Math 3450

4-9-24

Last time

$f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$

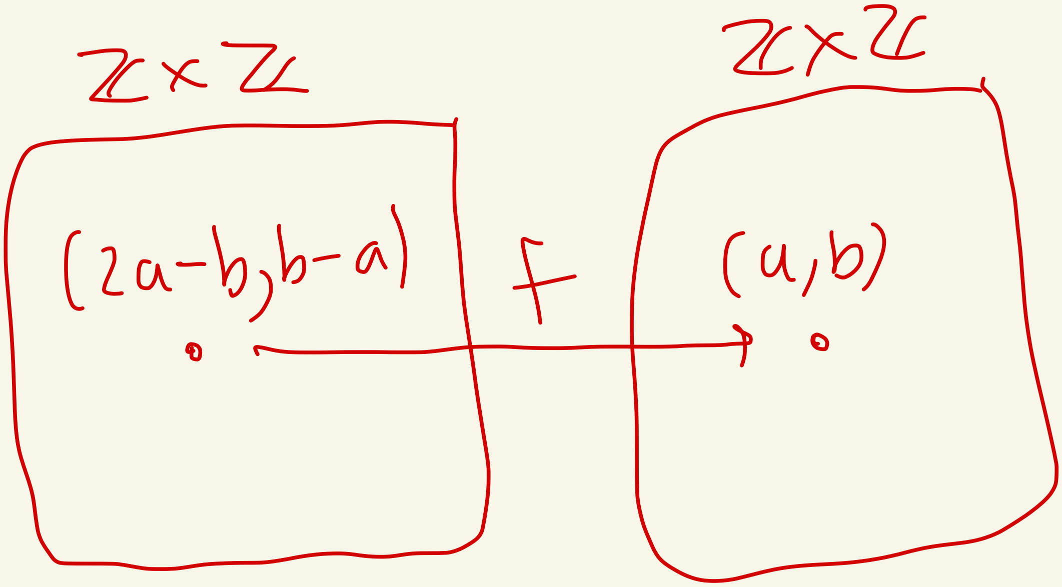
$f(m,n) = (m+n, m+2n)$

We showed

- f is 1-1
- $\boxed{f \text{ is onto}}$

Given $(a,b) \in \mathbb{Z} \times \mathbb{Z}$,

then $f(2a-b, b-a) = (a,b)$



From above we have that f is 1-1. Thus, f^{-1} exists.

And $\text{domain}(f^{-1}) = \text{range}(f) = \mathbb{Z} \times \mathbb{Z}$

f is onto

Claim: Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be defined by
$g(a,b) = (2a-b, b-a)$.
Then, $g = f^{-1}$.

Let's use thm from last time:
$f : A \to B$, f is 1-1, $C = \text{range}(f)$

$$\boxed{\begin{array}{l}\text{If } g: C \to A \text{ and } g \circ f = i_A \\ \text{then } g = f^{-1}\end{array}}$$

