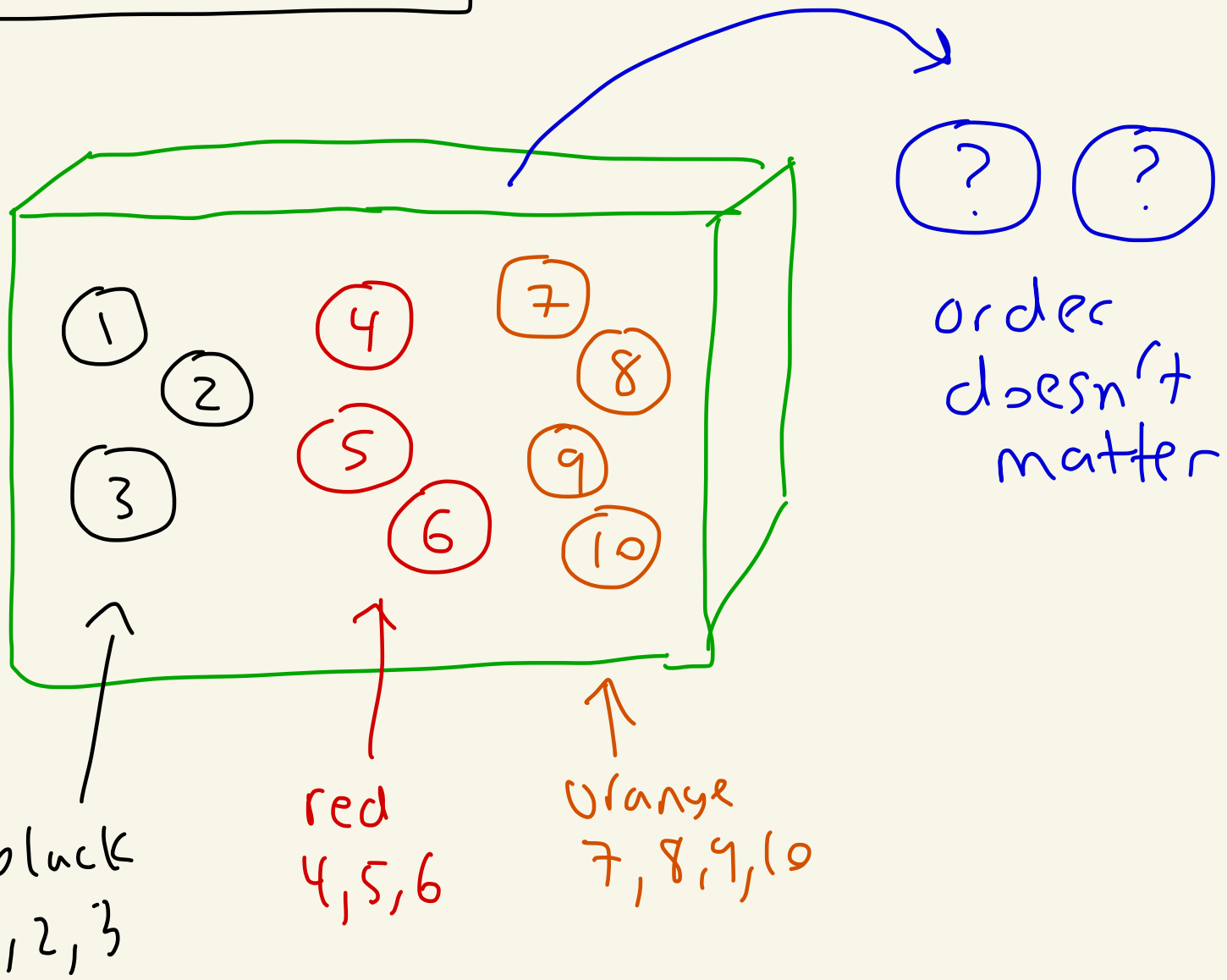
Math 4740

10/7/24

Practice Test #5



5(a) Size of sample space

$$|S| = \binom{10}{2} = \frac{10!}{2!\,8!} = \frac{10 \cdot 9 \cdot \cancel{8!}}{2 \cdot \cancel{8!}}$$

$$= 45$$

5(b) probability 1 black + 1 orange

$$\frac{\overbrace{\binom{3}{1}}^{\text{black}}\overbrace{\binom{4}{1}}^{\text{orange}}}{45} = \frac{3 \cdot 4}{45} = \boxed{\frac{12}{45}}.$$

$$\approx 0.267$$

$$\approx \boxed{26.7\%}$$

5(c) probability both are odd

choose 2 of the 5 balls: ①, ③, ⑤, ⑦, ⑨

$$\frac{\overbrace{\binom{5}{2}}^{\text{choose 2 of the 5 balls}}}{45} = \frac{\frac{5!}{2!3!}}{45} = \frac{\frac{5 \cdot 4 \cdot \cancel{3!}}{2 \cdot \cancel{3!}}}{45}$$

