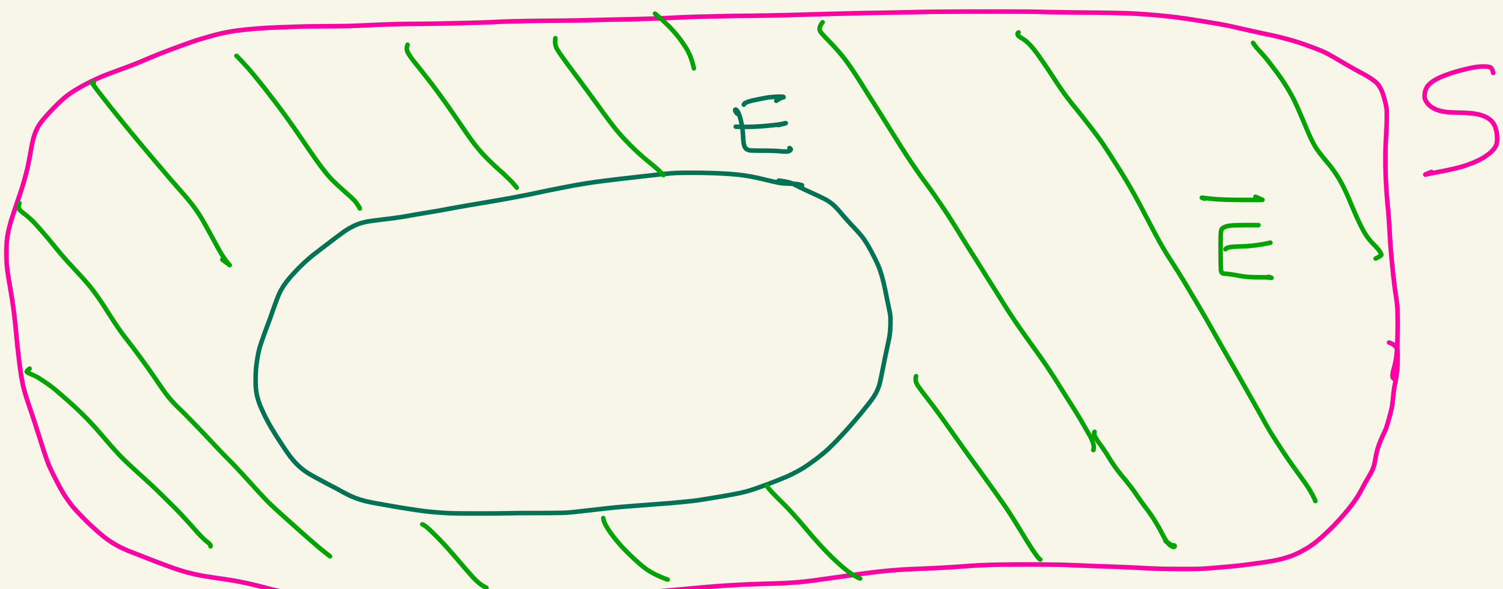
Math 4740
1/27/25

<u>Def:</u> Let S be a set and $E \subseteq S$. The <u>complement</u> of E in S is

$$\overline{E} = \underbrace{\{x \mid x \in S \text{ and } x \notin E\}}_{\text{read: "all } x \text{ where } x \text{ is in } S \text{ and } x \text{ is not in } E\text{"}}$$