<u>proof of claim:</u> We have

$$(g \circ f)(m,n) = g(f(m,n))$$

$$= g(m+n,\, m+2n)$$

$$= \big(2(m+n) - (m+2n),\, (m+2n) - (m+n)\big)$$

$$= (m,n)$$

$$= i_{\mathbb{Z}\times\mathbb{Z}}(m,n).$$

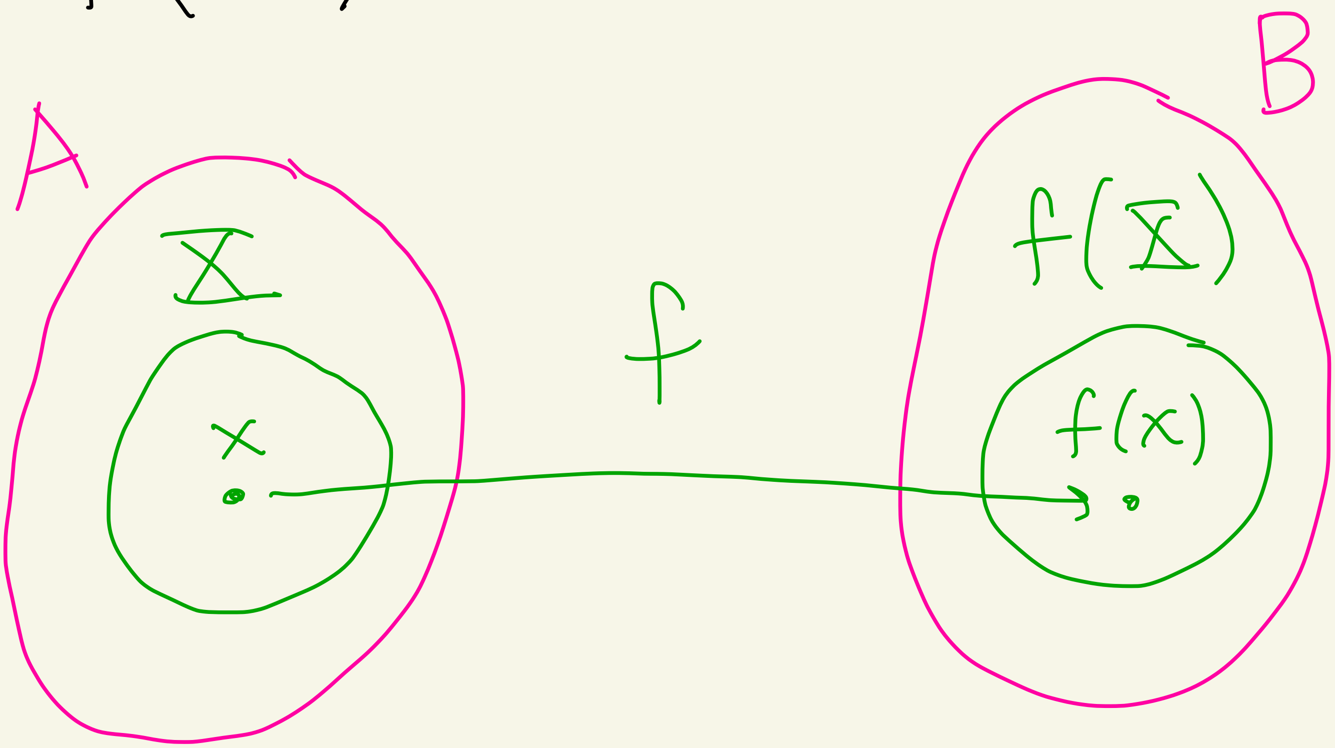
Since $g \circ f = i_{\mathbb{Z}\times\mathbb{Z}}$ we

have $g = f^{-1}$! $\boxed{\text{Claim}}$

Def: Let A and B be sets. Let $f: A \to B$.

