Math 3450

2/13/24

Ex: $S = \{1, 2, 3\}$

$\sim = \{(1,1), (2,2), (3,3), (1,3), (3,1)\}$

$1 \sim 1$ $2 \sim 2$ $3 \sim 3$ $1 \sim 3$ $3 \sim 1$

Reflexive? [$x \sim x$ for all $x \in S$]

Yes because $1 \sim 1$, $2 \sim 2$, $3 \sim 3$.

Symmetric? [If $x \sim y$, then $y \sim x$]

We have $1 \sim 1$ and also $1 \sim 1$.
We have $2 \sim 2$ and also $2 \sim 2$.
We have $3 \sim 3$ and also $3 \sim 3$.
We have $1 \sim 3$ and also $3 \sim 1$.
We have $3 \sim 1$ and also $1 \sim 3$.

Transitive [If $x \sim y$ and $y \sim z$, then $x \sim z$]

We have $1 \sim 1$ and $1 \sim 3$, and also $1 \sim 3$.
We have $1 \sim 1$ and $1 \sim 1$, and also $1 \sim 1$.
We have $2 \sim 2$ and $2 \sim 2$, and also $2 \sim 2$.
We have $3 \sim 3$ and $3 \sim 3$, and also $3 \sim 3$.

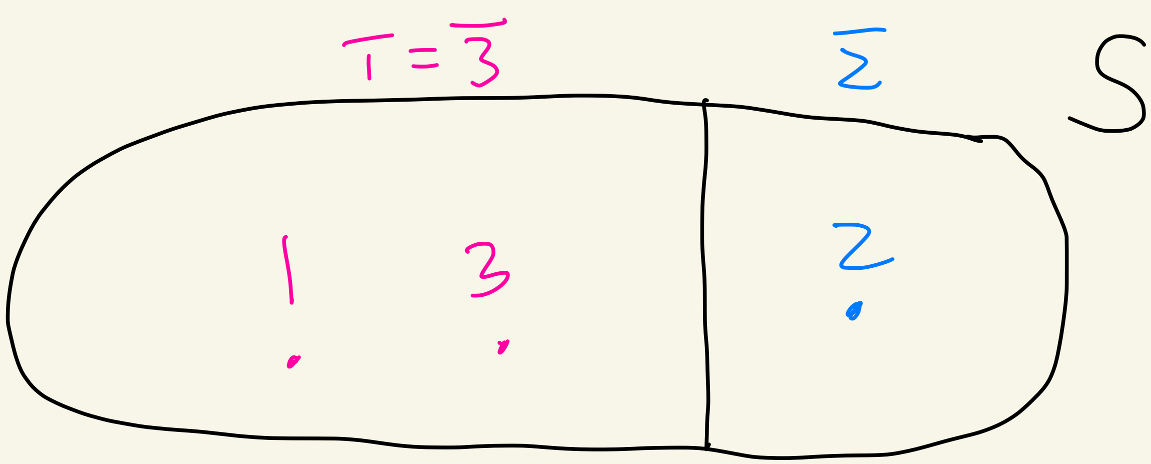
We have $3 \sim 3$ and $3 \sim 1$, and also $3 \sim 1$.
We have $1 \sim 3$ and $3 \sim 3$, and also $1 \sim 3$
We have $1 \sim 3$ and $3 \sim 1$, and also $1 \sim 1$.
We have $3 \sim 1$ and $1 \sim 3$, and also $3 \sim 3$.
We have $3 \sim 1$ and $1 \sim 1$, and also $3 \sim 1$.

Thus, $\sim$ is an equivalence relation on $S = \{1, 2, 3\}$.

