Math 4300
10/30/23

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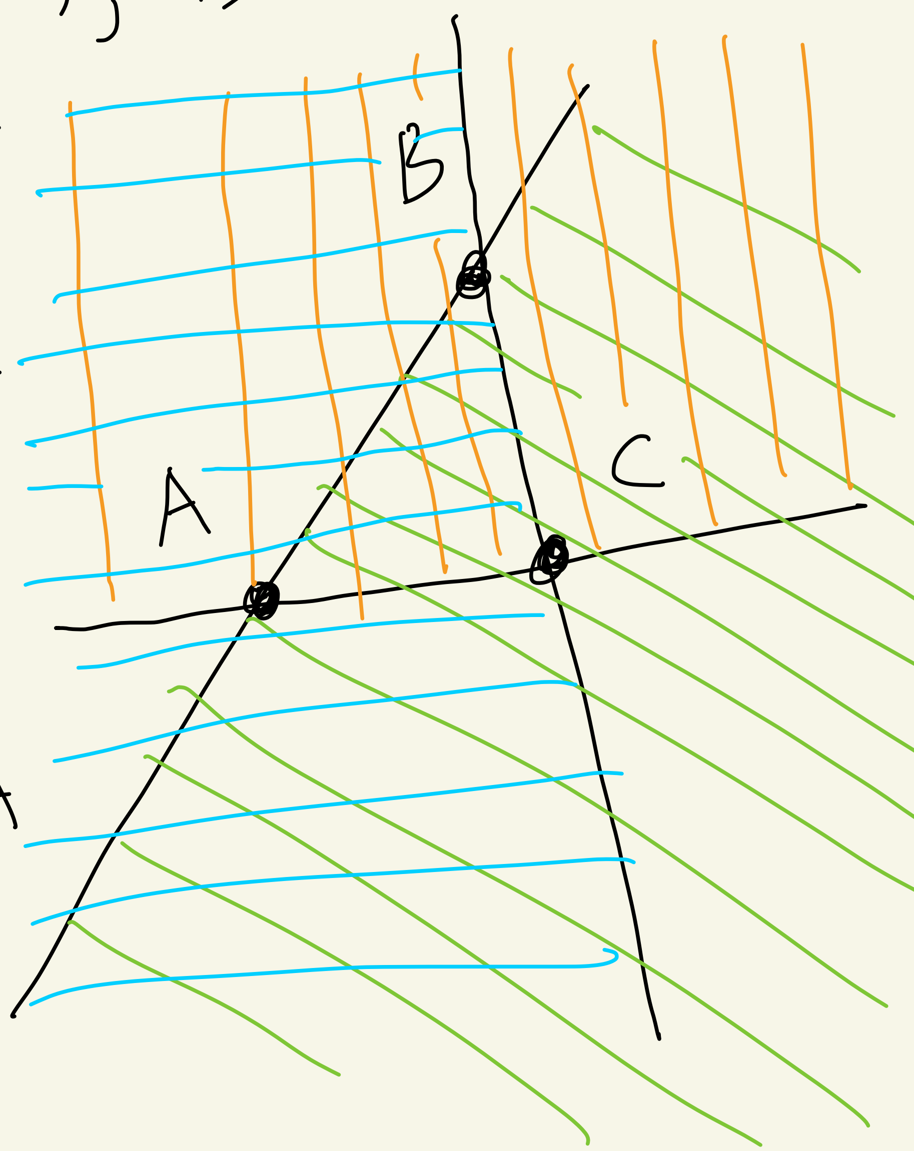
(Topic 9 continued...)

Def: Let $(\mathscr{P}, \mathscr{L}, d)$ be Pasch geometry. Let A, B, C be noncollinear points. The <u>interior</u> of $\triangle ABC$, written as $\mathrm{int}(\triangle ABC)$, is the intersection of three sets:

- the side of $\overleftrightarrow{AB}$ that contains C
- the side of $\overleftrightarrow{AC}$ that contains B
- the side of $\overleftrightarrow{BC}$ that contains A



