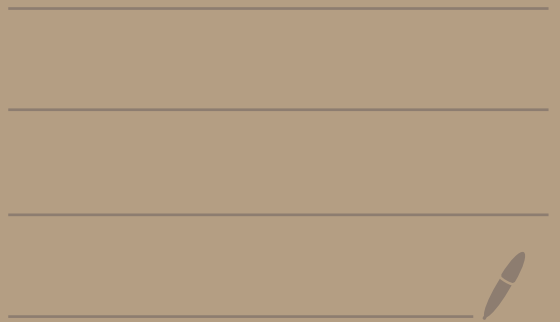# Math 4740
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Topic 1 – Probability spaces
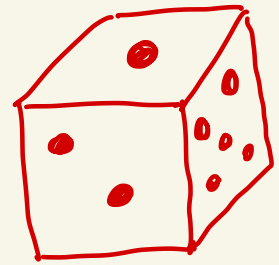
Def: A <u>set</u> is a collection of objects or elements.

If x is an element of a set S then we write $\underline{x \in S.}$

read: "x is in S"

If x is not in S, then we write $\underline{x \notin S.}$

read: "x is not in S"

If S has a finite number of elements, then we write $|S|$ for the number of elements in S.

Ex: Let's make a set that models the possible outcomes of rolling a 6-sided die with sides labelled $1, 2, 3, 4, 5, 6$.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$|S| = 6$$

$$4 \in S$$
$$2 \in S$$

$$\tfrac{1}{2} \notin S$$
$$\pi \notin S$$

Later we will call S the sample space of our experiment.

Note: Order doesn't matter in a set.
  Ex: $\{1,2,3,4,5,6\} = \{2,4,1,6,3,5\}$

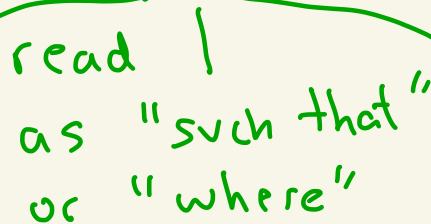Note: Sets can't have duplicates.
  $\{1,1,2\}$ is not a set.
  duplicates

General way to describe sets

$\{$ description of what the elements look like | condition the elements must satisfy to be in the set $\}$

read | as "such that" or "where"

some people use : here

Ex: Let's model rolling two 6-sided die, one green and one red.

$$S = \left\{ (r,g) \;\middle|\; \begin{matrix} r = 1,2,3,4,5,6 \\ g = 1,2,3,4,5,6 \end{matrix} \right\}$$

$$= \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

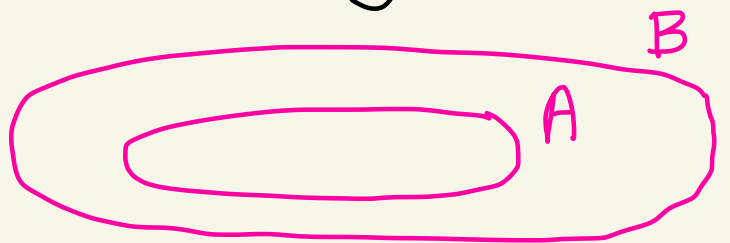$(1,2)$ ⟵ means red die is 1, green die is 2

$(5,3)$ ⟵ red die is 5, green die is 3

$|S| = 36$

Def: Let A and B be sets.
We say that A is a <u>subset</u> of B,
and write $A \subseteq B$, if every element
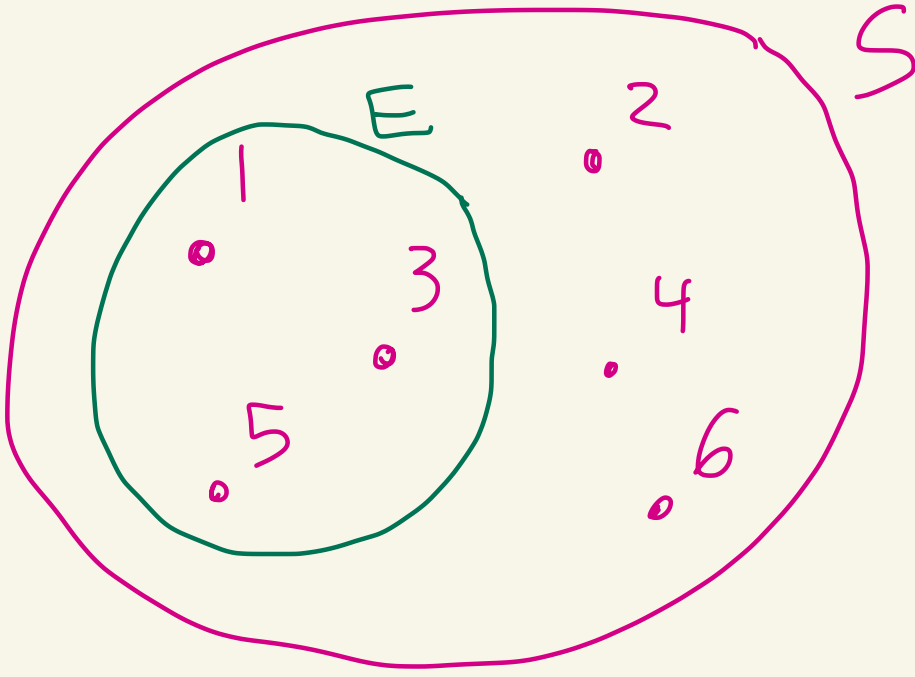of A is also
an element of B.

Ex:

$S = \{1, 2, 3, 4, 5, 6\}$ ← outcomes of rolling a 6-sided die

$E = \{1, 3, 5\}$

Here we have $E \subseteq S$

S

E

1

2

3

4

5

6

Here E will be called the event that either 1 or 3 or 5 occurs.