① Let $X \subseteq A$.
The <u>image of X under f</u> is

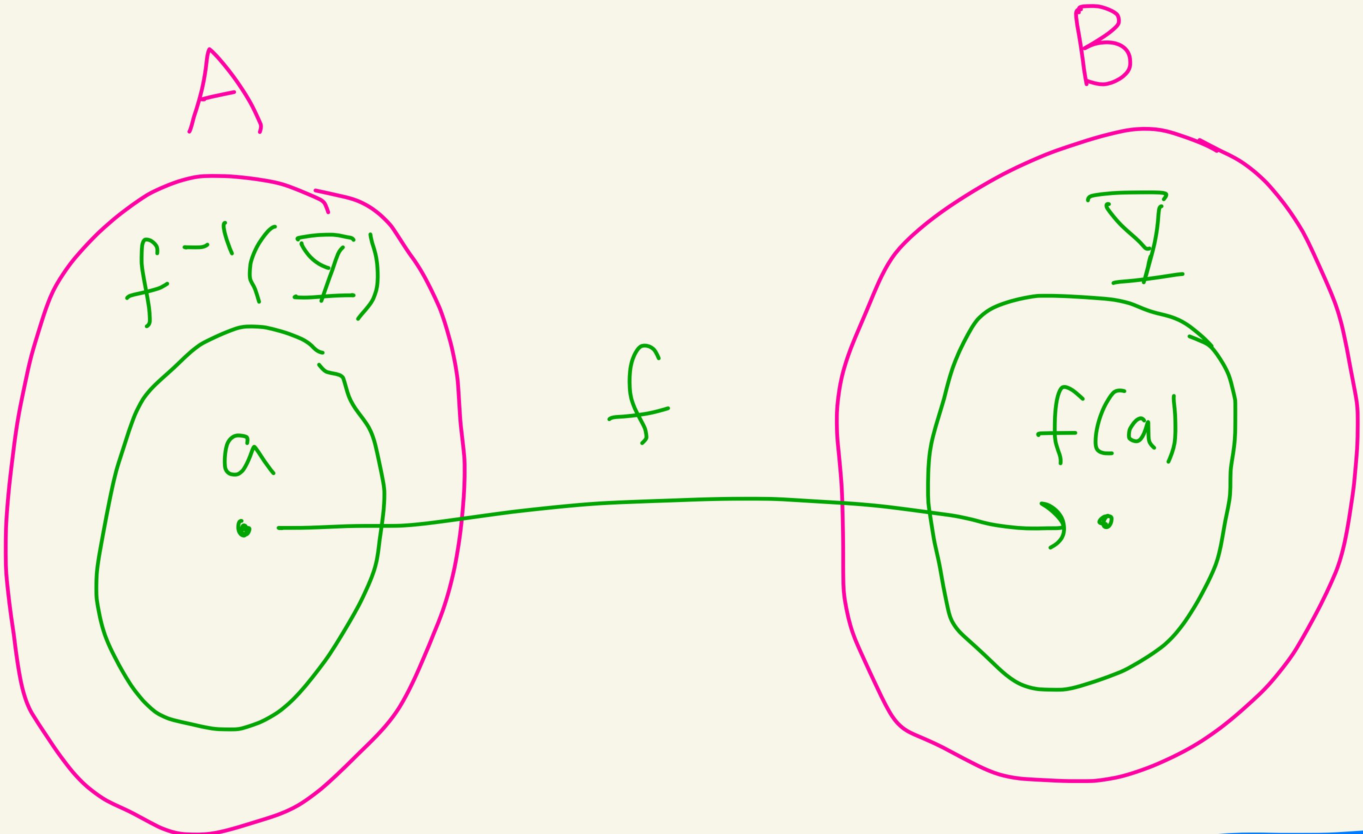
$$f(X) = \{ f(x) \mid x \in X \}$$



② Let $Y \subseteq B$.
The <u>inverse image of Y under f</u>

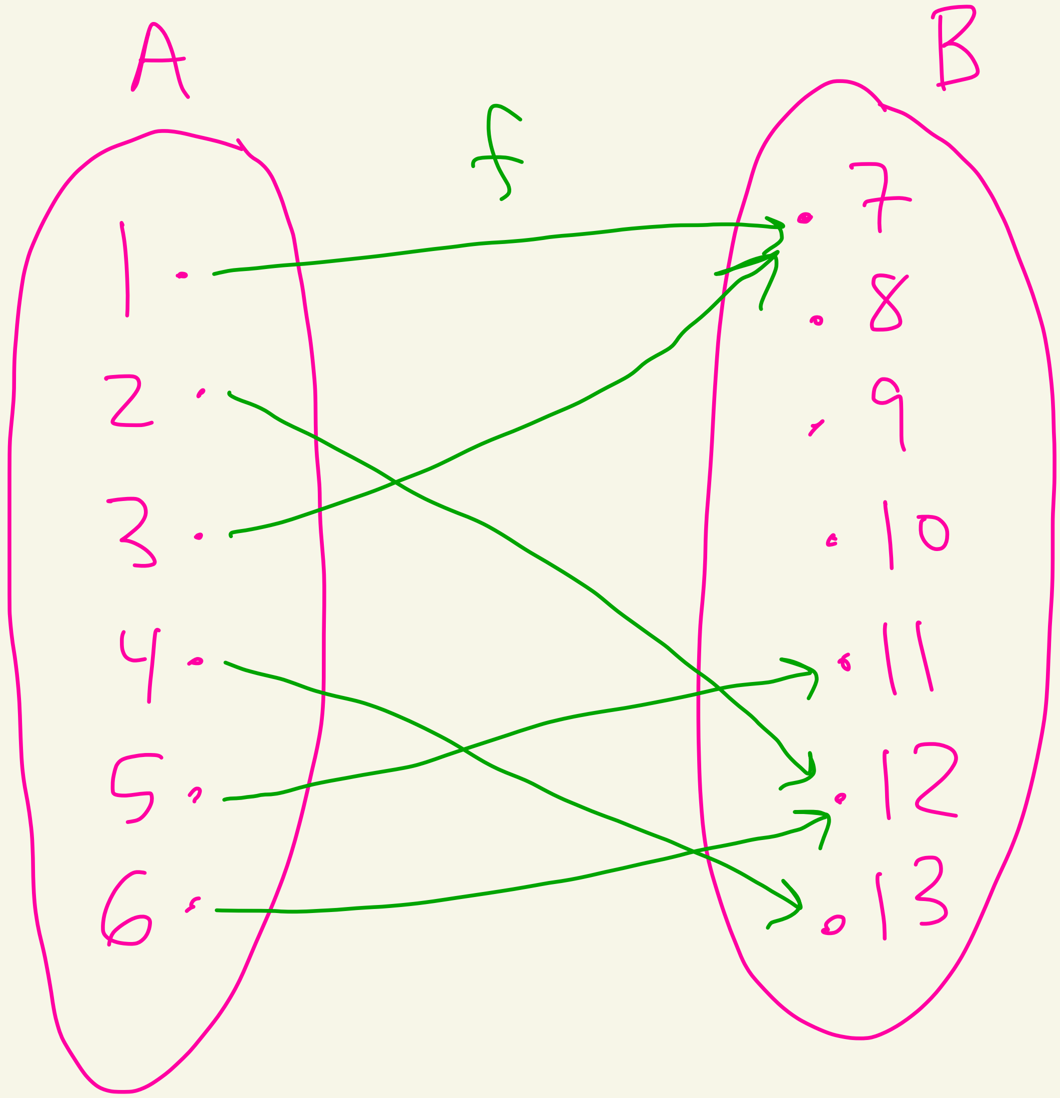
is the set

$$f^{-1}(Y) = \{a \in A \mid f(a) \in Y\}$$



<u>Note:</u> We use f^{-1} notation, but it doesn't necessarily mean inverse function because f^{-1} might not exist

Ex: Consider the following function.



$$\boxed{\begin{aligned} f(1)&=7\\ f(2)&=12\\ f(3)&=7\\ f(4)&=13\\ f(5)&=11\\ f(6)&=12 \end{aligned}}$$

