MATH 4650 **FINAL EXAMINATION** **V. AKIS** **SPRING 21**

This is a closed book/notes multiple choice exam. You must work alone, and not get help from anywhere except from Dr. Akis.

For each of the following problems circle with pencil the correct answer in your Answer Form. Note: There may be more than one correct choice per question but you must circle ONLY one. Unless otherwise stated, all sets are subsets of the set of real numbers $\mathbb{R}$. All intervals are infinite.

1. Which of the following sets is denumerable (countably infinite}.

   a. $\{1,2,3\}$ b. $\left\{\frac{n+1}{n} : n \in \mathbb{N}\right\}$ c. $[0,\pi]$ d. cl $\mathbb{Q} \cap [0,\pi]$

   e. none of the above

2. Which of the following sets is NOT open?

   a. $\bigcap_{n=1}^{50}(-1/n, 1/n)$ b. $\bigcap_{n=1}^{\infty}(-1/n, 1/n)$ c. $\bigcap_{n=1}^{\infty}(0, 1/n)$

   d. $\bigcup_{n=1}^{\infty}[-n, n]$ e. none of the above (they are all open)

3. Suppose $K = [-1,1] \setminus \bigcap_{n=1}^{\infty}(-1/n, 1/n)$. Then

   a. K is open b. K is compact c. K is closed

   d. bd $K = \{-1,1\}$ e. none of the above

4. Suppose $f: \mathbb{R} \to \mathbb{R}$ is continuous, such that, $f(3) = \pi$ and $f(4) = \sqrt{2}$. Then there exists $x \in [3,4]$, such that

   a. $f(x) > \pi$ b. $f(x) < \sqrt{2}$ c. $f(x) = 1$ d. $f(x) = 2$ e. none of the above

5. Suppose $I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots \supseteq I_k \supseteq \cdots$ is an infinite decreasing sequence of closed intervals. Then their intersection $\bigcap_{k=0}^{\infty} I_k$ is always

   a. a closed interval b. empty c. non-empty d. a single point

   e. none of the above

6. Suppose $I_1 \subseteq I_2 \subseteq \cdots \subseteq I_k \subseteq \cdots$ is an infinite increasing sequence of open intervals. Then their union $\bigcup_{k=1}^{\infty} I_k$ is

a. unbounded b. open c. closed d. equal to $\mathbb{R}$

e. none of the above

7. Suppose $S \subseteq [0,1]$ be a non-empty closed set, and let bd S denote the set of all boundary points of S. Which of the following is False?

a. bd $S \neq \emptyset$ b. bd $S \subseteq S$ c. bd S is compact d. $[0,1] \backslash S$ is open

e. none of the above (they are all true)

8. The set $\mathbb{Q} \cap [\sqrt{2}, \sqrt{5}]$

a. is compact b. has no infimum c. has no maximum

d. has no supremum e. none of the above

9. Consider the set $\mathbb{Q}^2 = \{(x, y) \in \mathbb{R}^2 : x, y \text{ are rational numbers}\}$, i.e. $\mathbb{Q}^2$ is the set of all points in the plane $\mathbb{R}^2$ with rational coordinates. Which of the following is False?

a. $\mathbb{Q}^2$ is countable b. $\mathbb{R}^2 \backslash \mathbb{Q}^2$ is uncountable c. bd $\mathbb{Q}^2 = \mathbb{R}^2$

d. cl $\mathbb{Q}^2 = \mathbb{R}^2$ e. none of the above (they are all true)

10. Let (X, d) be a metric space, and fix $p \in X$. For $k = 1, 2, 3, 4$, which of the following functions $\rho_k : X \times X \to \mathbb{R}$, is NOT a metric for X?

a. $\rho_1(x, y) = max\{d(x, p), d(y, p)\}$ b. $\rho_2(x, y) = min\{d(x, p), d(y, p)\}$

c. $\rho_3(x, y) = d(x, p) + d(y, p)$ d. $\rho_4(x, y) = d(x, p) \cdot d(y, p)$

e. none of the above (every ρ_k is a metric for X)

11. Let $\mathcal{G}$ be an open cover of a closed and bounded set K. Then $\mathcal{G}$ contains a subcover $\mathcal{G}_0$ of K, such that

a. $\mathcal{G}_0$ is finite b. $\mathcal{G}_0$ is denumerable c. $\mathcal{G}_0$ consists of open intervals

d. $\mathcal{G}_0$ consists of bounded open sets e. none of the above

12. Suppose $\{s_n\}$ is a Cauchy sequence. Which of the following is false?

a. every subsequence of $\{s_n\}$ is a Cauchy. b. lim inf $s_n =$ lim sup s_n c. $\{s_n\}$ converges

d. every subsequence of $\{s_n\}$ converges to the same value s

e. none of the above (they are all true)

13. Suppose $\{s_n\}$ is a bounded sequence of real numbers where $s_n < s_{n+1}$, for all $n \in \mathbb{N}$. Then

a. $\{s_n\}$ has no supremum b. $\{s_n\}$ has no infimum c. lim inf $s_n <$ lim sup s_n

d. $\{s_n\}$ has no convergent subsequence e. none of the above

14. Suppose $f: D \to \mathbb{R}$ is continuous and $\{s_n\}$ is a sequence in D. Then

a. If $\{s_n\}$ converges in $\mathbb{R}$, then so does $\{f(s_n)\}$ b. if $\{s_n\}$ diverges, then so does $\{f(s_n)\}$

c. if $s_n \to s \in D$, then $f(s_n) \to f(s)$ d. if $\{s_n\}$ is Cauchy, then so is $\{f(s_n)\}$

e. none of the above

15. Let $s_n = (-1)^n \, (n+1)/n$. Then

a. $\{s_n\}$ is Cauchy b. lim inf $s_n =$ lim sup s_n c. has no infimum

d. has no convergent subsequence e. none of the above

16. Suppose $f: D \to \mathbb{R}$ is continuous, where $D = [0,1] \cup \{2,3,4,5\}$. Then

a. f is continuous on $[0,5]$ b. f is uniformly continuous c. f is discontinuous at 5

d. f is discontinuous at 0. e. none of the above

17. Suppose $f: \mathbb{R} \to \mathbb{R}$ is uniformly continuous. Which of the following sets is a Cauchy sequence?

a. $\{f(x): x \in \mathbb{Q}\}$ b. $\{f(x): x \in \mathbb{N}\}$ c. $\{f(x): x \in \mathbb{Z}\}$ d. $\{f(1/n): n \in \mathbb{N}\}$

e. none of the above

18. Suppose $\{s_n\}$ and $\{t_n\}$ are sequences converging to the same limit, and let

$u_n = \begin{cases} s_n, & \text{if } n \text{ is even} \\ t_n, & \text{if } n \text{ is odd} \end{cases}$. Then

a. $\{u_n\}$ is Cauchy b. lim inf $u_n <$ lim sup u_n c. lim $s_n <$ lim sup u_n

d. $\{u_n\}$ has no convergent subsequence e. none of the above

19. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 2, & \text{if } x \in \mathbb{Q} \\ -2, & \text{if } x \notin \mathbb{Q} \end{cases}$. Then

a. f is continuous at 0 b. f is continuous on $\mathbb{Q}$ c. f is continuous on $\mathbb{R} \backslash \mathbb{Q}$

d. f is uniformly continuous e. none of the above

20. The current USA president's name is

a. Donald Trump b. Donald Trump c. Donald Trump d. Donald Trump

e. none of the above (hint...)