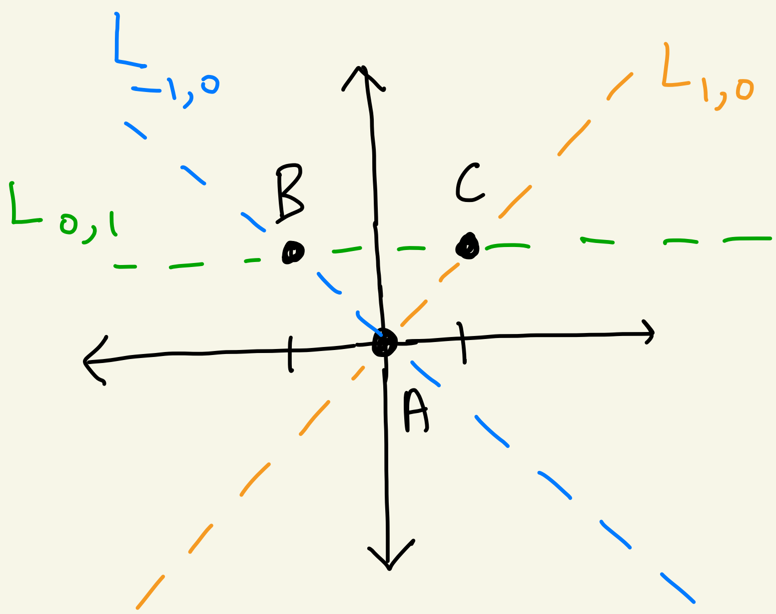
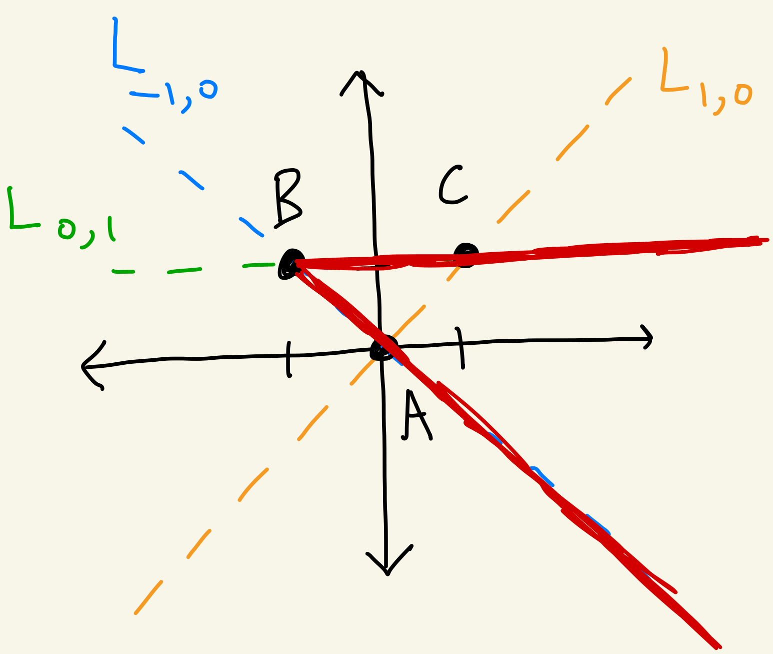
Math 4300
Homework 6
Solutions

$\boxed{1}$ $A=(0,0)$, $B=(-1,1)$, $C=(1,1)$

Note that $\overleftrightarrow{AB} = L_{-1,0}$, $\overleftrightarrow{AC} = L_{1,0}$, $\overleftrightarrow{BC} = L_{0,1}$

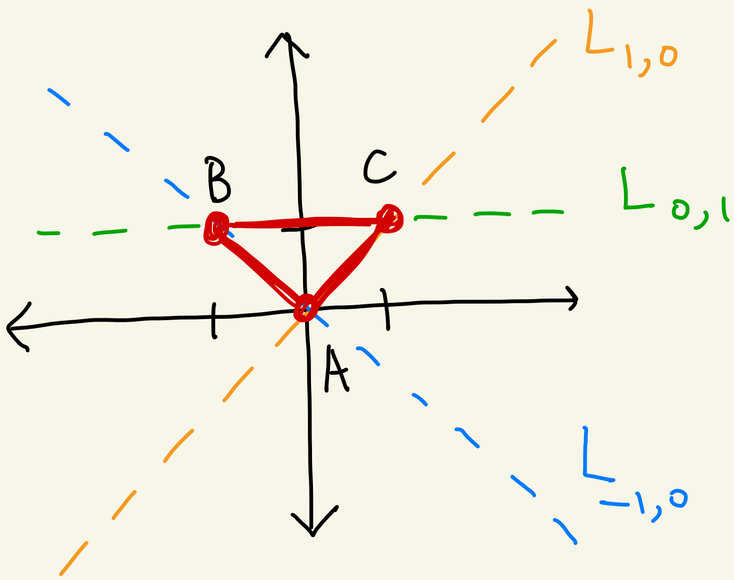


(a)