$$= \frac{10}{45} \approx 0.2\overline{2}$$

$$\approx \boxed{22.22\%}$$

Practice Test

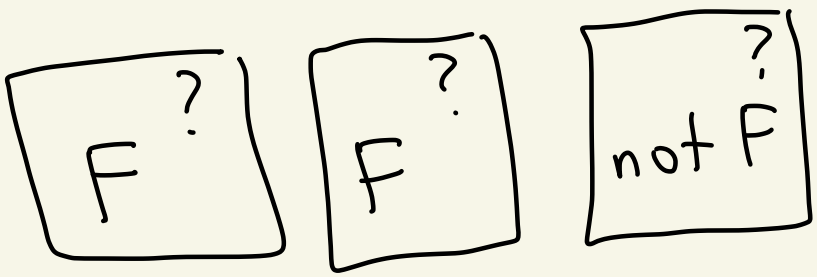
(4) Dealt 3 cards from standard 52 card deck.

(a) $|S| = \binom{52}{3} = \frac{52!}{3!\,49!}$

$= \frac{52 \cdot 51 \cdot 50 \cdot 49!}{6 \cdot 49!}$

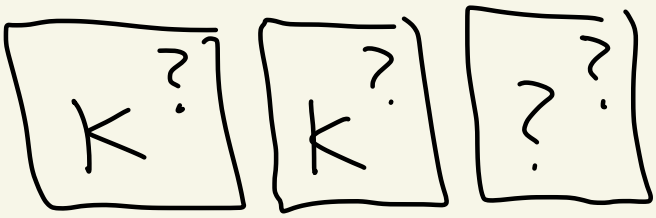
$= \boxed{22,100}$

(b) probability of pair.



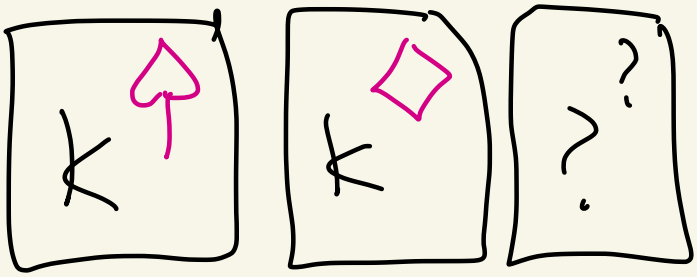
Count pairs

step 1: Pick the face value for the pair



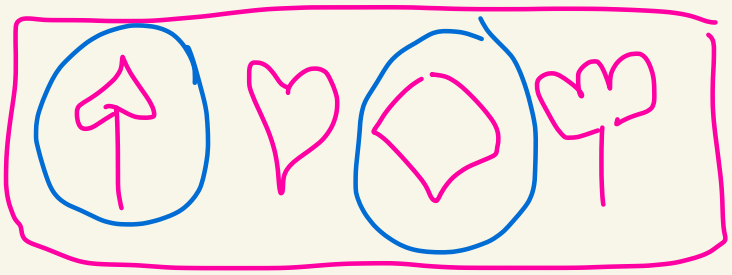
$$\binom{13}{1} = 13$$

Step 2: Fill in the suits for the pair.



$$\binom{4}{2} = 6$$

choose 2

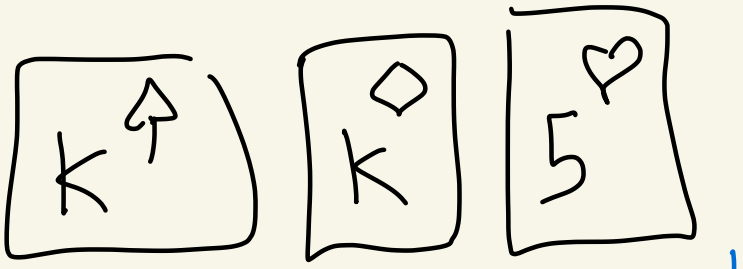


Step 3: Fill in the last card. It can't be the same face value as the pair. So there are

$52 - 4 = 48$ to choose from.

remove the 4 that are the pairs face value

$\binom{48}{1} = 48$



non king

$$\text{Answer} = \frac{13 \cdot 6 \cdot 48}{22,100}$$

$$= \frac{3,744}{22,100} \approx 0.169$$

$$\approx 16.9\%$$

Practice Test

(3) You roll four 8-sided dice. What's the probability you get exactly two 3's?

