Math 4300

11/29/23

HW 6

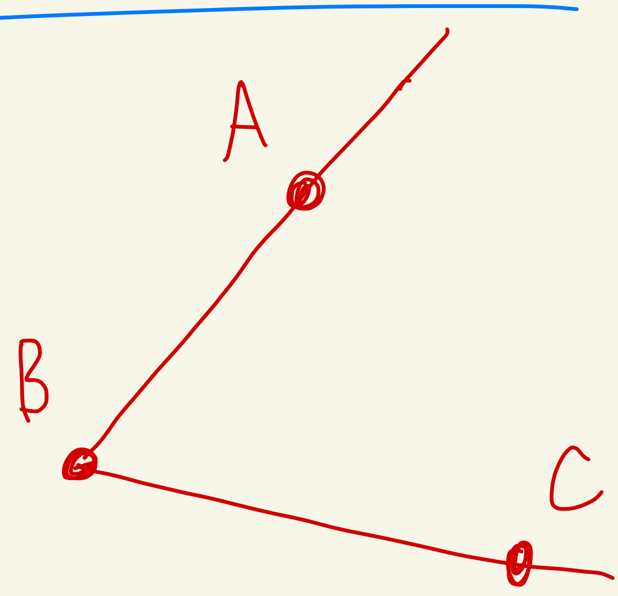
(3) Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A, B, C be three non-collinear points.

(a) Prove $\angle ABC = \angle CBA$

(b) Prove

$$\triangle ABC = \triangle ACB = \triangle BAC = \triangle BCA = \triangle CBA = \triangle CAB$$

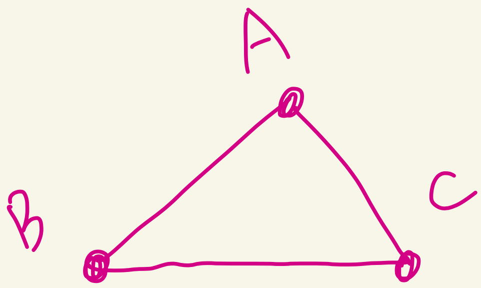
(a) We have



$$\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC} = \overrightarrow{BC} \cup \overrightarrow{BA} = \angle CBA$$

