Math 3450
1/25/24

Set theory review

$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$ ← set of natural numbers

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ ← set of integers

general way to describe a set

$$\{ \text{description of element} \mid \text{condition on element to be in the set} \}$$

read: "where" "such that" "given"

set of rational #s

$$\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p, q \text{ are integers and } q \neq 0 \right\}$$

$$= \left\{ \frac{1}{2}, \frac{2}{7}, \frac{-1}{23}, \frac{1}{137}, \frac{5}{1}, \ldots \right\}$$

Notation: We write $x \in S$ to mean that x is an element of S. Read it "x is in S".

We write $x \notin S$ if x is not in S.

Ex: $5 \in \mathbb{Q}$ $\quad \sqrt{2} \notin \mathbb{Q}$

$5 \in \mathbb{Z}$

$\frac{1}{2} \notin \mathbb{Z}$

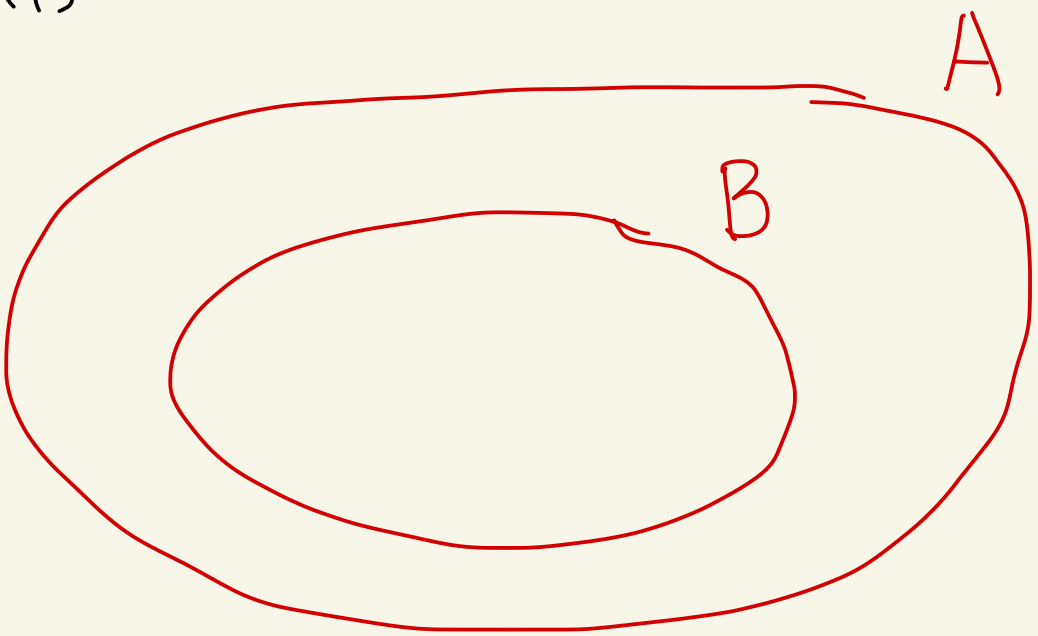
$$\mathbb{R} = \{ x \mid x \text{ has a decimal expansion} \}$$

$\mathbb{R}$: set of real numbers

$$= \left\{ 1, \frac{-5}{2}, \sqrt{2}, \pi, \ldots \right\}$$

$1 = 1.0$, $\frac{-5}{2} = -2.5$, $\sqrt{2} = 1.414\ldots$, $\pi = 3.14159\ldots$

Def: Let A and B be sets. We say that B is a subset of A, and write $B \subseteq A$, if every element of B is also an element of A.