Let's find the equivalence classes.

$$\bar{1} = \{x \in S \mid 1 \sim x\} = \{1, 3\}$$
$$\bar{2} = \{x \in S \mid 2 \sim x\} = \{2\}$$
$$\bar{3} = \{x \in S \mid 3 \sim x\} = \{1, 3\}$$

$\bar{1} = \bar{3}$



Super-duper Equivalence relation theorem

Let $\sim$ be an equivalence relation on a set S.
Let $x, y \in S$.
Then:

1. $x \in \overline{x}$
2. $\overline{x} = \overline{y}$ iff $x \in \overline{y}$
3. $\overline{x} = \overline{y}$ iff $x \sim y$
4. $\overline{x} \cap \overline{y} = \emptyset$ iff $x \nsim y$

Ex from above

1 $2 \in \overline{2}$
$\overline{2} = \{2\}$

2 / 3
$\overline{1} = \{1,3\} = \overline{3}$
$1 \in \overline{3}$
$3 \in \overline{1}$
$1 \sim 3,\ 3 \sim 1$

4
$\overline{1} = \{1,3\}$
$\overline{2} = \{2\}$
$\overline{1} \cap \overline{2} = \emptyset$
$1 \nsim 2$

proof:

1 We know $\overline{x} = \{y \in S \mid x \sim y\}$
Since $\sim$ is reflexive, $x \sim x$.
So, $x \in \overline{x}$.

1

2 ($\Rightarrow$) Suppose $\overline{x} = \overline{y}$.
By 1, $x \in \overline{x}$.

Thus, since $x \in \bar{x}$ and $\bar{x} = \bar{y}$ we get $x \in \bar{y}$.

($\Leftarrow$) Now suppose $x \in \bar{y}$.

Why is $\bar{x} = \bar{y}$?

$\bar{x} = \{ b \in S \mid x \sim b \}$

Claim 1: $\bar{x} \subseteq \bar{y}$

pf of claim 1: Let $z \in \bar{x}$

$\bar{y} = \{ b \in S \mid y \sim b \}$

Thus, $x \sim z$.

Also, since $x \in \bar{y}$ we know $y \sim x$.

Since $y \sim x$ and $x \sim z$, then by transitivity we get $y \sim z$.

Thus, $z \in \bar{y}$.

claim 1

Claim 2: $\bar{y} \subseteq \bar{x}$

pf of claim 2: Let $z \in \bar{y}$.

Then, $y \sim z$.

Since $x \in \bar{y}$ we know $y \sim x$.

Since $y \sim x$, by reflexivity

we get $x \sim y$.

Since $x \sim y$ and $y \sim z$, by transitivity we get $x \sim z$.

Since $x \sim z$ we know $z \in \overline{x}$

$\boxed{\text{Claim 2}}$

By claim 1 and claim 2 we get $\overline{x} = \overline{y}$. $\boxed{2}$

3

($\Rightarrow$) Suppose $\overline{x} = \overline{y}$.

Then by 2 we get $x \in \overline{y}$.

Thus, $y \sim x$.

By symmetry we get $x \sim y$.

($\Leftarrow$) Suppose $x \sim y$.

By def, get $y \in \overline{x}$

By 2 we get $\overline{y} = \overline{x}$.

3

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4 Instead of proving

"$\overline{x} \cap \overline{y} = \phi$ iff $x \nsim y$"

let's prove the contrapositive

"$\overline{x} \cap \overline{y} \neq \phi$ iff $x \sim y$"

P iff Q is equivalent to ¬P iff ¬Q

<u>pf:</u>

($\Rightarrow$) Suppose $\overline{x} \cap \overline{y} \neq \phi$.

Then there exists $z \in \overline{x} \cap \overline{y}$.

P	Q	¬P	¬Q	P iff Q	¬P iff ¬Q
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

So, $z \in \overline{x}$ and $z \in \overline{y}$.

Then, $x \sim z$ and $y \sim z$.

By symmetry we get $z \sim y$.

Thus, since $x \sim z$ and $z \sim y$,
by transitivity we get $x \sim y$.

($\Leftarrow$) Suppose $x \sim y$.

Then by 3, we get $\overline{x} = \overline{y}$.

By 1, $x \in \overline{x}$.

So, since $\overline{x} = \overline{y}$ and $x \in \overline{x}$
we have $x \in \underbrace{\overline{x} \cap \overline{y}}_{\text{this is just } \overline{x}}$.

So, $\overline{x} \cap \overline{y} \neq \phi$.