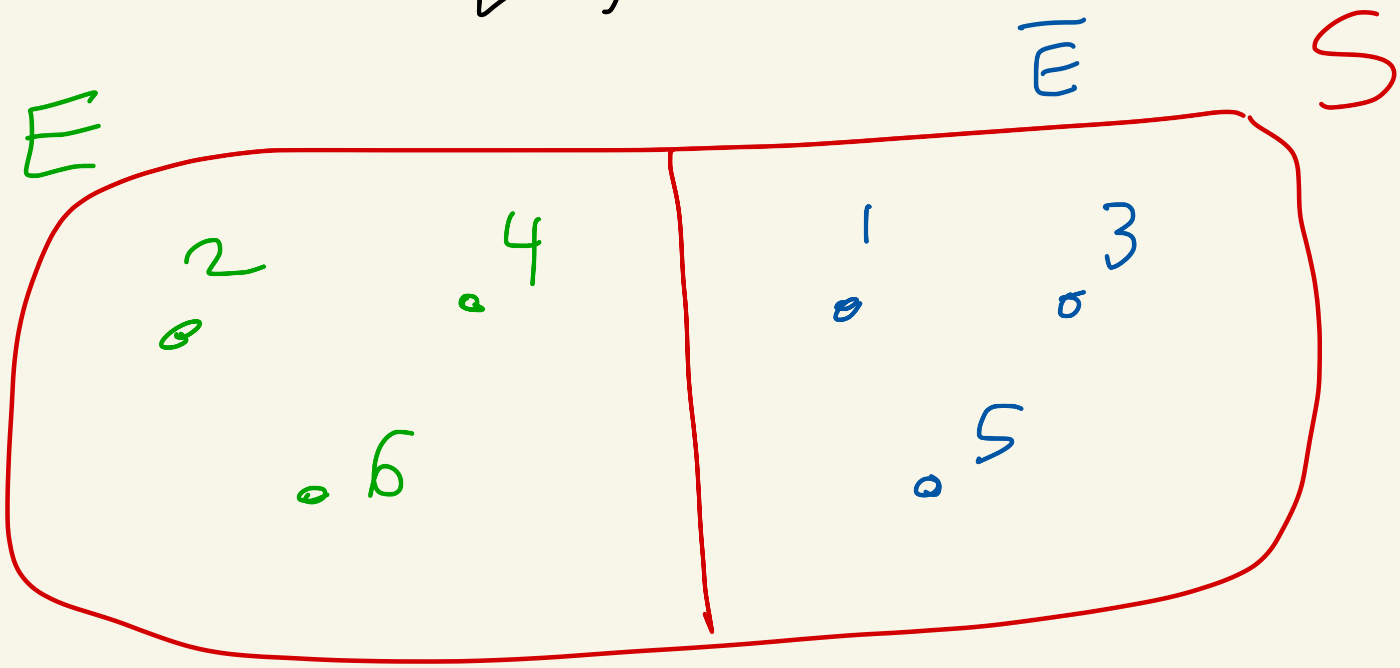
Other notations are:

E^c or $S - E$ or $S \setminus E$

<u>Ex:</u> $S = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6\}$

$\bar{E} = \{1, 3, 5\}$

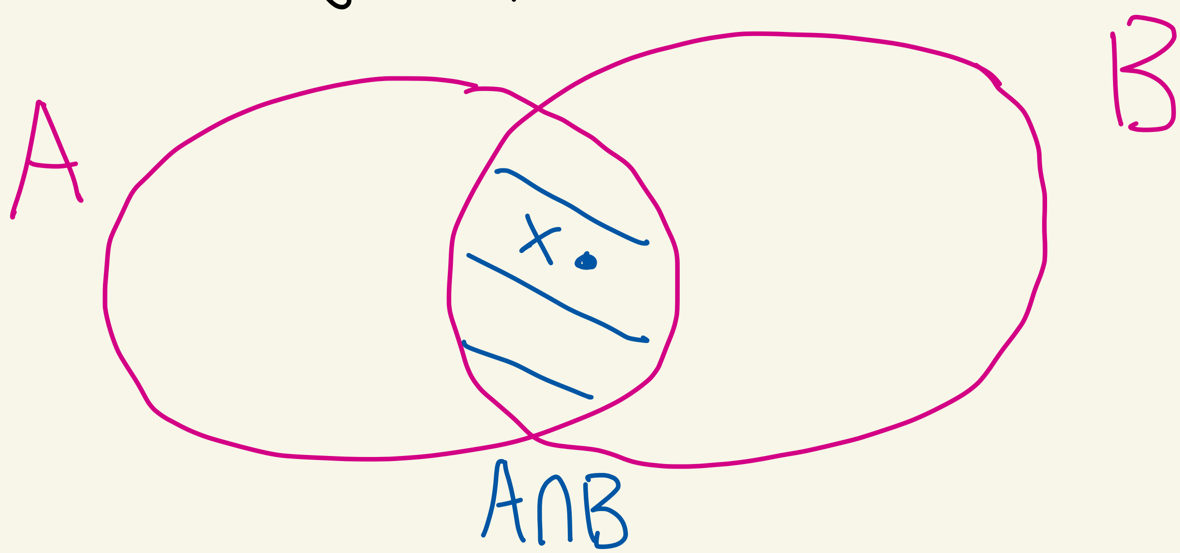


Def: The *empty set* is the set with no elements. It's denoted by ϕ.