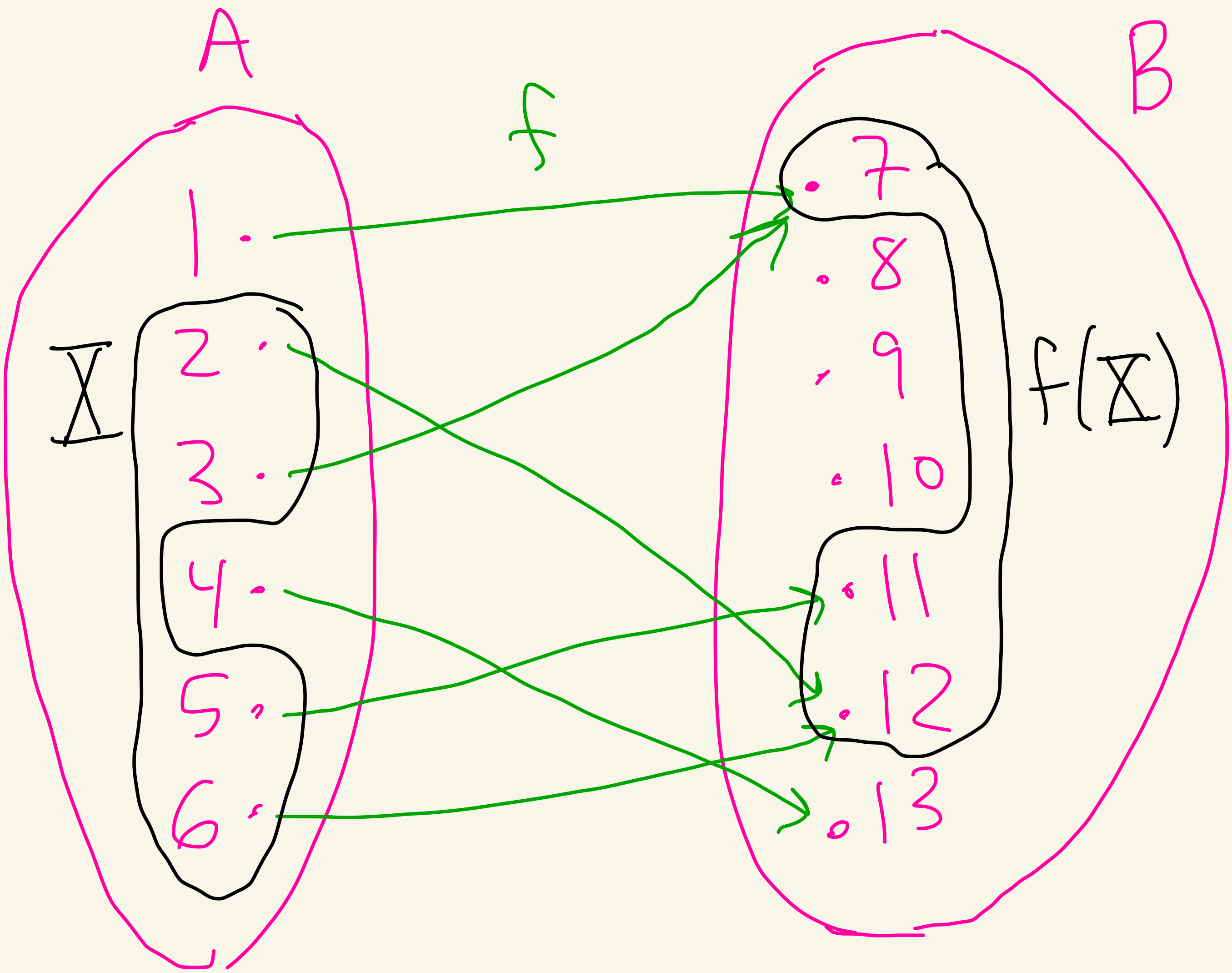
Let $X = \{2, 3, 5, 6\}$

Then,

$$f(X) = \{f(2), f(3), f(5), f(6)\}$$

$$= \{12, 7, 11, 12\}$$

