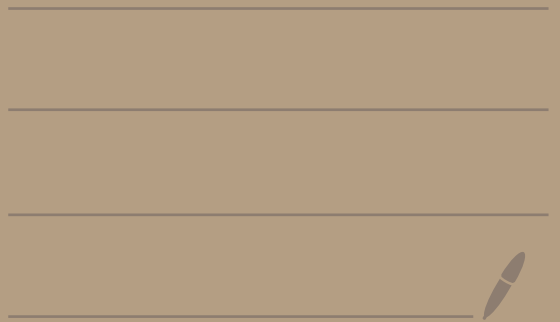
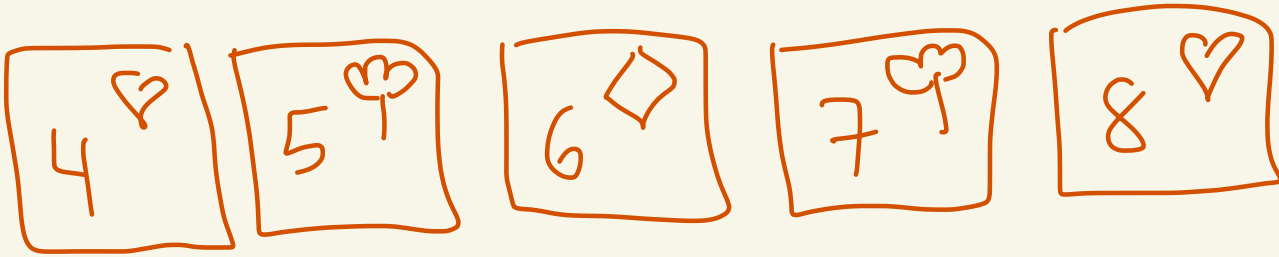


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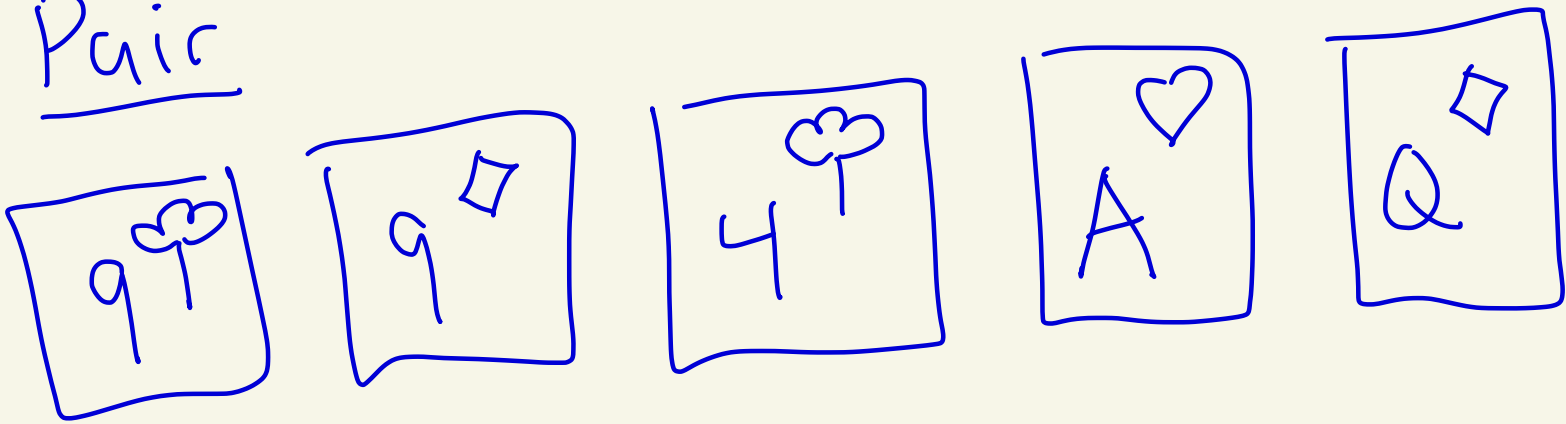


Ex:

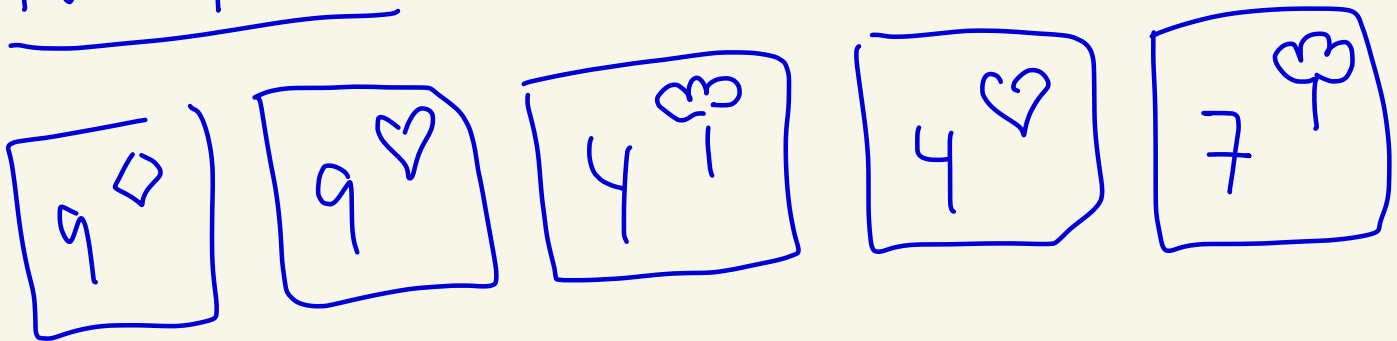
Straight



Pair



Two pair

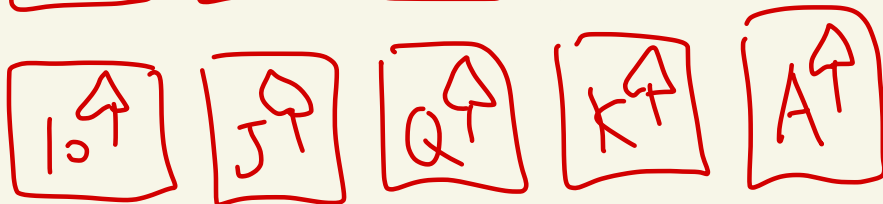
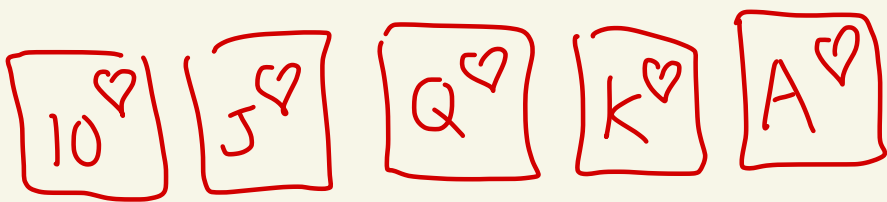


Recall: The number of 5-card
poker hands that exist

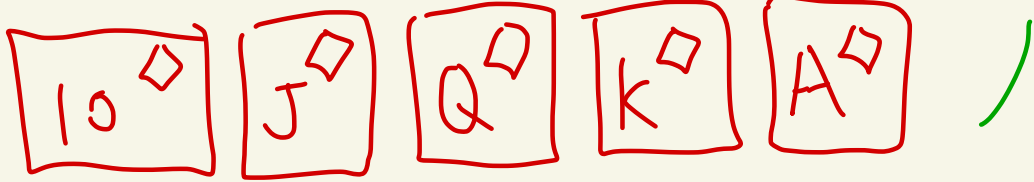
$$\binom{52}{5} = 2,598,960$$

Ex: Suppose you are dealt
5 cards from a standard
52-card deck. What's the
probability you get a royal flush?