$\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC}$

(b)



$\triangle ABC$

$= \overline{AB} \cup \overline{BC} \cup \overline{CA}$

$\boxed{2}$ $A = (1,2)$, $B = (1,4)$, $C = (3,4)$

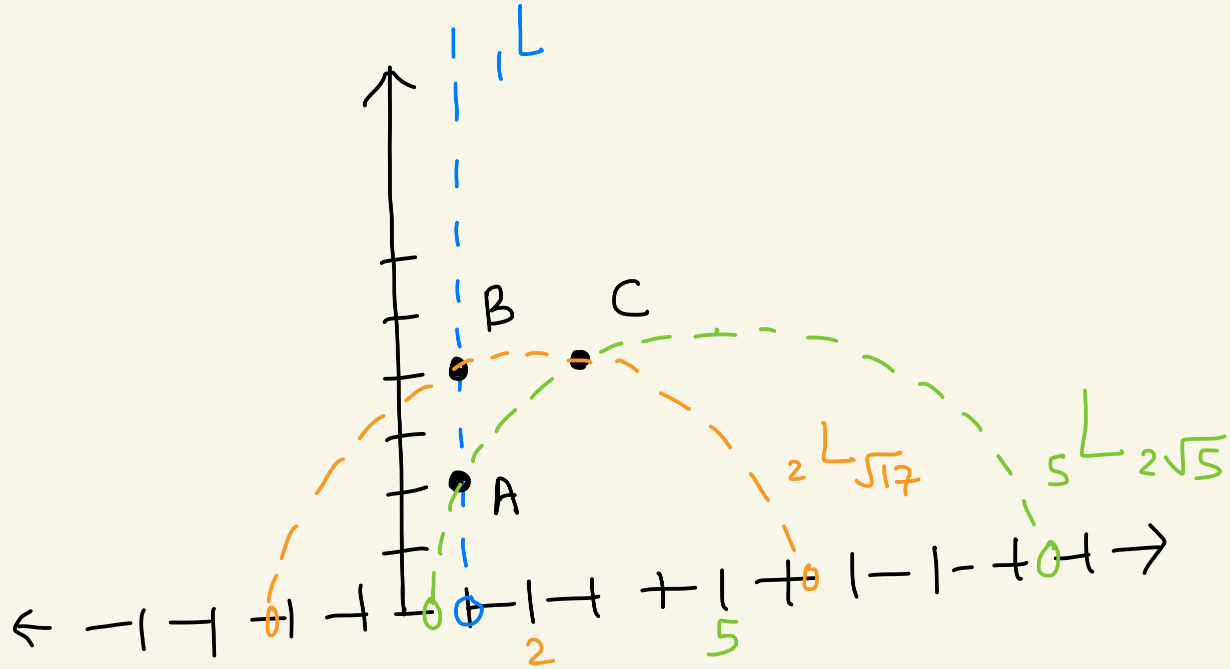
Note that $\overleftrightarrow{AB} = {}_1L$