$|S| = 8^4$

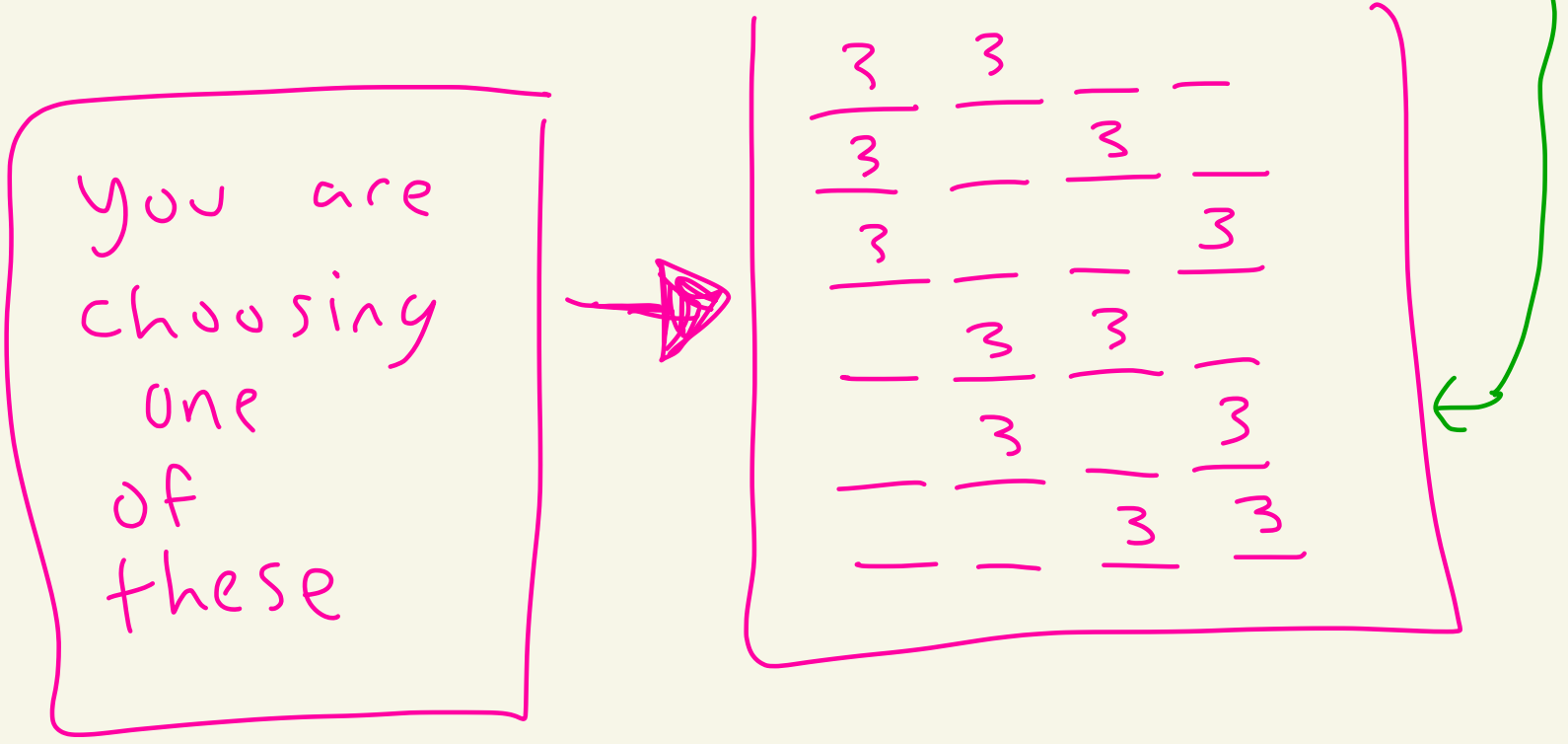
1-8	1-8	1-8	1-8
die 1	die 2	die 3	die 4

Count # ways to get exactly two 3's

Step 1: Pick the two spots for the 3's.

$\binom{4}{2} = 6$

	3		3
die 1	die 2	die 3	die 4



Step 2: Fill in the remaining two spots with non-3's.

$$\underbrace{2}_{\text{not } 3} \quad 3 \quad \underbrace{8}_{\text{not } 3} \quad 3$$