royal flushes



} 4 royal
flushes



$$\text{probability of a royal flush} = \frac{\# \text{ royal flushes}}{\# \text{ possible 5-card hands}}$$

$$= \frac{4}{2,598,960}$$

$$\approx 0.000001539\dots$$

$$\approx 0.0001539\%$$

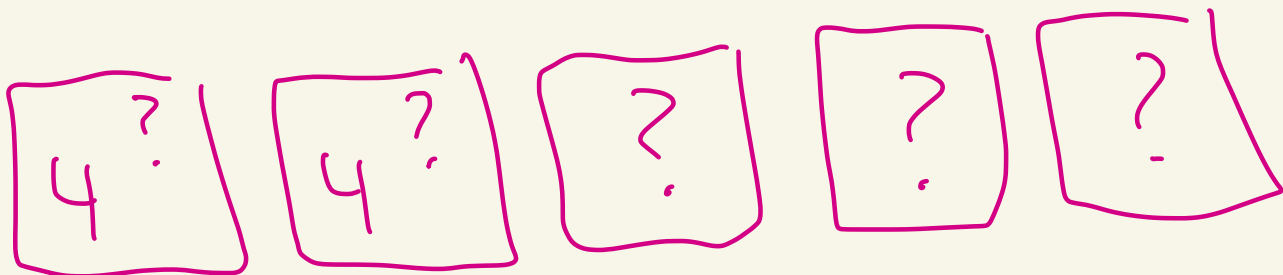
Ex: What's the probability you get a pair and nothing better?

Let's count how many pairs there are.

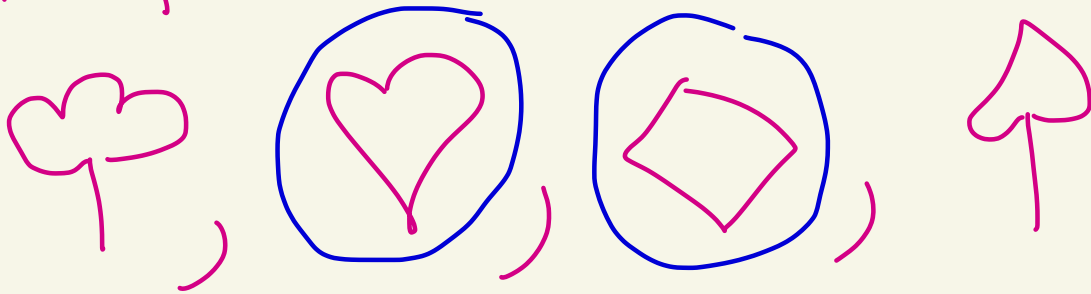
Step 1: Pick a face value for the pair.

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

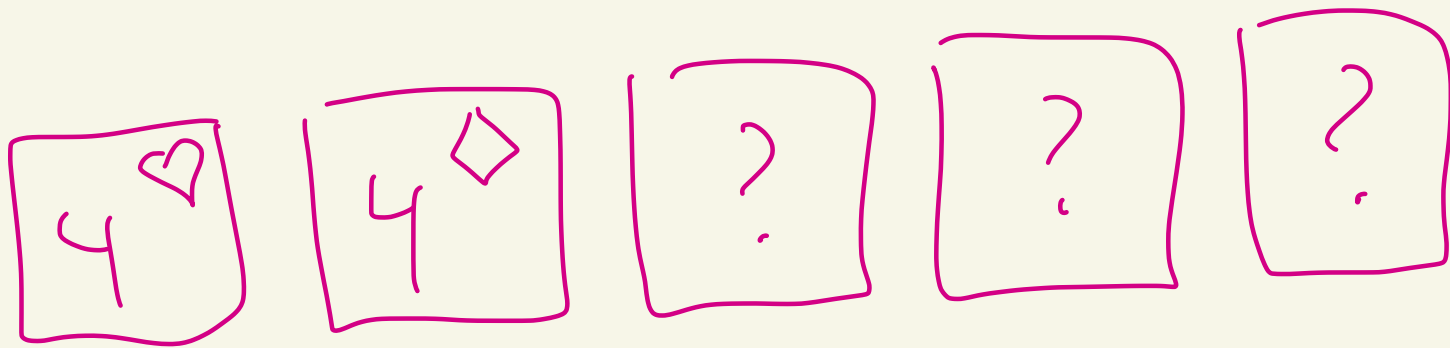
$$\# \text{ ways} = \binom{13}{1} = 13$$



Step 2: Pick 2 suits for
the pair.