Def: Let A and B be sets.

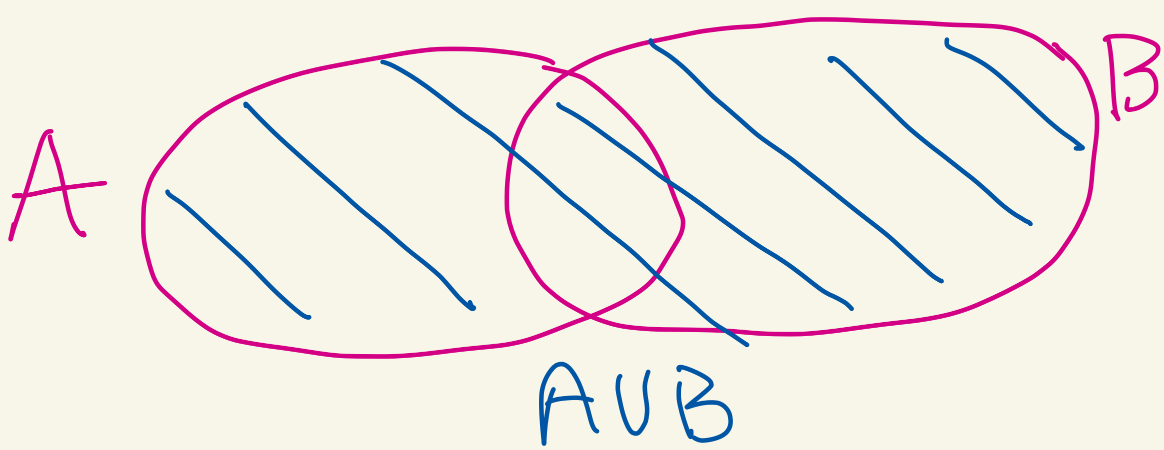
The intersection of A and B is

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$



The union of A and B is

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$



Ex: Let's make a sample space of all outcomes of flipping a coin 3 times in a row.

$S = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)\}$

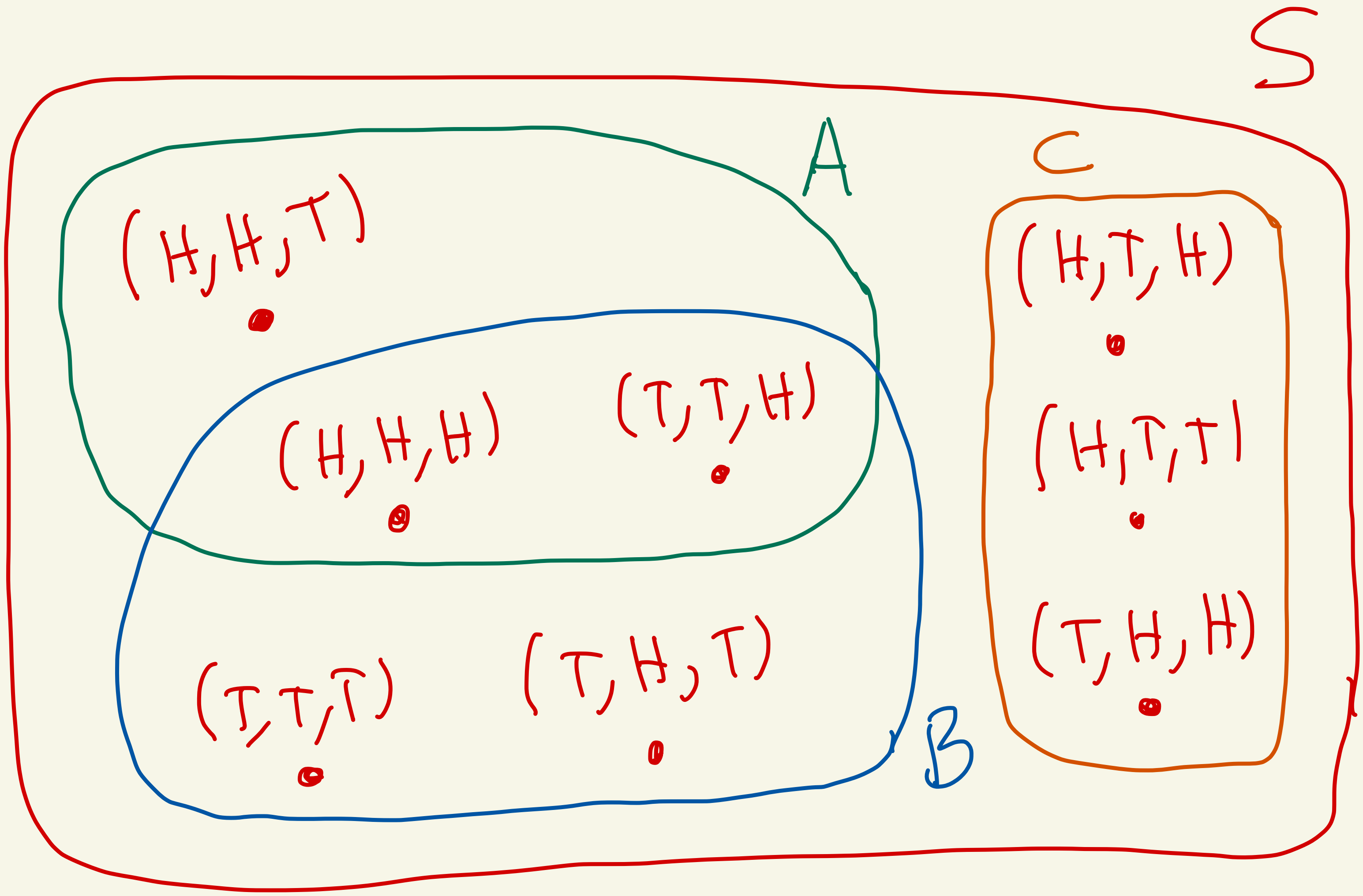
$(T,T,H) \leftarrow$ means:
flip 1 is Tails
flip 2 is Tails
flip 3 is Heads