From HW 5 we saw that $\overleftrightarrow{AC} = {}_5L_{2\sqrt{5}}$

One can check that $\overleftrightarrow{BC} = {}_2L_{\sqrt{17}}$

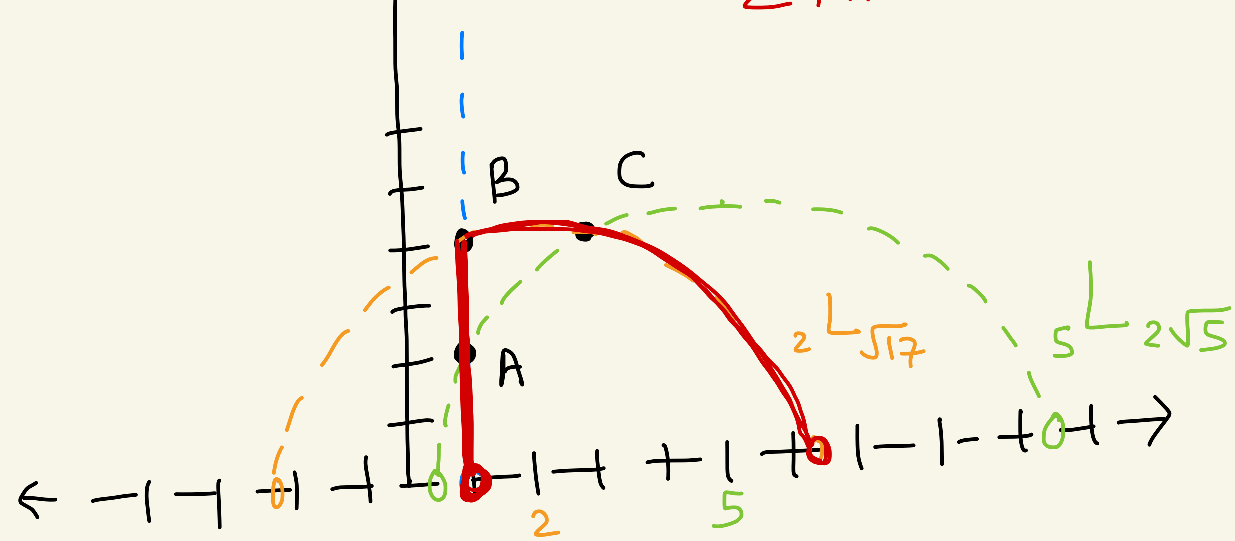
$\boxed{2\sqrt{5} \approx 4.47}$

$\boxed{\sqrt{17} \approx 4.12}$

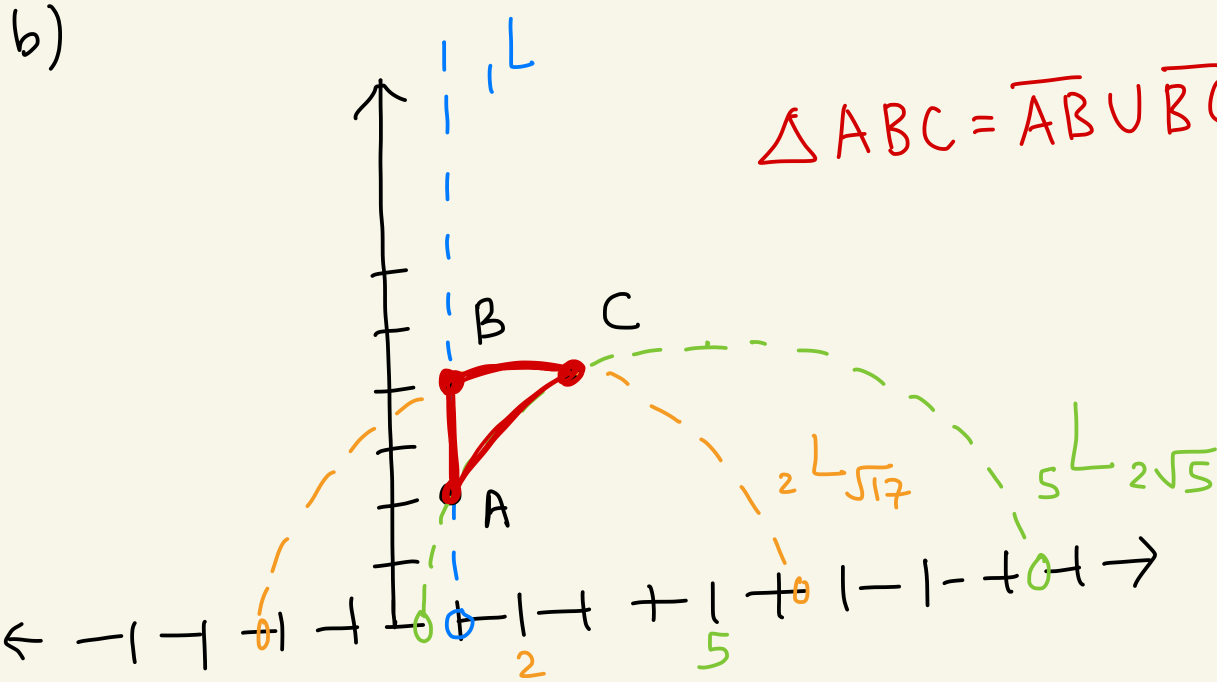


(a)

$\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC}$



(b)



$\Delta ABC = \overline{AB} \cup \overline{BC} \cup \overline{CA}$

③ (a)

We have that

$$\angle ABC \underset{\text{by def}}{=} \overrightarrow{BA} \cup \overrightarrow{BC} \underset{\text{property of union of sets}}{=} \overrightarrow{BC} \cup \overrightarrow{BA} \underset{\text{by def}}{=} \angle CBA$$

③ (b) We have that

$$\triangle ABC \underset{\text{by def}}{=} \overline{AB} \cup \overline{BC} \cup \overline{CA}$$

$$\triangle ACB \underset{\text{by def}}{=} \overline{AC} \cup \overline{CB} \cup \overline{BA} \overset{\text{HW 5}}{=} \overline{CA} \cup \overline{BC} \cup \overline{AB}$$

$$\underset{\text{property of union of sets}}{=} \overline{AB} \cup \overline{BC} \cup \overline{CA}$$

Thus, $\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{CA} = \triangle ACB$.

The other equalities all have a similar proof. Try it out.

You try the rest

4. Let B, Z be points with $B \neq Z$.

Let $\ell = \overleftrightarrow{BZ}$.

Let $f : \ell \to \mathbb{R}$ be a ruler with

$f(B) = 0$ and $f(Z) > 0$.

Let $t = f(Z) + 1$.

Since f is onto, there exists a point $D \in \ell$ with $f(D) = t$.

Then,

$$\underbrace{f(B)}_{0} < f(Z) < \underbrace{f(D)}_{f(Z)+1}$$

Thus, $B - Z - D$.