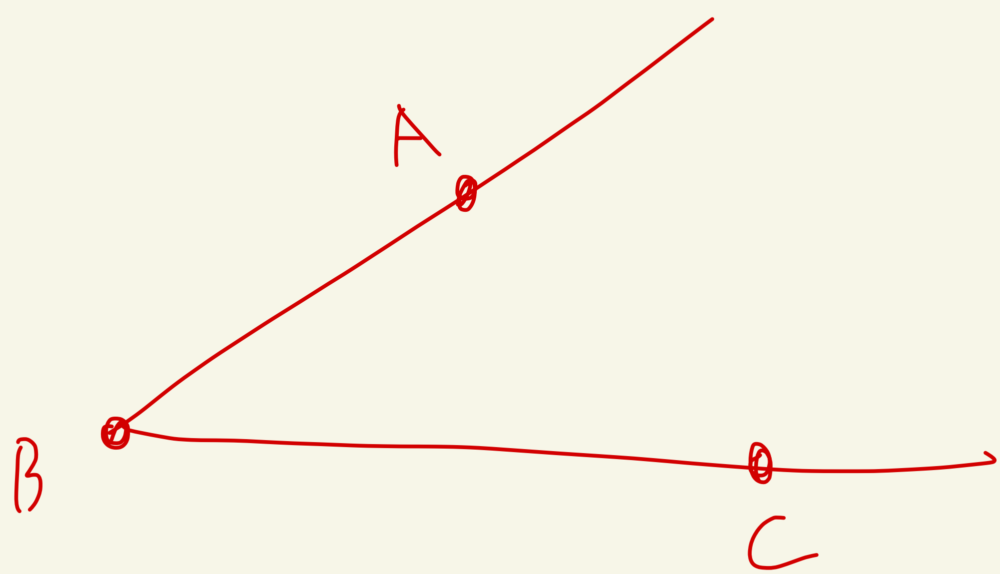
Theorem: int$(\triangle ABC)$ is convex

proof: HW. $\blacksquare$

Topic 10 - Angle Measure

Recall if A, B, C are noncollinear then

$$\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC}$$



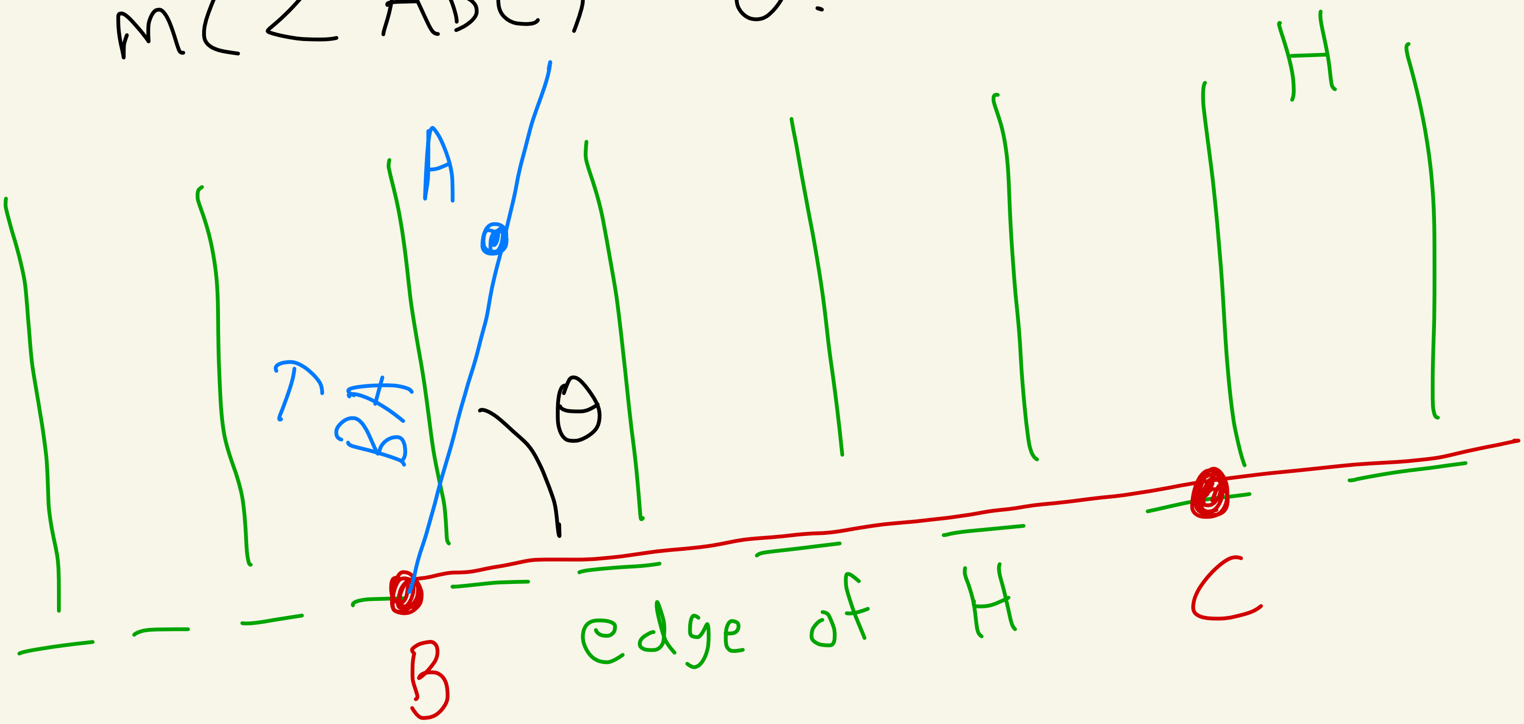
Def: Let $(\mathcal{P}, \mathcal{L}, d)$ be a Pasch geometry. Let $r_0 > 0$ be a real number.