(b) $\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{CA}$

$\overline{XY} = \overline{YX}$



$= \overline{BA} \cup \overline{CB} \cup \overline{AC}$

$= \overline{AC} \cup \overline{CB} \cup \overline{BA}$

$= \triangle ACB$

<u>Lemma:</u> If $X \neq Y$, then $\overline{XY} = \overline{YX}$

<u>pf:</u> We have

$\overline{XY} = \{X, Y\} \cup \{Z \mid \text{where } X - Z - Y\}$

$= \{Y, X\} \cup \{Z \mid \text{where } Y - Z - X\}$

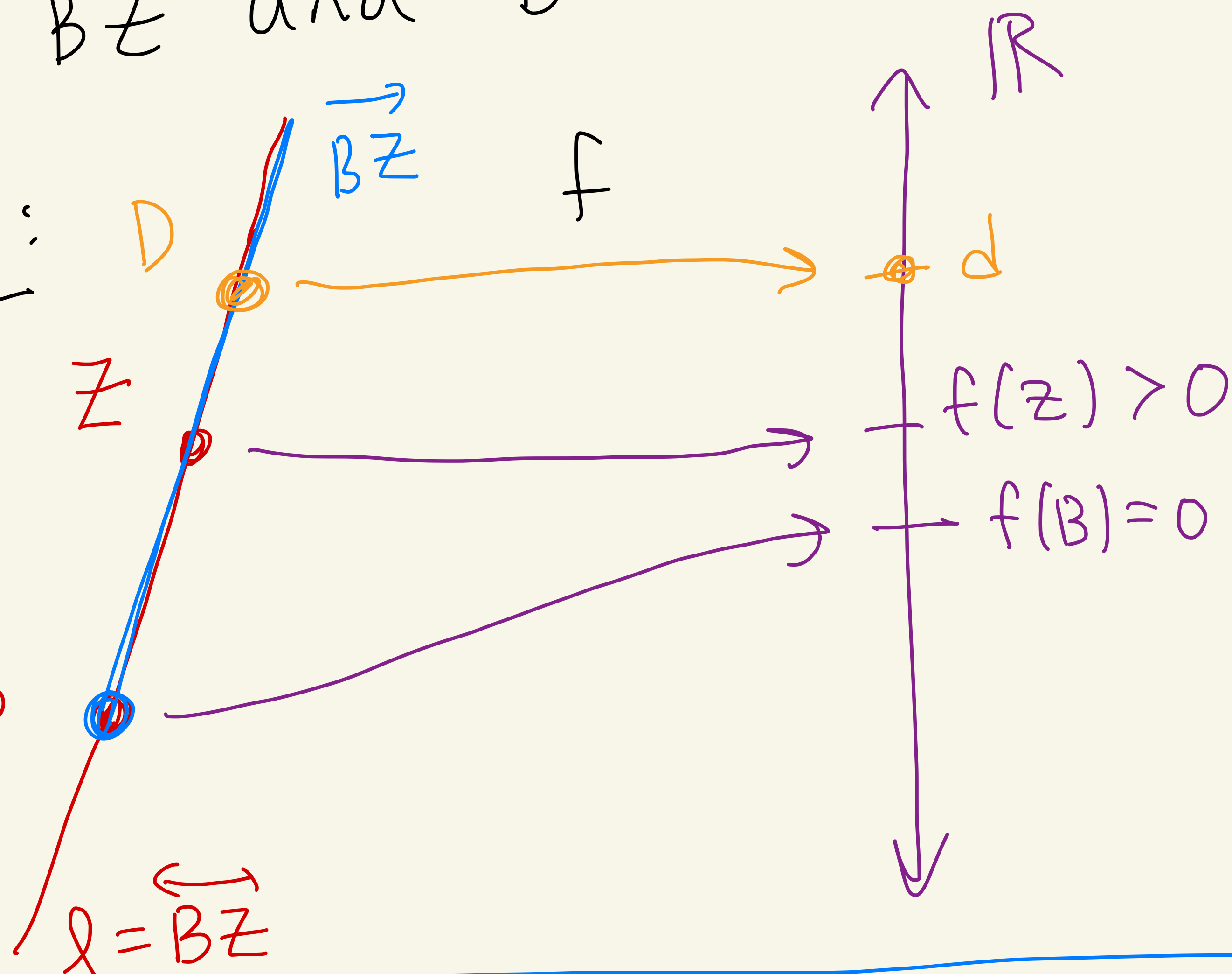
$= \overline{YX}$

$\blacksquare$

$\boxed{\text{HW 6}}$

④ Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let B, Z be points with $B \neq Z$. Prove there exists a point D such that $D \in \overrightarrow{BZ}$ and $B-Z-D$.

<u>proof:</u>



Recall: $\overrightarrow{BZ} = \overline{BZ} \cup \{C \mid B-Z-C\}$

$= \{B, Z\} \cup \{C \mid B-C-Z\}$
$\cup \{C \mid B-Z-C\}$

<u>proof:</u> Let $f : \overleftrightarrow{BZ} \to \mathbb{R}$ be a ruler where $f(B)=0$ and $f(Z)>0$.

Pick $d \in \mathbb{R}$ with $d > f(Z)$.

Since f is onto there exists $D \in \overleftrightarrow{BZ}$ where $f(D)=d$.

Then, $f(B) < f(Z) < f(D)$

$[\,0 < f(Z) < d\,]$

So, $B-Z-D$.

This also implies $D \in \overrightarrow{BZ}$. ∎

$\boxed{\text{HW 7}}$

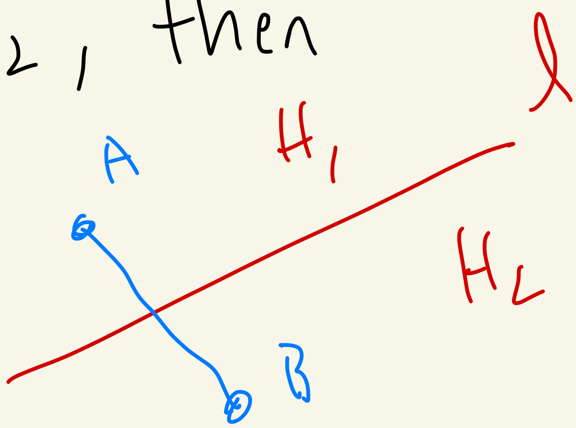
⑥ Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry satisfying the PSA. Let ℓ be a line from $\mathscr{L}$. Let $P, Q \in \mathscr{P}$ with $P \neq Q$ and $P \notin \ell$ and $Q \notin \ell$.

(a) Prove: P, Q are on opposite sides of ℓ iff $\overline{PQ} \cap \ell \neq \phi$

(b) Prove: P, Q are on the same side of ℓ iff $\overline{PQ} \cap \ell = \phi$

$\boxed{\text{proof:}}$ Since our geometry satisfies the PSA, there exist two half planes H_1 and H_2 where

- $\mathcal{P} - \ell = H_1 \cup H_2$
- $H_1 \cap H_2 = \phi$
- H_1 is convex and H_2 is convex
- If $A \in H_1$ and $B \in H_2$, then $\overline{AB} \cap \ell \neq \phi$.



(a)

($\Rightarrow$) Suppose P, Q are on opposite sides of ℓ. Then either

$P \in H_1$ and $Q \in H_2$

or $P \in H_2$ and $Q \in H_1$.

If $P \in H_1$ and $Q \in H_2$ by the 4th property of PSA we get $\overline{PQ} \cap \ell \neq \phi$.

Same thing for $P \in H_2$ and $Q \in H_1$.

($\Leftarrow$) Suppose $\overline{PQ} \cap \ell \neq \phi$.

Why are P, Q on opposite sides of ℓ?

What if P, Q were on the same side of ℓ?

Suppose $P, Q \in H_1$.

Then since H_1 is convex we would get that $\overline{PQ} \subseteq H_1$.

Then $\overline{PQ} \cap \ell = \phi$ because $H_1 \cap \ell = \phi$ by the 1st property of PSA.

Same idea if $P, Q \in H_2$.

Thus P, Q are on opposite sides of ℓ.

$\boxed{(a)}$