Math 4740

10/16/24

(Topic 4 continued...)

<u>Ex</u>: Consider the example (S, Ω, P) from last time where S is rolling two 6-sided dice and X is the sum of the dice. Let's calculate the objects from last time.

First let's calculate the probability function

$$p(i) = P(X = i)$$

$$p(2) = P(X=2) = P(\{(1,1)\}) = \frac{1}{36}$$

$$p(3) = P(X=3) = P(\{(1,2),(2,1)\}) = \frac{2}{36}$$

$$p(4) = P(X=4) = P(\{(1,3),(2,2),(3,1)\})$$
$$= \frac{3}{36}$$

$$p(5) = P(X=5) = P(\{(1,4),(2,3),(3,2),(4,1)\})$$
$$= \frac{4}{36}$$

$$p(6) = P(X=6) =$$
$$P(\{(1,5),(2,4),(3,3),(4,2),(5,1)\})$$
$$= 5/36$$

$$p(7) = P(X=7)$$
$$= P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})$$

$= 6/36$

$$p(8) = P(X=8)$$
$$= P(\{(2,6),(3,5),(4,4),(5,3),(6,2)\})$$
$$= 5/36$$

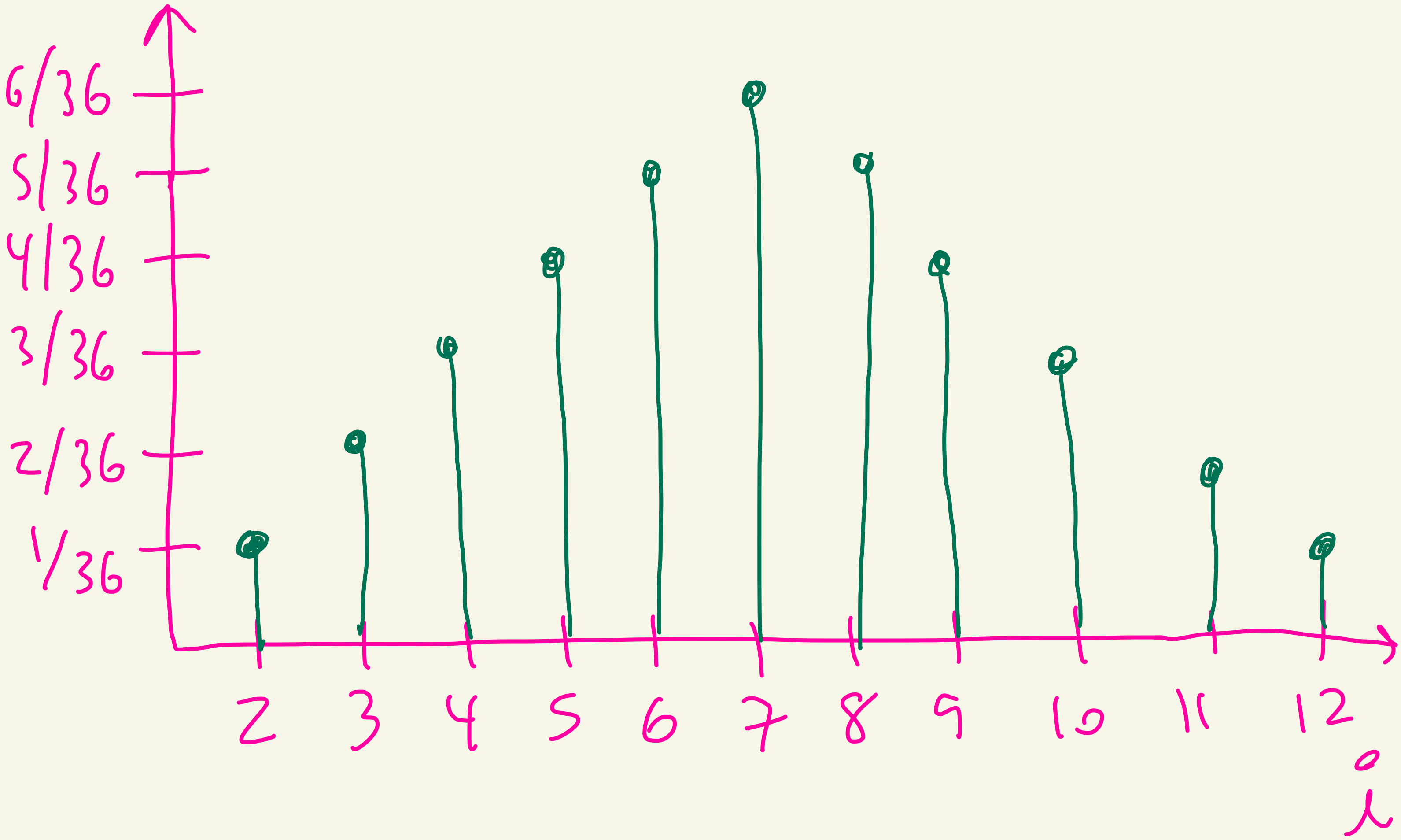
$$p(9) = P(X=9) = 4/36$$

$$p(10) = P(X=10) = 3/36$$

$$p(11) = P(X=11) = 2/36$$

$$p(12) = P(X=12) = 1/36$$

$p(i) = P(X = i)$



Let's now calculate

$F(i) = P(X \leq i)$

(cumulative distribution function)