$\boxed{4}$

Workshop

HW 2 | 9 (c)

Calculate $\bigcup_{n=3}^{\infty} A_n$ and $\bigcap_{n=3}^{\infty} A_n$

where $A_n = \left(2+\frac{1}{n}, n\right)$ interval in $\mathbb{R}$

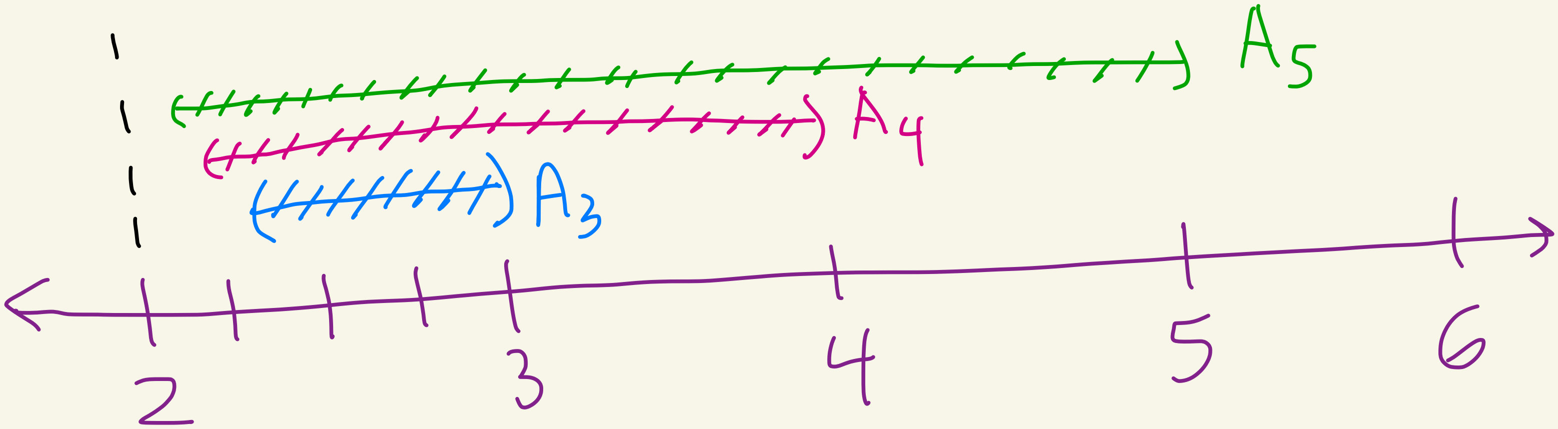
$A_3 = \left(2+\frac{1}{3}, 3\right)$

$A_4 = \left(2+\frac{1}{4}, 4\right)$

$A_5 = \left(2+\frac{1}{5}, 5\right)$

$\bigcup_{n=3}^{\infty} A_n = (2, \infty)$

$\bigcap_{n=3}^{\infty} A_n = A_3 = \left(\frac{7}{3}, 3\right)$



$\boxed{\text{Hw 3}}$

1 (a) $\boxed{\begin{array}{l} S = \mathbb{R} \\ a \sim b \text{ means } a \le b \end{array}}$

<u>Ex:</u> $1 \sim 2$ since $1 \le 2$

$2 \not\sim -1$ since $2 \not\le -1$

$\boxed{\text{Reflexive?}}$ Let $x \in \mathbb{R}$

Then, $x \le x$

So, $x \sim x$.

$\boxed{\text{Symmetric?}}$ We know $1 \sim 2$ since $1 \le 2$.

But $2 \not\sim 1$ since $2 \not\le 1$.

So, $\sim$ is not symmetric

$\boxed{\text{Transitive:}}$ Let $x, y, z \in \mathbb{R}$.

Suppose $x \sim y$ and $y \sim z$.

Then, $x \le y$ and $y \le z$.

This implies $x \le z$.

So, $x \sim z$.

$\boxed{\text{HW 2}}$

6 Show $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{4, 5\}$

$B \cap C = \{4\}$

$A \times (B \cap C) = \{(1,4), (2,4)\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$A \times C = \{(1,4), (1,5), (2,4), (2,5)\}$

$(A \times B) \cap (A \times C) = \{(1,4), (2,4)\}$

pf:

($\Rightarrow$) Let $y \in A \times (B \cap C)$.

So, $y = (m, n)$ where

$m \in A$ and $n \in B \cap C$.

So, $m \in A$ and $n \in B$ and $n \in C$.

Thus, $(m, n) \in A \times B$

and $(m, n) \in A \times C$.