Let $m : \mathcal{A} \to \mathbb{R}$ where $\mathcal{A}$ is the set of all angles.

We say that m is an angle measure or protractor based on r_0 if the following three conditions are true:

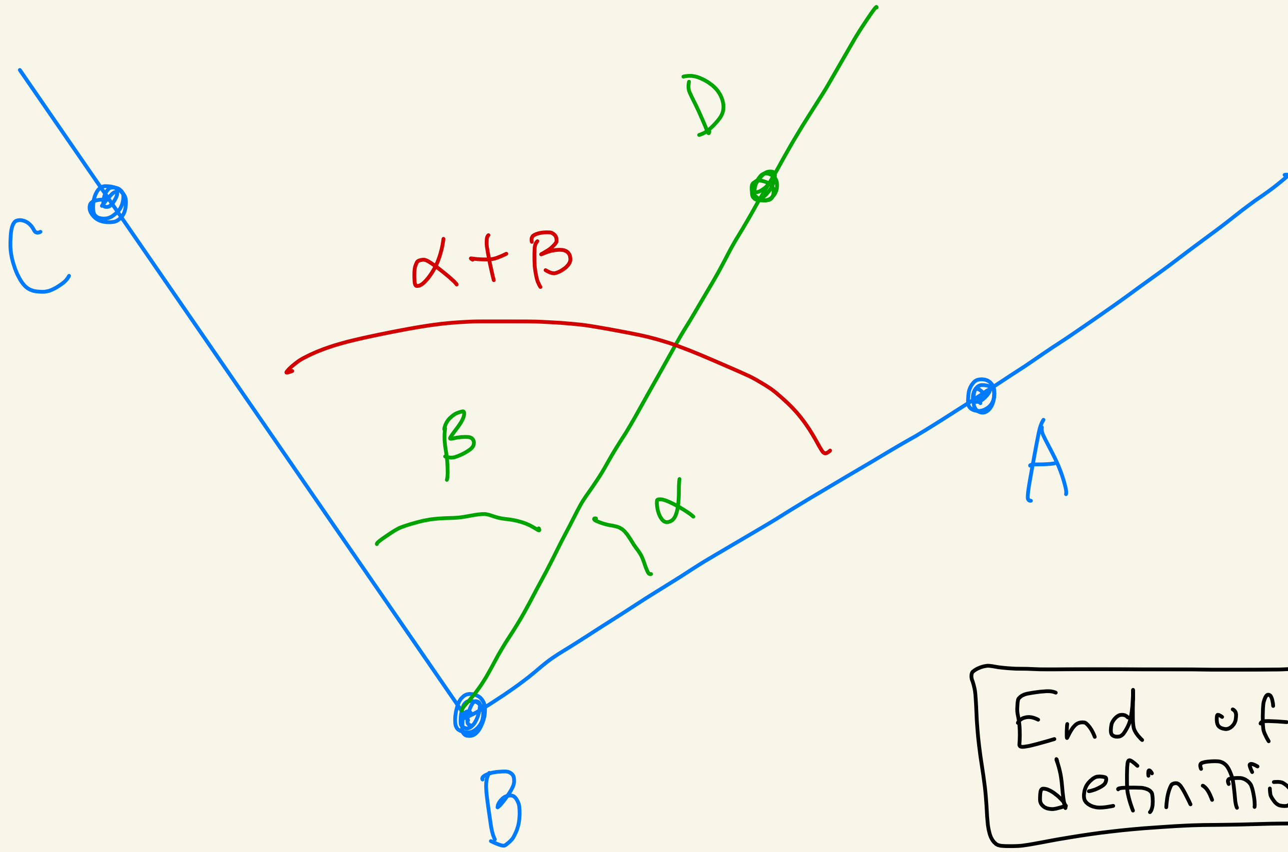
(i) If $\angle ABC$ is an angle, then $0 < m(\angle ABC) < r_0$