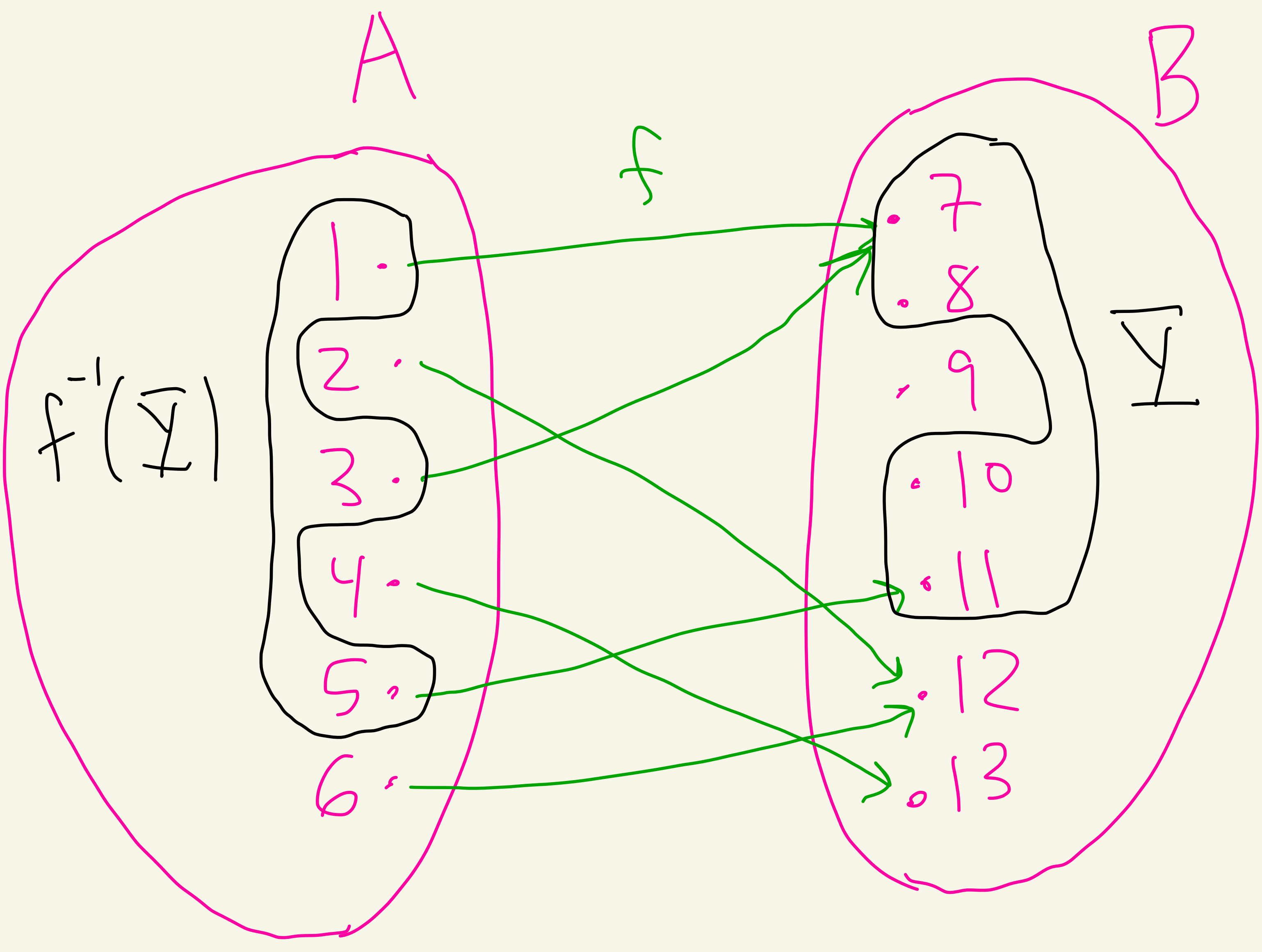
$$= \{7, 11, 12\}$$



Let $Y = \{7, 8, 10, 11\}$

Then,

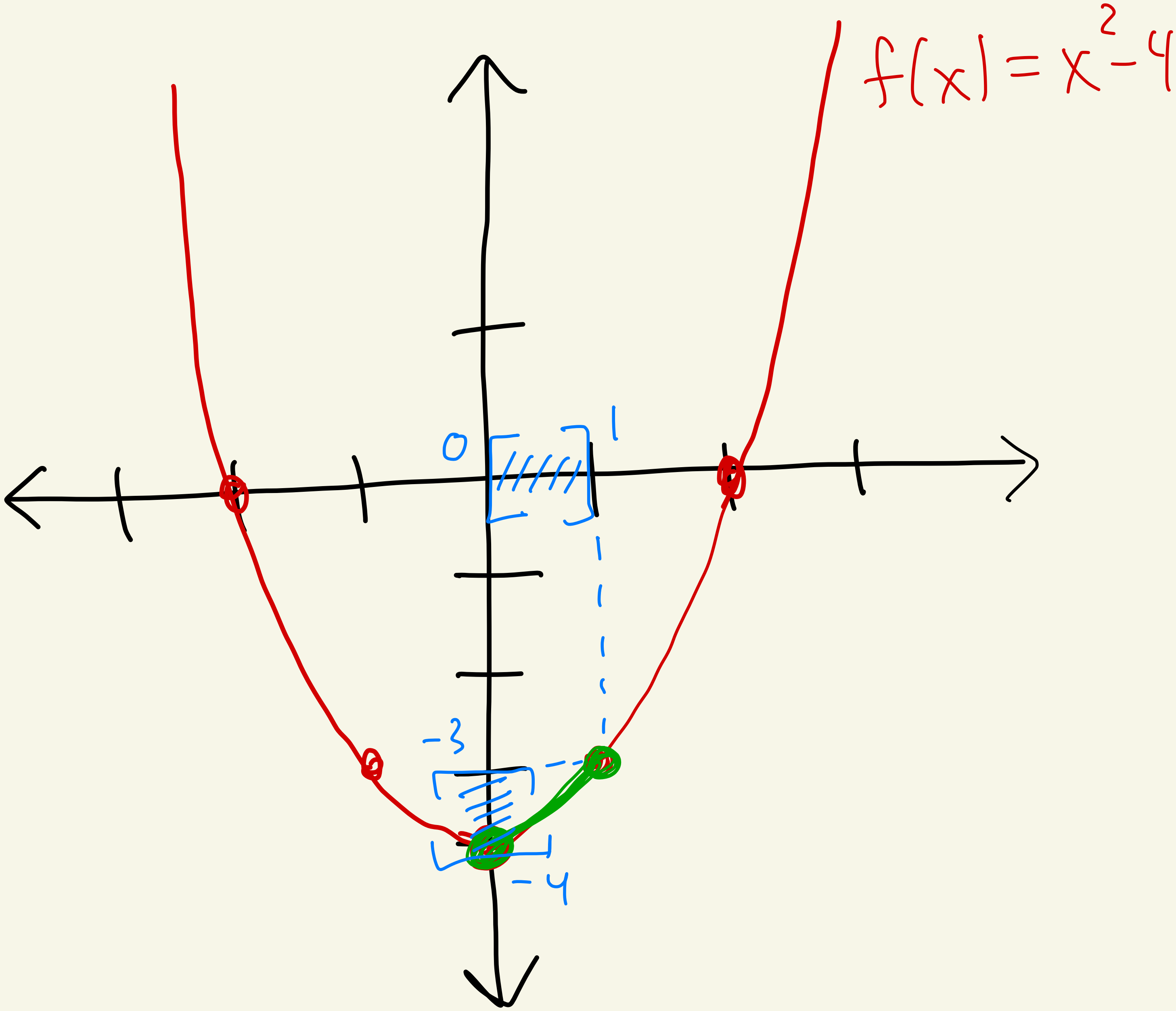