Let

$A = \{(H,H,T), (H,H,H), (T,T,H)\}$

$B = \{(T,T,T), (T,T,H), (H,H,H), (T,H,T)\}$

$C = \{(H,T,H), (H,T,T), (T,H,H)\}$



Then

$$A \cup B = \{(H,H,T),(H,H,H),(T,T,H),(T,T,T),(T,H,T)\}$$

$$A \cap B = \{(H,H,H),(T,T,H)\}$$

$$A \cap C = \phi$$

$A \cup C = \{(H,H,T), (H,H,H), (T,T,H),$
$(H,T,H), (H,T,T), (T,H,H)\}$

$\overline{B} = \{(H,H,T), (H,T,H), (H,T,T),$
$(T,H,H)\}$

$\overline{S} = \phi$

<u>Def:</u> If $X \cap Y = \phi$, we say that X and Y are <u>disjoint</u> sets

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<u>Ex:</u> A and C are disjoint in the previous example.

<u>Def:</u> Let $A_1, A_2, \ldots, A_n$ be sets.

The <u>intersection</u> of $A_1, A_2, \ldots, A_n$ is

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$= \{ x \mid x \in A_i \text{ for all } 1 \le i \le n \}$$

$$= \{ x \mid x \in A_1 \text{ and } x \in A_2 \text{ and } \cdots \text{ and } x \in A_n \}$$

The <u>union</u> of $A_1, A_2, \ldots, A_n$ is

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

$$= \{ x \mid x \in A_1 \text{ or } x \in A_2 \text{ or } \cdots \text{ or } x \in A_n \}$$

$$= \{ x \mid x \text{ is in at least one of the } A_i \}$$

Ex: Let

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

represent rolling a 12-sided die.

Let

$A_1 = \{1, 2, 3\}$ $\quad A_3 = \{5, 6, 7, 4\}$

$A_2 = \{3, 4, 5\}$ $\quad A_4 = \{8, 3\}$

$A_1 \cup A_2 \cup A_3 \cup A_4$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A_2 \cup A_3 \cup A_4$

