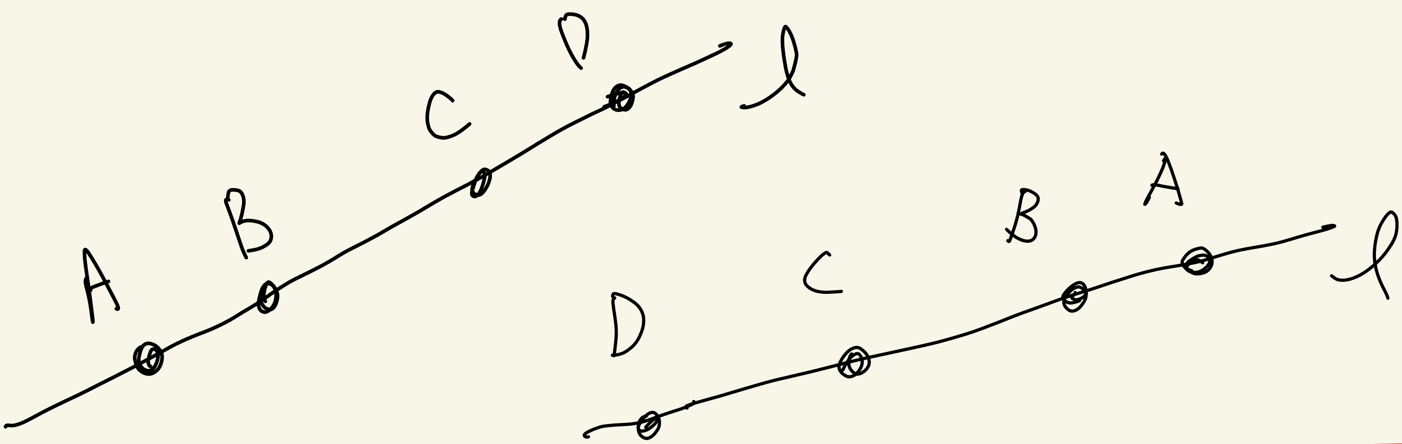
Math 4300

9/27/23

Last thing from topic 4:

Def: $A-B-C-D$ means:

$A-B-C$, $A-B-D$, $A-C-D$, and $B-C-D$



Say A, B, C, D lie on ℓ and $f: \ell \to \mathbb{R}$ is a ruler. Then the above says:

$$f(A) < f(B) < f(C) < f(D)$$

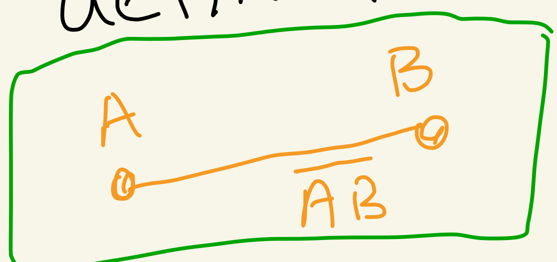
or

$$f(D) < f(C) < f(B) < f(A)$$

See HW 4 problem 7 for a simplification of the above definition

Topic 5 – Line Segment and rays

<u>Def:</u> Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A and B be distinct points from $\mathscr{P}$. The <u>line segment from A to B</u> is defined to be the set