$$f^{-1}(Y) = \{1, 3, 5\}$$



HW 4 problem modified

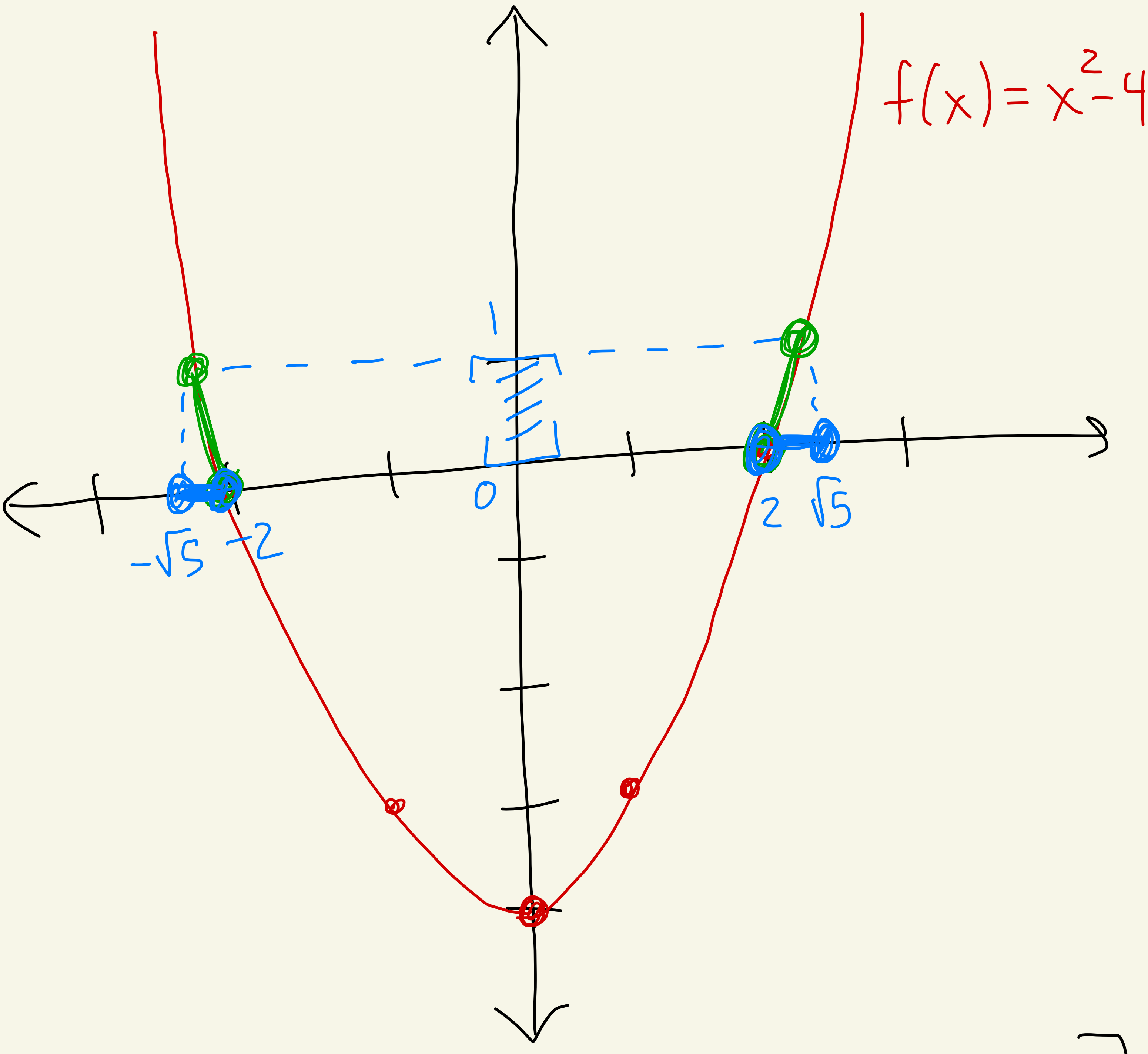
Let $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2 - 4$