$F(1) = P(X \leq 1) = 0$

$F(2) = P(X \leq 2) = P(\{(1,1)\})$

$$= P(X = 2) = 1/36$$

$$F(2.5) = P(X \le 2.5) = P(\{(1,1)\})$$

$$= P(X = 2) = 1/36$$

$$F(3) = P(X \le 3) =$$

$$= P(X=2) + P(X=3)$$

$$= 1/36 + 2/36 = 3/36$$

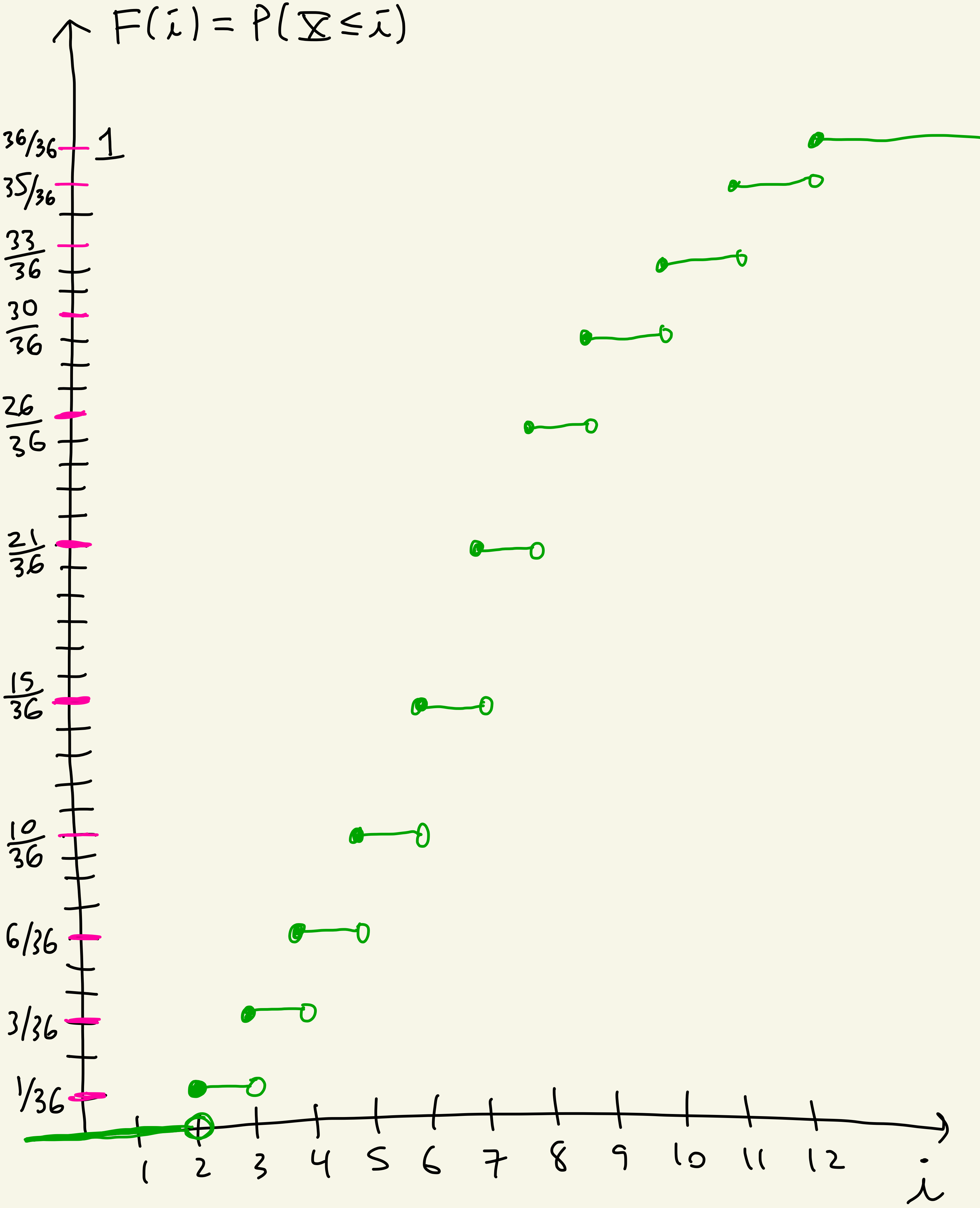
$$F(4) = P(X \le 4)$$

$$= P(X=2) + P(X=3) + P(X=4)$$

$$= 1/36 + 2/36 + 3/36 = 6/36$$

$$F(7) = P(X \le 7)$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} = \frac{21}{36}$$



$F(i) = P(X \leq i)$
36/36
1
35/36
33/36
30/36
26/36
21/36
15/36
10/36
6/36
3/36
1/36
1
2
3
4
5
6
7
8
9
10
11
12
i

Def: Let X be a discrete random variable on a probability space (S, Ω, P). Let $x_1, x_2, x_3, \ldots$ be the range of X (outputs of X)

The expected value of X is

$$E[X] = \sum_i x_i \cdot P(X = x_i)$$

Ex: Let S represent rolling two 6-sided dice and X be the sum of the dice.

$$E[X] = (2)\underbrace{\left(\frac{1}{36}\right)}_{P(X=2)} + (3)\underbrace{\left(\frac{2}{36}\right)}_{P(X=3)}$$

$$+(4)\left(\frac{3}{36}\right)+(5)\left(\frac{4}{36}\right)+(6)\left(\frac{5}{36}\right)$$