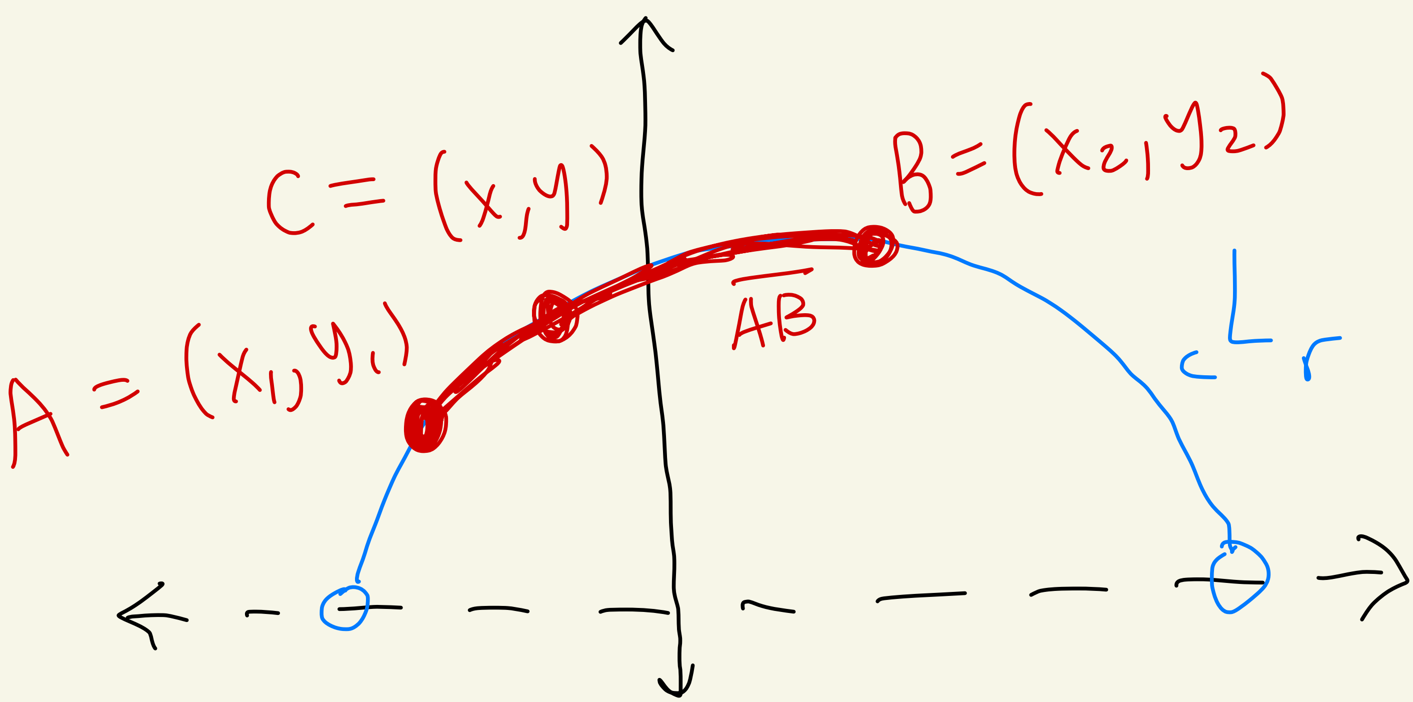
$$\overline{AB} = \{A\} \cup \{B\} \cup \{C \in \mathscr{P} \mid A-C-B\}$$

$$= \{C \in \mathscr{P} \mid C = A \text{ or } C = B \text{ or } A-C-B\}$$

Ex: Consider the hyperbolic plane $\mathcal{H} = (\mathbb{H}, \mathcal{L}_H, d_H)$.
Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$ be in $\mathbb{H}$ where $x_1 < x_2$ and A, B lie on ${}_cL_r$.

Then

$$\overline{AB} = \left\{ C = (x,y) \;\middle|\; \begin{array}{l} C \in {}_cL_r \text{ where} \\ x_1 \le x \le x_2 \end{array} \right\}$$



proof: See notes and HW 5 #11 $\blacksquare$

Theorem: Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry.
Let $A, B, C, D \in \mathscr{P}$ with $A \neq B$ and $C \neq D$.
If $\overline{AB} = \overline{CD}$, then $\underbrace{\{A,B\} = \{C,D\}}$

says either:
$\boxed{A = C \text{ and } B = D}$
or
$\boxed{A = D \text{ and } B = C}$

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proof: See notes $\blacksquare$

This theorem makes the following definition well-defined.

<u>Def:</u> Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let $A, B \in \mathscr{P}$ with $A \neq B$.

The <u>endpoints</u> of $\overline{AB}$ are A and B.