(ii) If $\overrightarrow{BC}$ lies on the edge of a halfplane H and θ is a real number with $0 < \theta < r_0$, then there exists a unique ray $\overrightarrow{BA}$ where $A \in H$ and $m(\angle ABC) = \theta$.



(iii) If $D \in \text{int}(\angle ABC)$, then

$$\underbrace{m(\angle ABD)}_{\alpha} + \underbrace{m(\angle DBC)}_{\beta} = \underbrace{m(\angle ABC)}_{\alpha+\beta}$$



End of definition

Note: If $r_0 = \pi$, then m is called a radian measure. If $r_0 = 180$, then m is called a degree measure.

In this class we will assume $r_0 = 180$ from this point forward.

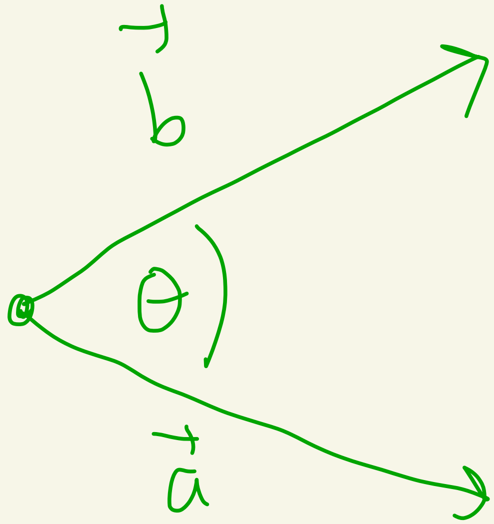
<u>Def:</u> A <u>proctractor geometry</u> $(\mathcal{P}, \mathcal{L}, d, m)$ is a Pasch geometry $(\mathcal{P}, \mathcal{L}, d)$ together with an angle measure m.

<u>Note:</u> We want to create an angle measure on the Euclidean plane. But to do so we need the inverse cosine function. But the cosine function is usually defined in terms of angles. In Millman/Parker book in section 5.4 they show how to construct the inverse

cosine function without angles. They use an integral. Then they prove all the properties that cosine and inverse cosine have.

We will just assume cosine and inverse cosine exist and satisfy their usual properties.

Recall from Calculus:



$$\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}\right)$$

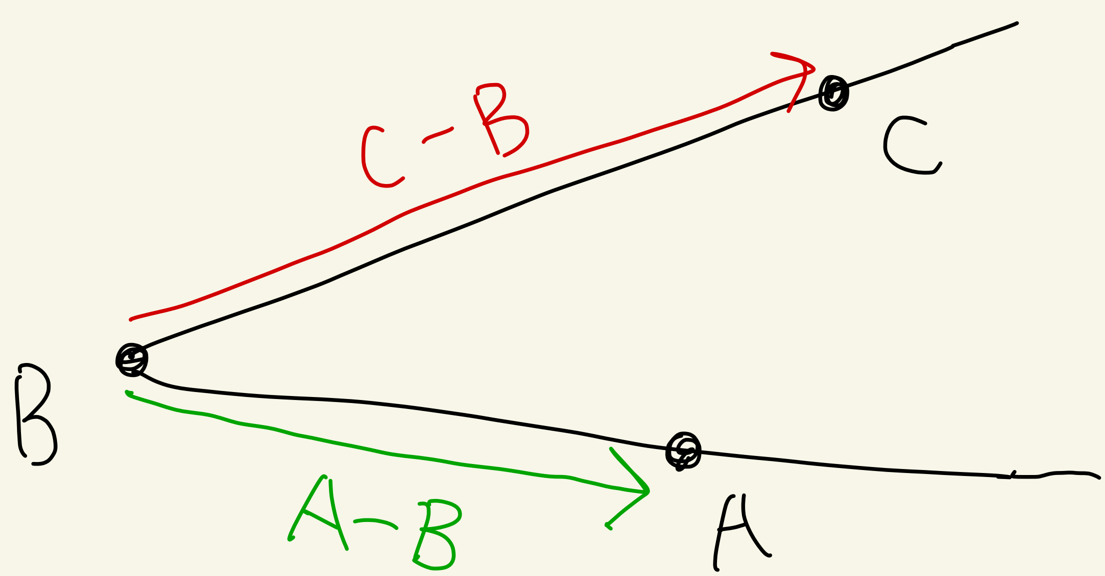
This gives us an idea to define angle measure

<u>Def:</u> Consider the Euclidean plane $\mathcal{E} = (\mathbb{R}^2, \mathcal{L}_E, d_E)$.

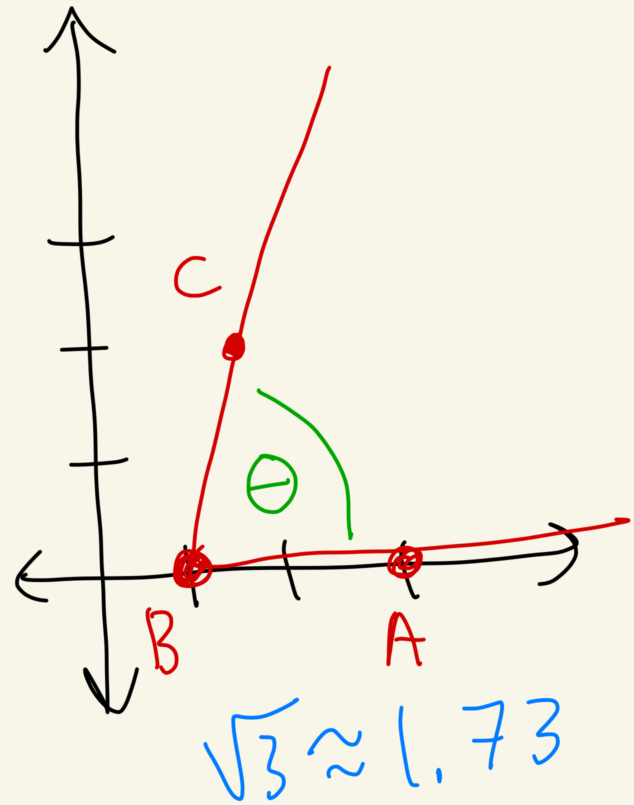
Let $\angle ABC$ be an angle in $\mathcal{E}$.

Define the <u>Euclidean angle measure</u> m_E as follows:

$$m_E(\angle ABC) = \cos^{-1}\left(\frac{\langle A-B, C-B\rangle}{\|A-B\| \cdot \|C-B\|}\right)$$



Ex: In $\mathcal{E}$, measure $m(\angle ABC)$ where $A=(0,3)$, $B=(0,1)$, $C=(\sqrt{3},2)$.



$$m(\angle ABC)$$

$$= \cos^{-1}\left(\frac{\langle A-B, C-B\rangle}{\|A-B\|\,\|C-B\|}\right)$$

$$= \cos^{-1}\left(\frac{\langle (0,2), (\sqrt{3},1)\rangle}{\|(0,2)\|\,\|(\sqrt{3},1)\|}\right)$$

$$= \cos^{-1}\left(\frac{0+2}{\sqrt{0^2+2^2}\sqrt{\sqrt{3}^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Theorem: m_E is an angle measure in the Euclidean plane. Thus, $\mathcal{E} = (\mathbb{R}^2, \mathcal{L}_E, d_E, m_E)$ is a protractor geometry.

Proof: Millman/Parker 5.4 $\boxtimes$