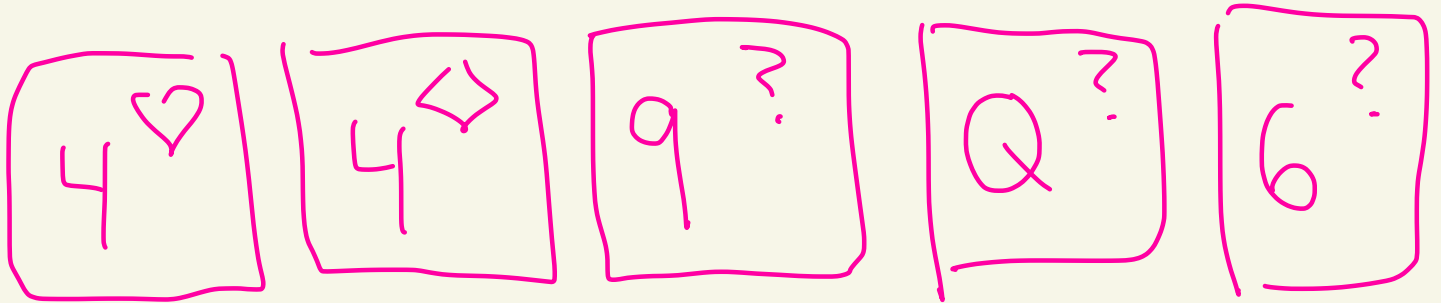
$$\# \text{ ways} = \binom{4}{2} = \frac{4!}{2!2!} = 6$$



Step 3: Pick the remaining
3 face values. They can't
be the same & they can't
be the pairs face value

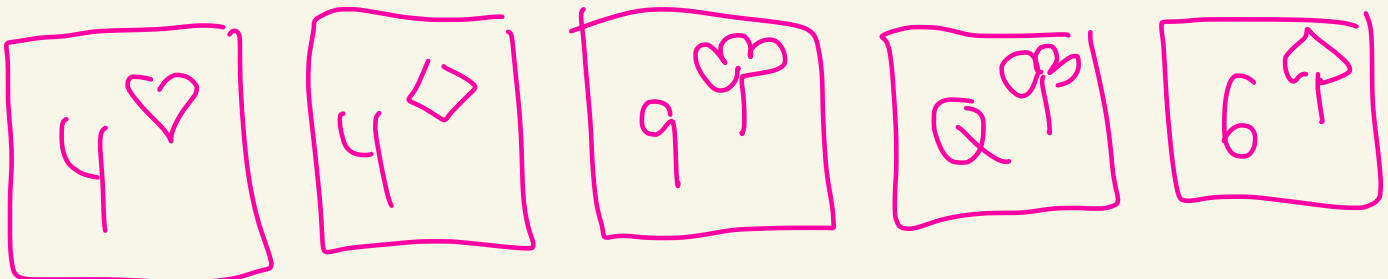
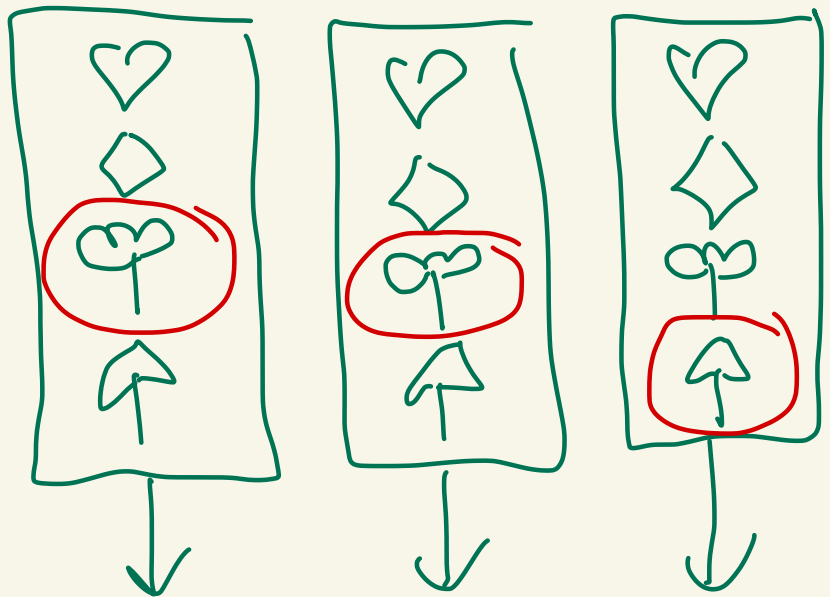
A, 2, 3, ~~4~~, 5, 6, 7, 8, 9, 10, J, Q, K