The <u>length</u> of $\overline{AB}$ is

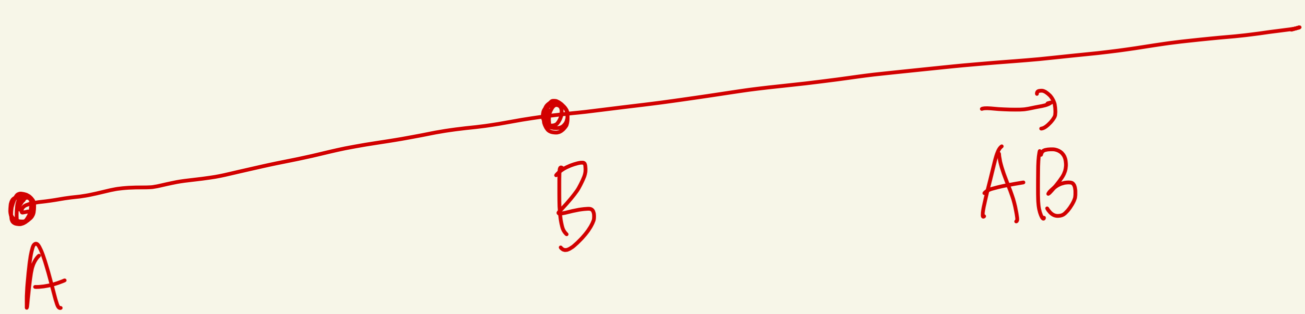
$$AB = d(A, B).$$

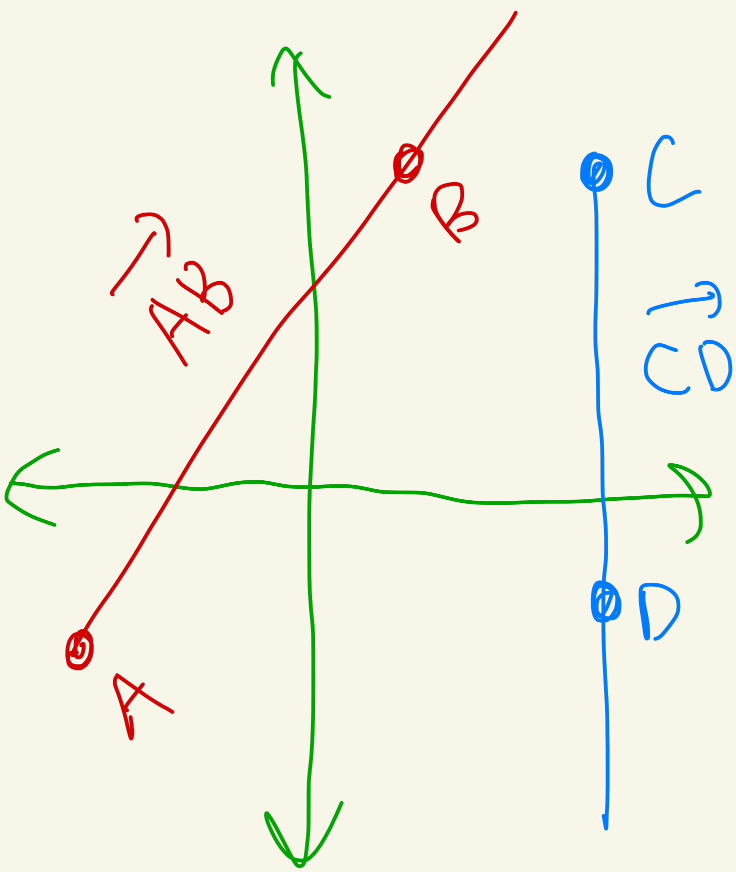
Def: Let $(\mathcal{P}, \mathcal{L}, d)$ be a metric geometry. Let $A, B \in \mathcal{P}$ with $A \neq B$.

The ray from A towards B is defined to be the set

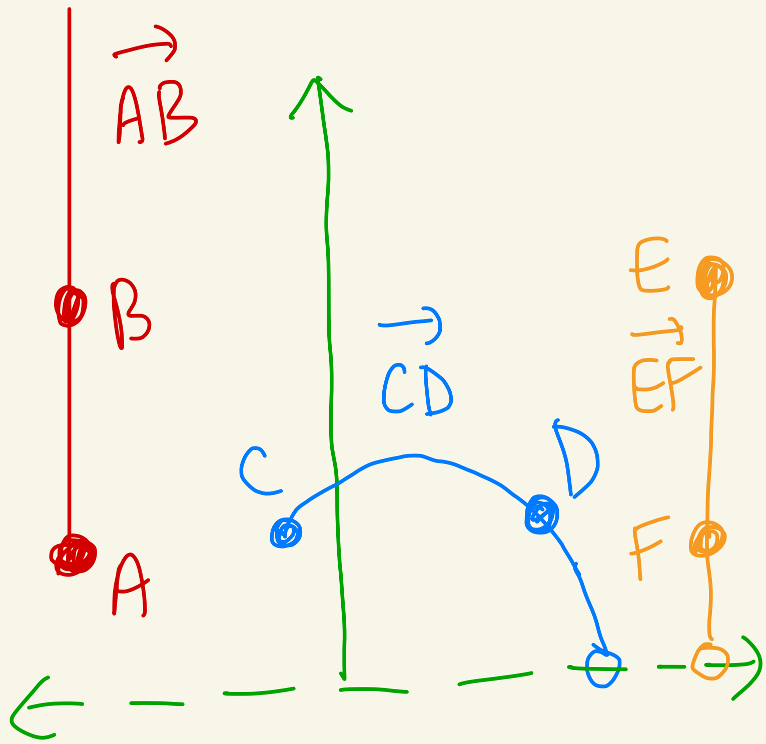
$$\overrightarrow{AB} = \overline{AB} \cup \{C \in \mathcal{P} \mid A-B-C\}$$

$$= \left\{ C \in \mathcal{P} \;\middle|\; \begin{array}{l} C=A \text{ or } A-C-B \text{ or} \\ C=B \text{ or } A-B-C \end{array} \right\}$$