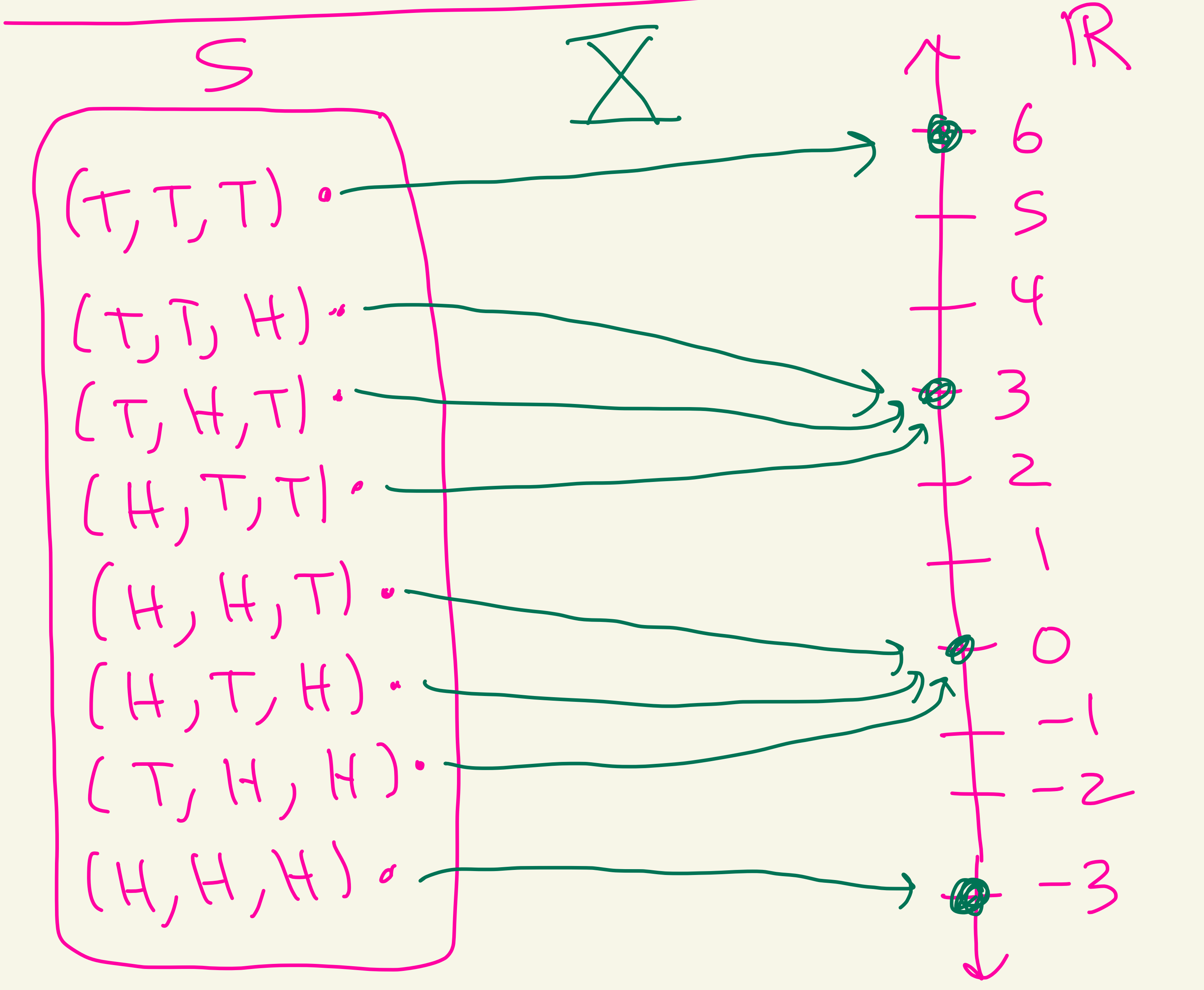
$$+(7)\left(\frac{6}{36}\right)+(8)\left(\frac{5}{36}\right)+(9)\left(\frac{4}{36}\right)$$