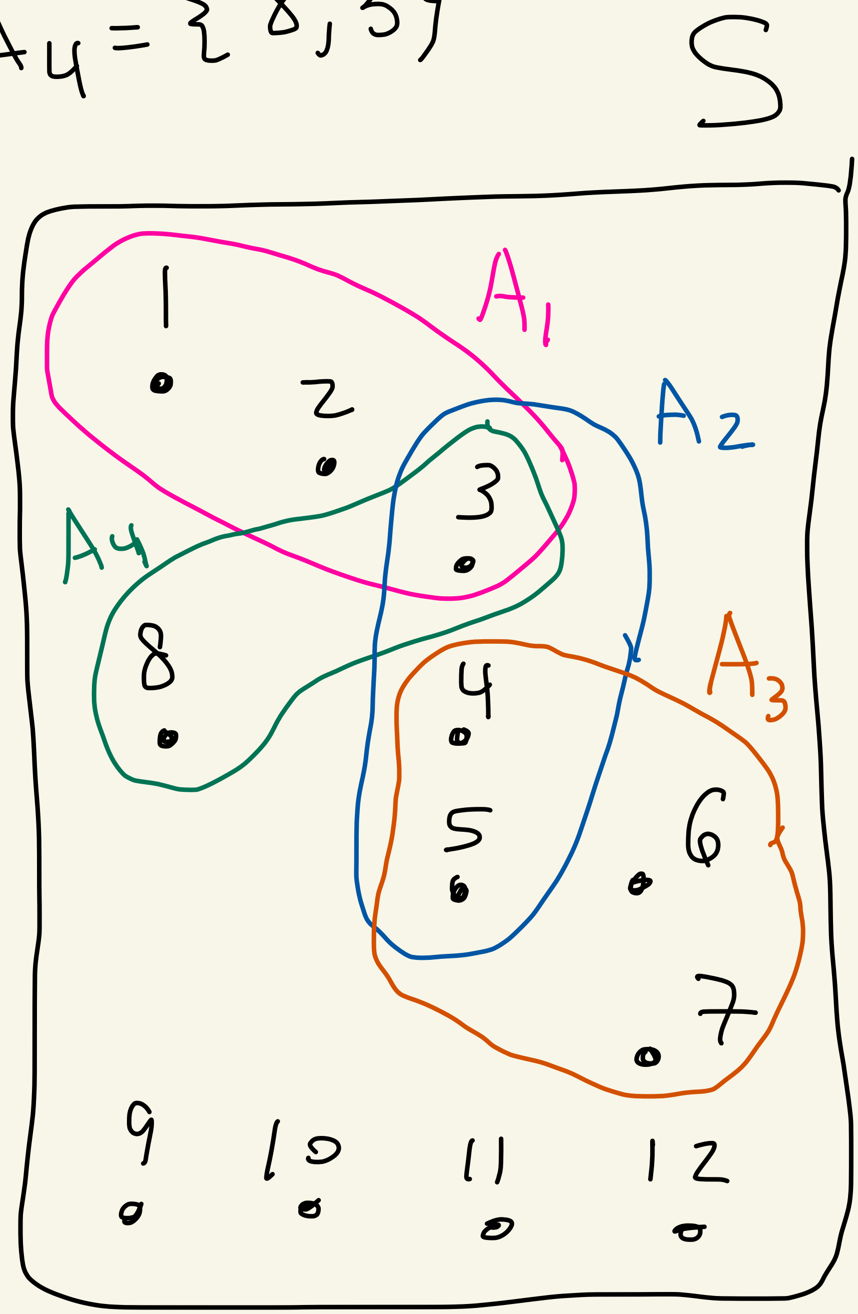
$= \{3, 4, 5, 6, 7, 8\}$

$A_1 \cap A_2 \cap A_3 \cap A_4$

$= \phi$

$A_1 \cap A_2 \cap A_4$

$= \{3\}$



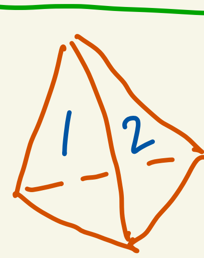
$$A_2 \cap A_3 \cap A_4 = \phi$$

Def: Let A and B be sets. The Cartesian product of A and B is

$$\underbrace{A \times B}_{\text{read: "A cross B"}} = \{ (a,b) \mid a \in A \text{ and } b \in B \}$$

Ex: $A = \{H, T\}$ ← flipping a coin

$B = \{1, 2, 3, 4\}$ ← rolling a 4-sided die



$$A \times B = \{(H,1),(H,2),(H,3),(H,4),(T,1),(T,2),(T,3),(T,4)\}$$

$$B \times A = \{(1,H),(1,T),(2,H),(2,T),(3,H),(3,T),(4,H),(4,T)\}$$

$$A \times A = \{(H,H),(H,T),(T,H),(T,T)\}$$

$$B \times B = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$$

Def: Let A and B be sets. A function f from A to B, written $f: A \longrightarrow B$, is a rule that assigns to each element of A a unique element of B.