(a) Calculate $f([0,1])$



$f([0,1]) = [-4, -3]$

(b) Calculate $f^{-1}([0,1])$

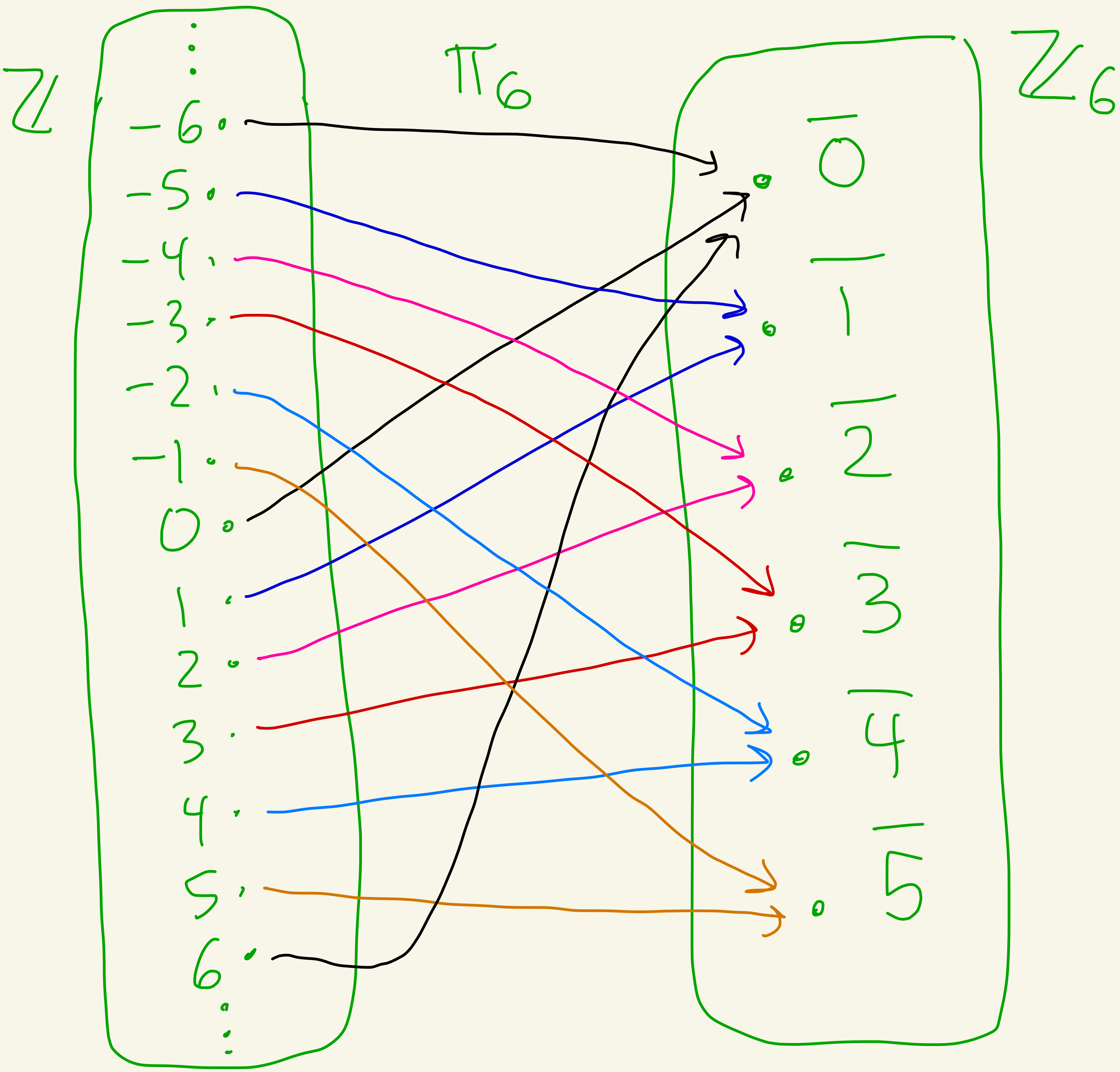


$$f^{-1}([0,1]) = [-\sqrt{5},-2] \cup [2,\sqrt{5}]$$

$\boxed{\text{HW 4 \#11}}$

$\pi_6 : \mathbb{Z} \to \mathbb{Z}_6$ where $\pi_6(x) = \overline{x}$

(a) Draw a picture of π_6



(b) Let $\mathbb{X} = \{1, 3, -5, 10, 102\}$

Then,

$$\pi_6(\mathbb{X}) = \{\pi_6(1), \pi_6(3), \pi_6(-5), \pi_6(10), \pi_6(102)\}$$

$$= \{\overline{1}, \overline{3}, \overline{-5}, \overline{10}, \overline{102}\}$$

$$= \{\overline{1}, \overline{3}, \overline{1}, \overline{4}, \overline{0}\}$$

$$\begin{array}{r} 17 \\ 6\overline{)102} \\ -6 \\ \hline 42 \\ -42 \\ \hline 0 \end{array}$$

$$= \{\overline{0}, \overline{1}, \overline{3}, \overline{4}\}$$

(c) Next time let's calculate $\pi_6^{-1}(\mathbb{Y})$ where $\mathbb{Y} = \{\overline{1}\}$.