Math 2550-01
11/6/24

Hw 6

① (c) In $\mathbb{R}^3$, let

$\vec{v} = \langle 2, -1, 3 \rangle$, $\vec{u} = \langle 4, 1, 2 \rangle$, $\vec{w} = \langle 8, -1, 8 \rangle$

Show they are lin. dep. and write one as a lin. combo. of the others.

$$c_1 \vec{v} + c_2 \vec{u} + c_3 \vec{w} = \vec{0}$$

$$c_1 \langle 2, -1, 3 \rangle + c_2 \langle 4, 1, 2 \rangle + c_3 \langle 8, -1, 8 \rangle = \langle 0, 0, 0 \rangle$$

$$\langle 2c_1, -c_1, 3c_1 \rangle + \langle 4c_2, c_2, 2c_2 \rangle + \langle 8c_3, -c_3, 8c_3 \rangle = \langle 0, 0, 0 \rangle$$

$$\langle 2c_1 + 4c_2 + 8c_3, -c_1 + c_2 - c_3, 3c_1 + 2c_2 + 8c_3 \rangle = \langle 0, 0, 0 \rangle$$

We get

$$2c_1 + 4c_2 + 8c_3 = 0$$
$$-c_1 + c_2 - c_3 = 0$$
$$3c_1 + 2c_2 + 8c_3 = 0$$

We get

$$\left(\begin{array}{ccc|c} 2 & 4 & 8 & 0 \\ -1 & 1 & -1 & 0 \\ 3 & 2 & 8 & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 2 & 4 & 8 & 0 \\ 3 & 2 & 8 & 0 \end{array}\right)$$

$$\xrightarrow{-R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 4 & 8 & 0 \\ 3 & 2 & 8 & 0 \end{array}\right)$$

$$\xrightarrow[-3R_1 + R_3 \to R_3]{-2R_1 + R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 6 & 6 & 0 \\ 0 & 5 & 5 & 0 \end{array}\right)$$

$$\xrightarrow{\frac{1}{6}R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & 5 & 0 \end{array}\right)$$

$$\xrightarrow{-5R_2 + R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

We get

$$\boxed{\begin{aligned} c_1 - c_2 + c_3 &= 0 \quad (1) \\ c_2 + c_3 &= 0 \quad (2) \\ 0 &= 0 \end{aligned}}$$

leading: c_1, c_2
free: c_3

$c_3 = t$

(2) $c_2 = -c_3 = -t$

(1) $c_1 = c_2 - c_3 = -t - t = -2t$

So,

$$c_1 \vec{v} + c_2 \vec{u} + c_3 \vec{w} = \vec{0}$$

becomes

$$(-2t)\vec{v} + (-t)\vec{u} + (t)\vec{w} = \vec{0}$$

Plug $t=1$ in to get

$$\boxed{-2\vec{v} - \vec{u} + \vec{w} = \vec{0}}$$

Shows that $\vec{v}, \vec{u}, \vec{w}$ are linearly dependent

And

$$\boxed{\vec{w} = 2\vec{v} + \vec{u}}$$

We wrote $\vec{w}$ as a linear combo of $\vec{v}$ and $\vec{u}$

$\boxed{\text{HW 6}}$

2) In $\mathbb{R}^2$, let $\vec{a} = \langle 1, 1 \rangle$, $\vec{b} = \langle -1, 1 \rangle$.

(a) Show $\vec{a}, \vec{b}$ are linearly independent.
Thus, $\beta = [\vec{a}, \vec{b}]$ is a basis for $\mathbb{R}^2$.

Solve $c_1 \vec{a} + c_2 \vec{b} = \vec{0}$

We get

$$c_1 \langle 1, 1 \rangle + c_2 \langle -1, 1 \rangle = \langle 0, 0 \rangle$$

$$\langle c_1, c_1 \rangle + \langle -c_2, c_2 \rangle = \langle 0, 0 \rangle$$

$$\langle c_1 - c_2, c_1 + c_2 \rangle = \langle 0, 0 \rangle$$

So,

$$\boxed{\begin{aligned} c_1 - c_2 &= 0 \\ c_1 + c_2 &= 0 \end{aligned}}$$

We get

$$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right) \xrightarrow{-R_1 + R_2 \to R_2} \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 2 & 0 \end{array}\right) \xrightarrow{\frac{1}{2}R_2 \to R_2} \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 1 & 0 \end{array}\right)$$

We get

$$\boxed{\begin{aligned} c_1 - c_2 &= 0 \quad (1) \\ c_2 &= 0 \quad (2) \end{aligned}} \longrightarrow \boxed{\begin{aligned} &(2)\ c_2 = 0 \\ &(1)\ c_1 = c_2 = 0 \end{aligned}}$$

The only sols to $c_1\vec{a} + c_2\vec{b} = \vec{0}$ are $c_1 = 0, c_2 = 0$. Thus, $\vec{a}, \vec{b}$ are linearly independent.

