$

$$c_1 = 1, c_2 = 0, c_3 = -1:$$

$$1 \cdot \langle 1, 0 \rangle + 0 \cdot \langle 2, -1 \rangle - \langle 5, 1 \rangle = \boxed{\langle -4, -1 \rangle}$$

$$c_1 = 0, c_2 = 0, c_3 = 0:$$

$$0 \cdot \langle 1, 0 \rangle + 0 \cdot \langle 2, -1 \rangle + 0 \cdot \langle 5, 1 \rangle = \boxed{\langle 0, 0 \rangle}$$

$$c_1 = 1, c_2 = 0, c_3 = 0:$$

$$1 \cdot \langle 1, 0 \rangle + 0 \cdot \langle 2, -1 \rangle + 0 \cdot \langle 5, 1 \rangle = \boxed{\langle 1, 0 \rangle}$$

HW 2 - Part 1

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 1 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

$$2A - B = \begin{pmatrix} 2 & -2 & 0 \\ 0 & 4 & 2 \\ -6 & 2 & -4 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ -2 & 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 & 0 \\ -1 & 4 & 1 \\ -8 & 2 & -6 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 1 & -2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

3×3 \downarrow 3×3

[]

answer is 3×3

$$= \begin{pmatrix} 0-1+0 & 1+0+0 & 0-1+0 \\ 0+2+2 & 0+0+0 & 0+2+2 \\ 0+1-4 & -3+0+0 & 0+1-4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 & -1 \\ 4 & 0 & 4 \\ -3 & -3 & -3 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 3 & 4 & 3 \end{array} \right) \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 1 & 3 & 1 \\ 1 & 3 & 1 \end{array} \right) \leftarrow \text{undefined}$$

$\left\{ \begin{array}{c} 3 \times 2 \\ 3 \times 2 \end{array} \right.$

\uparrow \uparrow
 not equal

HW 1 - Part 2

①(d) Let $\vec{u}, \vec{v} \in \mathbb{R}^2$
and $\alpha \in \mathbb{R}$.

Prove that

$$\alpha(\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$$

Let $\vec{u} = \langle a, b \rangle$, $\vec{v} = \langle c, d \rangle$.

Then,

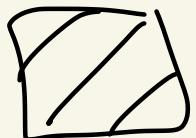
$$\begin{aligned}\alpha(\vec{u} \cdot \vec{v}) &= \alpha(\langle a, b \rangle \cdot \langle c, d \rangle) \\ &= \alpha(ac + bd) \\ &= \alpha ac + \alpha bd\end{aligned}$$

And,

$$\begin{aligned} (\alpha \vec{u}) \cdot \vec{v} &= (\alpha \langle a, b \rangle) \cdot \langle c, d \rangle \\ &= \langle \alpha a, \alpha b \rangle \cdot \langle c, d \rangle \\ &= \alpha ac + \alpha bd \end{aligned}$$

EQVA

$$\text{So, } \alpha (\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}.$$



HW 3

①(c) Solve

$$\begin{aligned}x - y + 2z - w &= -1 \\2x + y - 2z - 2w &= -2 \\-x + 2y - 4z + w &= 1 \\3x &\end{aligned}$$
$$-3w = -3$$

have 1 ✓

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array} \right)$$

make these 0

$$-2R_1 + R_2 \rightarrow R_2$$

$$R_1 + R_3 \rightarrow R_3$$

$$-3R_1 + R_4 \rightarrow R_4$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

make this 1

$R_2 \leftrightarrow R_3$



$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

make these 0

$-3R_2 + R_3 \rightarrow R_3$



$-3R_2 + R_4 \rightarrow R_4$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

in row echelon form

Back to the
land of
equations...

$$x - y + 2z - w = -1$$

$$y - 2z = 0$$

$$0 = 0$$

$$0 = 0$$

leading variables:
 x, y

free variables:
 z, w

Solve for leading variables and give the free variables a new name.

$$x = -1 + y - 2z + w$$

$$y = 2z$$

$$z = t$$

$$w = u$$

①

②

③

④

Back substitute:

④ $w = u$

$$③ z = t$$

$$② y = 2z = 2t$$

$$\begin{aligned}① x &= -1 + y - 2z + w \\&= -1 + 2t - 2t + u \\&= -1 + u\end{aligned}$$

Answer:

$$x = -1 + u$$

$$y = 2t$$

$$z = t$$

$$w = u$$

where t, u
can be any
real numbers