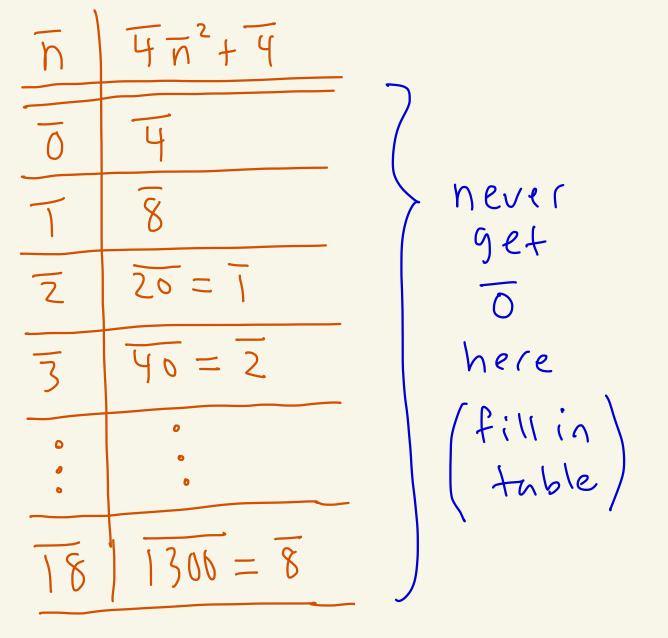
Math 4460 4/28/25

g(d(2,14)=2==1 HW 5 multides 14=2.7) ъ р | р | 14  $\mathbb{Z}_{14}^{\times} = \{\overline{1}, \overline{3}, \overline{5}, \overline{9}, \overline{11}, \overline{13}\}$ 78 42 56 inverses T=T 70 84  $\overline{3}^{-1} = \overline{5}^{-1} = \overline{3}^{-1} = \overline{5}^{-1} = \overline{5}^{-1} = \overline{3}^{-1}$ 98  $\overline{9}' = \overline{11}$   $\overline{7}$   $\overline{9} \cdot \overline{11} = \overline{99} = \overline{1}$  $\overline{11}' = \overline{9}$  $\overline{13} = \overline{13}$   $\overline{13}$   $\overline{13} = (-1)(-1) = \overline{13}$ 

primitive root  $\overline{3}^{4} = \overline{39} = 11$   $\overline{3}$  is  $\overline{3}^{5} = \overline{33} = \overline{5}$  primitive  $\overline{3}^{6} = \overline{15} = 1$  (oot 3 15  $\overline{3}' = (\overline{3})$  $\overline{3}^2 = \overline{9}$  $\overline{3}^{3} = \overline{27} = \overline{13}$ In Zir calculate II 1,000 Know: since ITEZX < gcd(13,14)=1 by Euler we get  $\Pi \varphi(I'I) = \Pi$ which is TIG = T because  $\varphi(14) = \mathbb{Z}_{14}^{\times} = [\{\overline{7}, \overline{3}, \overline{5}, \overline{9}, \overline{11}, 13\}]$ =6. 166 6 000 Then,

$$\begin{aligned}
\boxed{\left|\left|\begin{array}{c}1 \circ 000 \right|^{2} = \left|\left|\begin{array}{c}6 \left(166\right) + 4\right| \\ = \left|\left|\begin{array}{c}6 \right|^{166} \cdot \left|\left|\begin{array}{c}7 \right| \\ 4 \\ 9 \\ -36$$

HW 5 (13) Show that 19/442+4 fur all nEZ. Proof: Suppose, by Way of contradiction, that 19/4n2+4 fur some nEZ. Then,  $4n^2+4=19k$  where keZ. Then, in Zig we get  $\overline{4n^2} + \overline{4} = 0$ In 2419. There is no such n E Zig by the following table.



We thus have a contradiction and 19/4/2+4 for all n.

HW 5  $\begin{array}{c} (14) \ \text{Let} \ n \in \mathbb{Z}, \ n \geqslant 2. \\ \text{Let} \ a, b, c \in \mathbb{Z}. \end{array}$ If gcd(a,n)=1 and ab=ac in Zn, then  $\overline{b} = \overline{c}$  in Zn. Proof: Since gcd(a,n)=1 We Know that a has a multiplicative inverse  $\overline{a}^{-1}$  in  $\mathbb{Z}_{n}$ . Then, ab=ac gives a ab = a ac yielding TB=TZ Produciny b=c. X

Can you think of an example of ab=ac in Zn but Dite in Za?  $\bar{\alpha}=6$ ,  $\bar{b}=2$ ,  $\bar{c}=3$ , n=6

6=2  $Z_{4}; \frac{2}{2 \cdot 1} = \frac{6}{2 \cdot 3}$  $\overline{1 \neq 3}$ 

HW 4 If p is prime and  $x^2 \equiv y^2 (m \cdot d p)$ then pl(x+y) or pl(x-y). prouf: Suppose p is prime and  $x^2 \equiv y^2 \pmod{p}$ . Then,  $p\left(\begin{array}{cc} 2 & 2 \\ X - y^2 \right)$ . So, p (x+y)(x-y). Since p is prime, p (x+y) or p (x-y), VSED: p prime and plab, then pla or plb

(()) If p is prime and pla and plb then  $gcd(a,b) \ge p \ge 2.71$ p prime