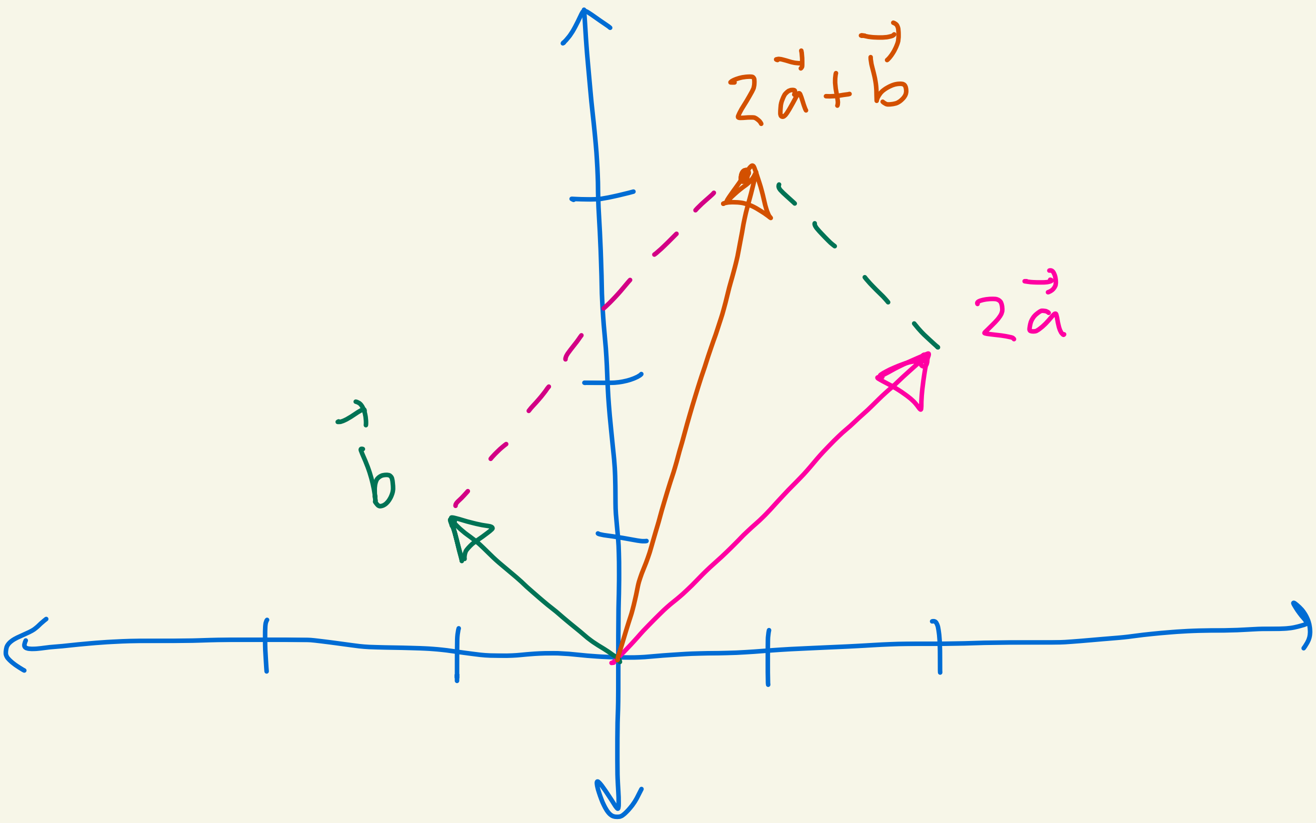
(c) Draw $2\vec{a}$, $\vec{b}$, $2\vec{a}+\vec{b}$ and the paralelegram they create.