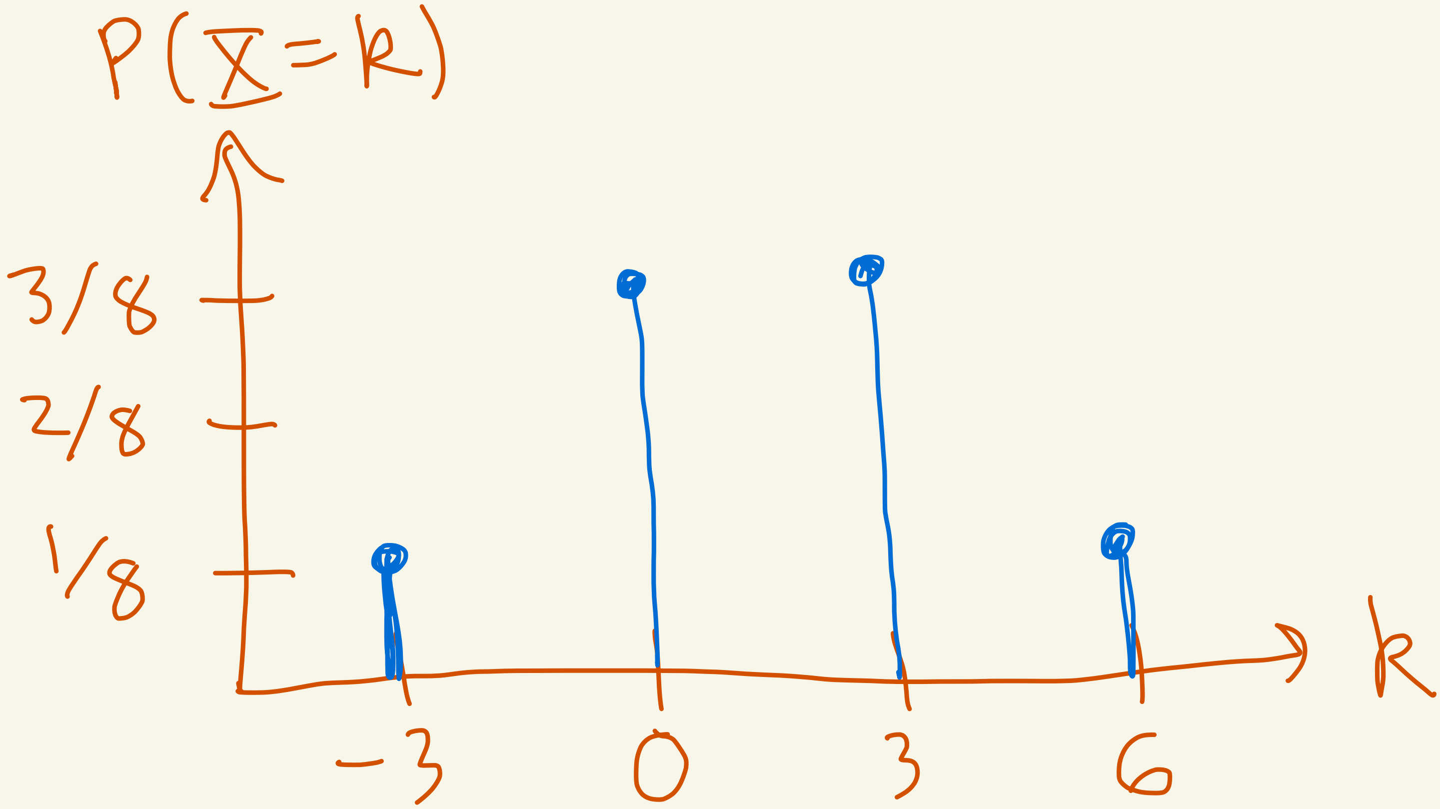
$$+(10)\left(\frac{3}{36}\right)+(11)\left(\frac{2}{36}\right)+(12)\left(\frac{1}{36}\right)$$

$$=\frac{\left(2+6+12+20+30+42+40+36+30+22+12\right)}{36}$$

$$=\frac{252}{36}=7$$

<u>Ex:</u> Suppose you flip a coin three times. For every head you lose $\$1$. For every tail you win $\$2$. Let X be the amount won or lost





Q: What's the probability you win something?

$$P(X>0) = P(X=3) + P(X=6)$$
$$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

Q: What is $E[X]$?

$$E[X] = (-3)\left(\frac{1}{8}\right) + (0)\left(\frac{3}{8}\right) + (3)\left(\frac{3}{8}\right) + (6)\left(\frac{1}{8}\right)$$

$= \$\frac{12}{8} = \$1.5$

This is saying that if you play the game many many times then on average you would expect to win about \$1.50 each time.

So if you played it 1,000,000 times you'd expect to win around $(1{,}000{,}000)(\$1.50)$

$= \$1{,}500{,}000$

Syllabus –

test 1 – 33.3%
test 2 – 33.3%
final – 33.3%

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drop 1 –

$\max\{\text{test 1}, \text{test 2}\} = 50\%$

$\text{final} = 50\%$

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no final –

test 1 – 50%
test 2 – 50%

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final

final – 100%