7 possibilities

7 possibilities

$7 \cdot 7 = 49$ possibilities

Answer

$$\frac{6 \cdot 49}{8^4} = \frac{294}{8^4}$$

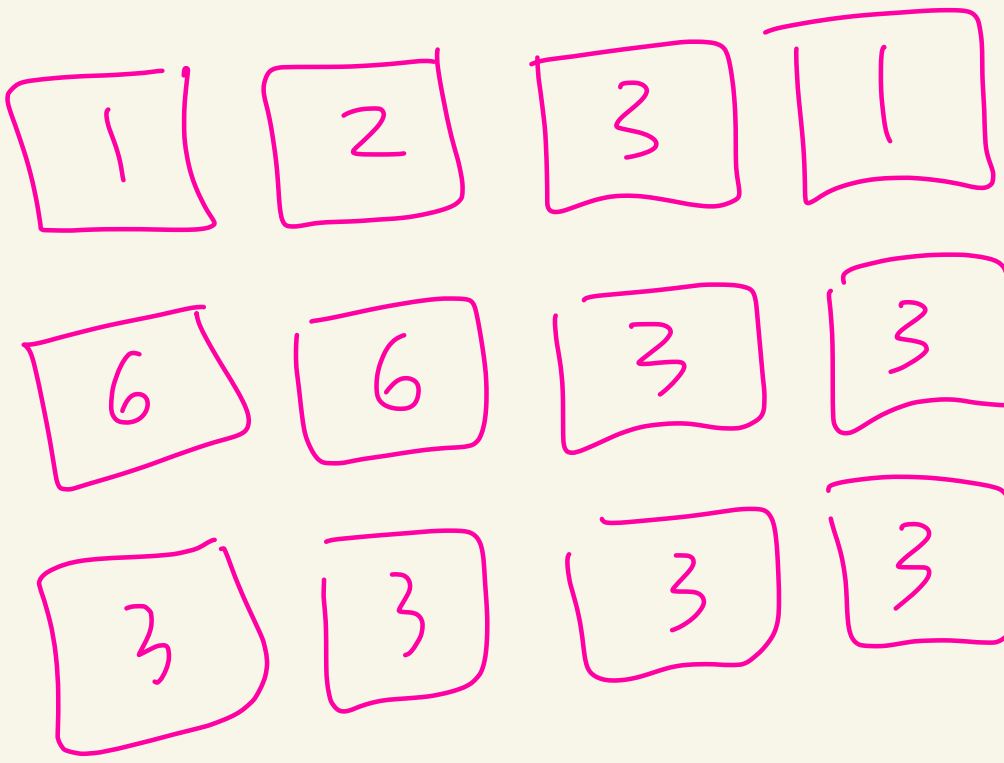
$$\approx 0.071\ldots$$

$$\approx \boxed{7.1\%}$$

Practice test

3(b) Roll 6-sided die 4 times probability 3 occurs at least once

Ex:



Let E be the event that we get at least one 3.

Then $\bar{E}$ there are no 3's.

Count $\overline{E}$

roll 1	roll 2	roll 3	roll 4
not 3	not 3	not 3	not 3

$$\#\text{ways} = 5 \cdot 5 \cdot 5 \cdot 5 = 5^4$$

probability of $\overline{E}$ is $\frac{5^4}{|S|} = \frac{5^4}{6^4}$

probability of E is $1 - \frac{5^4}{6^4}$

$$= \frac{6^4 - 5^4}{6^4}$$

$$\approx \boxed{51.77\%}$$

HW 2

(12) Toss a coin 20 times.

$$|S| = 2^{20}$$

$\underbrace{H/T \quad H/T \quad H/T \quad \cdots \quad H/T}_{20 \text{ times}}$

(a) probability at least two heads occurs, ie two or more heads occuring

E = $\underbrace{\text{at least two heads occuring}}_{\geq 2}$

$\overline{E}$ = $\underbrace{\text{zero or one head occuring}}_{< 2}$

Zero heads: 1 way

T T T T T ... T

One head:

H T T ... T
T H T ... T
T T H ... T
⋮ ⋮
T T T ... H

} 20 ways

So,

$$|\bar{E}| = 1 + 20 = 21$$

So,

$$P(E) = 1 - P(\bar{E}) = 1 - \frac{21}{2^{20}}$$

$\approx$ 99.997997... %

(b)

$P(\underbrace{\text{at most 3 heads}}_{\leq 3 \text{ heads}})$

$= P\left(\begin{matrix}\text{exactly}\\ 0\\ \text{head}\end{matrix}\right) + P\left(\begin{matrix}\text{exactly}\\ 1\\ \text{head}\end{matrix}\right) + P\left(\begin{matrix}\text{exactly}\\ 2\\ \text{heads}\end{matrix}\right)$

$+ P\left(\begin{matrix}\text{exactly}\\ 3\\ \text{heads}\end{matrix}\right)$

$= \frac{1}{2^{20}} + \frac{\binom{20}{1}}{2^{20}} + \boxed{\frac{\binom{20}{2}}{2^{20}}} + \frac{\binom{20}{3}}{2^{20}}$

T H T T H T T ... T T

pick two spots for the heads $\binom{20}{2}$. The rest have to be tails, no choices.