$[\vec{a} = \langle 1,1\rangle, \vec{b} = \langle -1,1\rangle]$

$2\vec{a} = 2\langle 1,1\rangle = \langle 2,2\rangle$

$\vec{b} = \langle -1,1\rangle$

$2\vec{a} + \vec{b} = \langle 2,2\rangle + \langle -1,1\rangle = \langle 1,3\rangle$



(2) (g) $\beta = [\vec{a}, \vec{b}]$, $\vec{a} = \langle 1,1 \rangle$, $\vec{b} = \langle -1, 1 \rangle$

Show β is an orthogonal basis, but not an orthonormal basis.

orthogonal

$$\left.\begin{aligned} \vec{a} \cdot \vec{b} &= \langle 1, 1 \rangle \cdot \langle -1, 1 \rangle \\ &= (1)(-1) + (1)(1) = 0 \end{aligned}\right] \checkmark$$

not orthonormal

$$\left.\begin{aligned} \|\vec{a}\| &= \sqrt{1^2 + 1^2} = \sqrt{2} \neq 1 \\ \|\vec{b}\| &= \sqrt{(-1)^2 + 1^2} = \sqrt{2} \neq 1 \end{aligned}\right]$$

not orthonormal since not length 1

② (h) $\beta = [\vec{a}, \vec{b}]$, $\vec{a} = \langle 1, 1 \rangle$, $\vec{b} = \langle -1, 1 \rangle$

Use the coordinate dot product theorem to find $\underbrace{[\vec{v}]_\beta}_{\vec{v}\text{'s } \beta\text{-coordinates}}$ where $\vec{v} = \langle 10, \frac{1}{2} \rangle$.

Since β is an orthogonal basis we get that

$$\vec{v} = \left(\frac{\vec{v} \cdot \vec{a}}{\|\vec{a}\|^2} \right) \vec{a} + \left(\frac{\vec{v} \cdot \vec{b}}{\|\vec{b}\|^2} \right) \vec{b}$$

$$= \left(\frac{\langle 10, \frac{1}{2} \rangle \cdot \langle 1, 1 \rangle}{(\sqrt{2})^2} \right) \vec{a} + \left(\frac{\langle 10, \frac{1}{2} \rangle \cdot \langle -1, 1 \rangle}{(\sqrt{2})^2} \right) \vec{b}$$

$$= \left(\frac{10 + 1/2}{2} \right) \vec{a} + \left(\frac{-10 + 1/2}{2} \right) \vec{b} = \frac{21}{4} \vec{a} - \frac{19}{4} \vec{b}$$

So, $\underbrace{\vec{v} = \frac{21}{4} \vec{a} - \frac{19}{4} \vec{b}}$ $\longleftarrow$ So, $[\vec{v}]_\beta = \langle \frac{21}{4}, \frac{-19}{4} \rangle$

Check: $\langle 10, \frac{1}{2} \rangle = \frac{21}{4} \langle 1, 1 \rangle - \frac{19}{4} \langle -1, 1 \rangle$

$\beta = [\vec{a}, \vec{b}]$ $\vec{a} = \langle 1,1 \rangle$, $\vec{b} = \langle -1,1 \rangle$

2 (j) Suppose you know that $[\vec{v}]_\beta = \langle 5,-4 \rangle$. What is $\vec{v}$?

This means: $\vec{v} = 5\vec{a} - 4\vec{b}$

So, $\vec{v} = 5\langle 1,1 \rangle - 4\langle -1,1 \rangle$

$= \langle 5,5 \rangle - \langle -4,4 \rangle$

$= \langle 9,1 \rangle$

So, $\vec{v} = \langle 9,1 \rangle$

Q: Calculate the determinant of $A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 3 & 3 \end{pmatrix}$.

Is A invertible?

$$\det\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 3 & 3 \end{pmatrix} \qquad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$= 1 \cdot \underbrace{\begin{vmatrix} 3 & 1 \\ 3 & 3 \end{vmatrix}}_{\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 3 & 3 \end{pmatrix}} - (-1) \cdot \underbrace{\begin{vmatrix} 0 & 2 \\ 3 & 3 \end{vmatrix}}_{\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 3 & 3 \end{pmatrix}} + 0$$

$$= 1 \cdot \left[(3)(3) - (1)(3)\right] + 1 \cdot \left[(0)(3) - (2)(3)\right]$$

$$= 9 - 3 - 6 = 0.$$

So, $\boxed{\det(A) = 0}$

Since $\det(A) = 0$, A^{-1} does not exist