Ex: Let

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$

represent flipping a coin twice.

Let

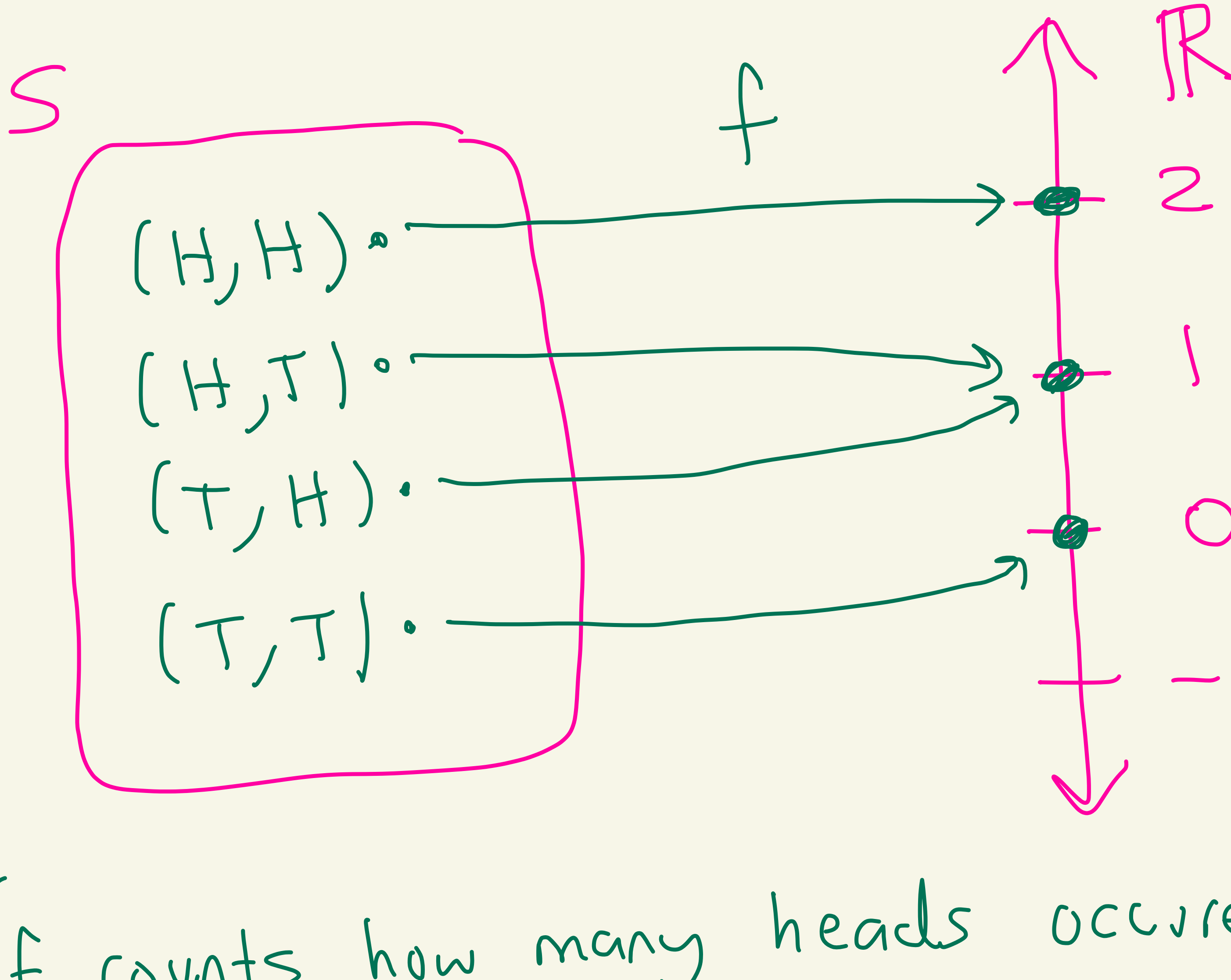
$$f : S \longrightarrow \mathbb{R}$$

count how many heads occured.

$\mathbb{R}$ denotes the set of all real numbers

So,

$$f(H,H) = 2 \qquad f(T,H) = 1$$
$$f(H,T) = 1 \qquad f(T,T) = 0$$



(f counts how many heads occured)