Some rays in the Euclidean plane

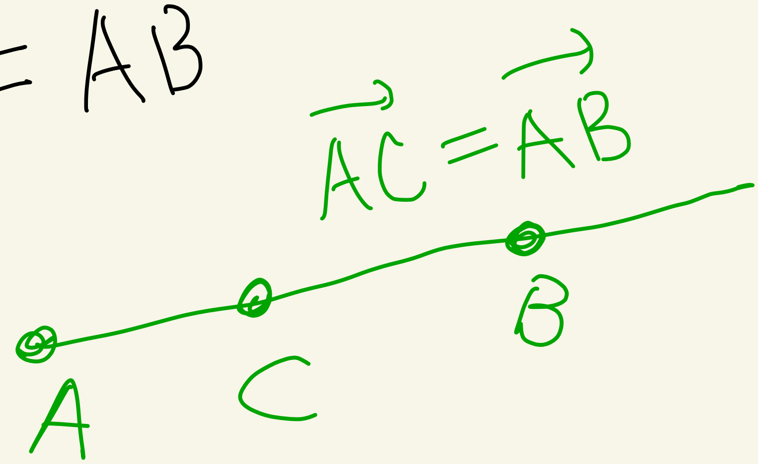


Some rays in the hyperbolic plane

<u>Theorem:</u> Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry.
Let $A, B, C, D \in \mathscr{P}$ with $A \neq B$ and $C \neq D$.

Then:

(i) If $C \in \overrightarrow{AB}$ and $C \neq A$, then $\overrightarrow{AC} = \overrightarrow{AB}$

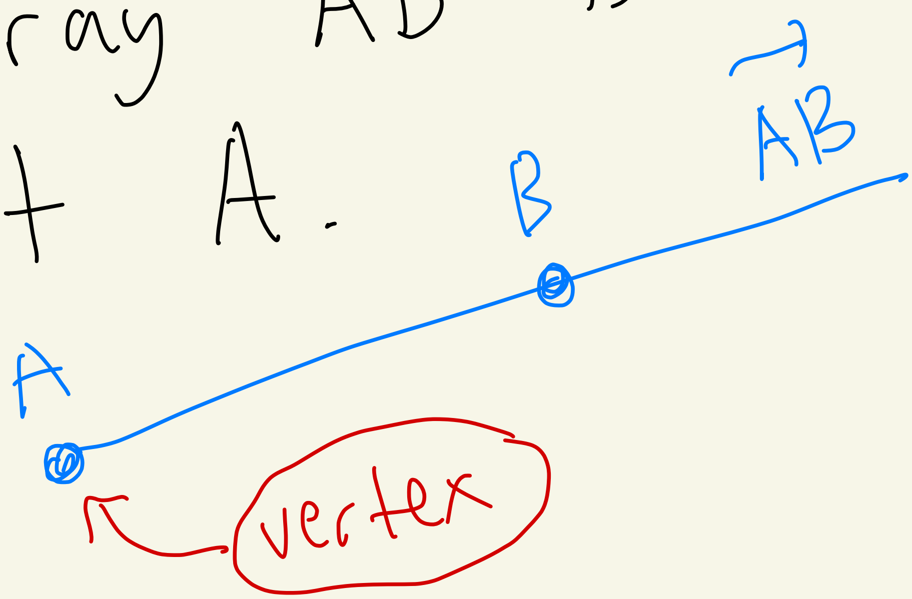


(ii) If $\overrightarrow{AB} = \overrightarrow{CD}$, then $A = C$

<u>proof:</u> See HW 5 #9 $\blacksquare$

The previous theorem makes the following definition well-defined

<u>Def:</u> Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry.
Let $A, B \in \mathscr{P}$ with $A \neq B$.
The <u>vertex</u> (or <u>initial point</u>) of the ray $\overrightarrow{AB}$ is the point A.



$\boxed{\text{HW 5 problem 6}}$:

Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let $A, B \in \mathscr{P}$ with $A \neq B$.

Then:

(a) $\overline{AB} = \overline{BA}$

(b) $\overline{AB} \subseteq \overrightarrow{AB} \subseteq \overleftrightarrow{AB}$

(c) $\overline{AB} = \overrightarrow{AB} \cap \overrightarrow{BA}$

(d) $\overleftrightarrow{AB} = \overrightarrow{AB} \cup \overrightarrow{BA}$

Theorem: Consider the Euclidean plane $\mathcal{E} = (\mathbb{R}^2, \mathcal{L}_E, d_E)$

Let $A, B \in \mathbb{R}^2$ with $A \neq B$.

Then:

$$\overline{AB} = \left\{ C \in \mathbb{R}^2 \;\middle|\; \begin{array}{c} C = A + t(B-A) \\ \text{where } 0 \leq t \leq 1 \end{array} \right\}$$

$$\overrightarrow{AB} = \left\{ C \in \mathbb{R}^2 \;\middle|\; \begin{array}{c} C = A + t(B-A) \\ \text{where } 0 \leq t \end{array} \right\}$$

proof: HW 5 #10

$\blacksquare$