Some people write $B \subset A$ for subset

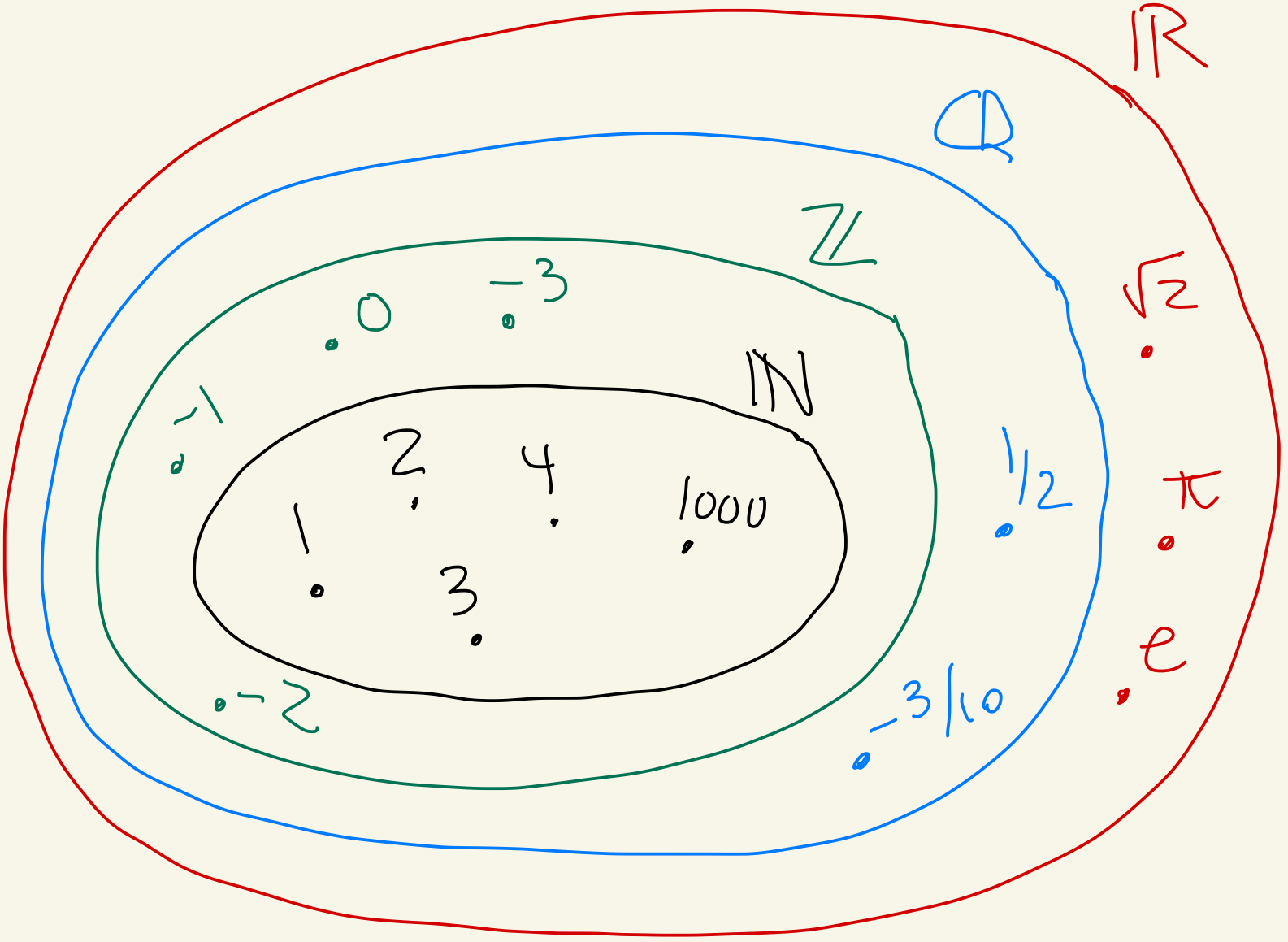
Ex:

$\mathbb{N} \subseteq \mathbb{Z}$

$\mathbb{Z} \subseteq \mathbb{Q}$

$\mathbb{Q} \subseteq \mathbb{R}$

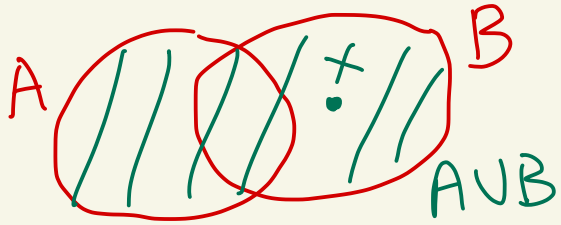
$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$



Def: Let A and B be sets.

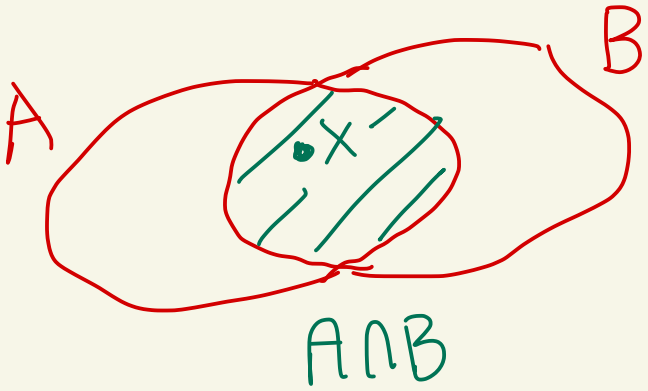
The union of A and B is

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



The intersection of A and B is

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



Ex:

$A = \{\sqrt{2}, 5, 9, 20, \frac{1}{3}, 2\}$

$B = \{-16, 9, \frac{1}{3}, 4\}$

$A \cup B = \{\underbrace{\sqrt{2}, 5, 9, 20, \frac{1}{3}, 2}_{\text{A stuff}}, \underbrace{-16, 4}_{\text{extra B stuff}}\}$

$A \cap B = \{9, \frac{1}{3}\}$

Method to show that $A = B$ when A and B are sets

① Show that $A \subseteq B$

② Show that $B \subseteq A$

Ex: Let C, D, E be sets

Prove that $C \cap (D \cup E) = (C \cap D) \cup (C \cap E)$

proof:

$\subseteq$: Let's show

$C \cap (D \cup E) \subseteq (C \cap D) \cup (C \cap E)$