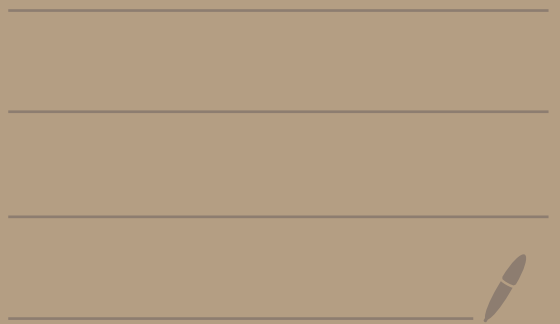


Math 4460

1/23/23



Assumptions for the class

We will assume that the set of integers

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, 4, -4, \dots\}$$

exists.

We will also assume the basic algebraic properties of \mathbb{Z} .

For example, if $a, b, c \in \mathbb{Z}$ we will assume:

- $a + b \in \mathbb{Z}$ (closed under +)

- $ab \in \mathbb{Z}$ (closed under \cdot)

- $a + (b + c) = (a + b) + c$

- $a(bc) = (ab)c$

- $a + b = b + a$

- $ab = ba$

- $0 + a = a + 0 = a$

- $1 \cdot a = a \cdot 1 = a$

- $a + (-a) = (-a) + a = 0$

- $a[b + c] = ab + ac$

- $[b + c]a = ba + ca$

ring
axioms

We will assume all the other usual basic algebra/arithmetic facts like

- If $a > b$ then $-a < -b$.

HW 1 TOPIC - Division and Primes

Def: Let a and b be integers with $a \neq 0$.

We say that a divides b if there exists an integer k where $b = ak$.

If a divides b then we say that a is a divisor of b and we write $a | b$.

If a does not divide b we say that a is not a divisor of b and we write $a \nmid b$.

read "a divides b"

read "a does not divide b"

Ex:

Divisors of 12: 1, 2, 3, 4, 6, 12
-1, -2, -3, -4, -6, -12

$7 \nmid 12$ because there is no $k \in \mathbb{Z}$ with $12 = 7k$. This would need $k = \frac{12}{7} \notin \mathbb{Z}$

$-3 | 12$
because $12 = (-3)(-4)$
 $b = ak$

$6 | 12$
because $12 = (6)(2)$
 $b = ak$

Def: Let $p \in \mathbb{Z}$ with $p > 1$.

We say that p is a prime if the only positive divisors of p are

1 and p . If p is not a prime, then we call it a composite number.

Ex: Let's circle the primes...

2, 3, 4, 5, 6, 7, 8, 9, 10,
11, 12, 13, 14, 15, 16, 17, 18, 19,
20, 21, 22, 23, 24, 25, 26, 27, ...

not prime since $4 = (2)(2)$

not prime since $6 = (2)(3)$

Proposition: Let x and y be positive integers.

If $x|y$, then $1 \leq x \leq y$.

proof: Suppose x and y are positive integers and that $x|y$.

Since x is a positive integer we know $1 \leq x$.

Since $x|y$ we know $y = xk$ where $k \in \mathbb{Z}$.

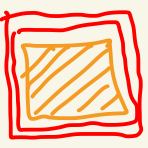
We know k is positive because
 $k = \frac{y}{x}$ and x and y are both positive.

using properties of the rational numbers \mathbb{Q}

Thus, $1 \leq k$. multiply by x on both sides

Since x is positive this gives $x \leq \frac{xk}{y}$.

So, $1 \leq x \leq xk = y$.

Thus, $1 \leq x \leq y$. 

Proposition: Let p and q be prime numbers.

If $p|q$, then $p=q$.

proof: Suppose p and q are primes and $p|q$.

Since q is prime its only positive divisors are 1 and q .

Since p is positive and $p|q$, then this means $p=1$ or $p=q$.

Since p is prime, we know $p > 1$.

So, $p=q$. 