$$\# \text{ ways} = \binom{12}{3} = \frac{12!}{3!9!} = 220$$



Step 4: Fill in the remaining 3 suits.

$$\begin{aligned} \# \text{ ways} &= \\ & \binom{4}{1} \binom{4}{1} \binom{4}{1} \\ & = 4^3 = 64 \end{aligned}$$



total # of pairs (and not better)

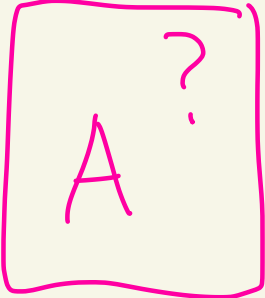
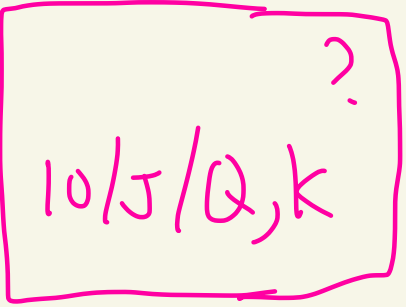
$$= \underbrace{13}_{\text{step 1}} \cdot \underbrace{6}_{\text{step 2}} \cdot \underbrace{220}_{\text{step 3}} \cdot \underbrace{64}_{\text{step 4}} = 1,098,240$$

probability is $\frac{1,098,240}{2,598,960}$

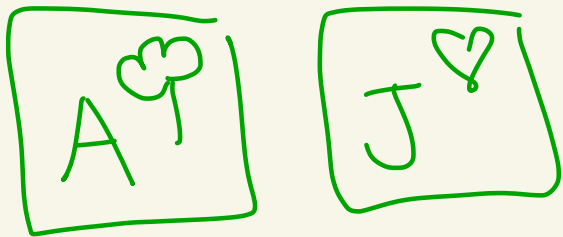
$$\approx 0.422569\dots$$

$$\approx 42.2569\%$$

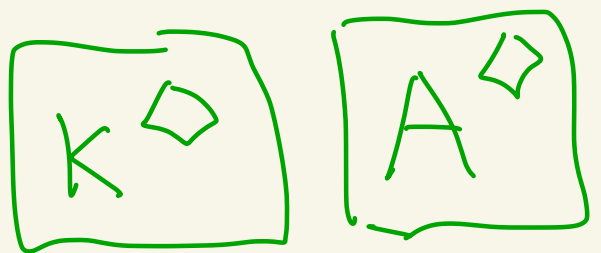
Ex: Suppose you are dealt 2 cards from a standard 52-card deck. What's the probability you get a blackjack?

Blackjack =  

Ex:



Ex:



Sample space size

$$\binom{52}{2} = \frac{52!}{2!50!} = \frac{52 \cdot 51 \cdot \cancel{50!}}{2! \cdot \cancel{50!}}$$

$$= \frac{52 \cdot 51}{2} = 1326$$

of possible
2 card hands

How many blackjacks are there?

Step 1: Pick the ace.

$A\heartsuit, A\spadesuit, A\clubsuit, A\spadesuit$

$$\# \text{ ways} = \binom{4}{1} = 4$$

$A\spadesuit$

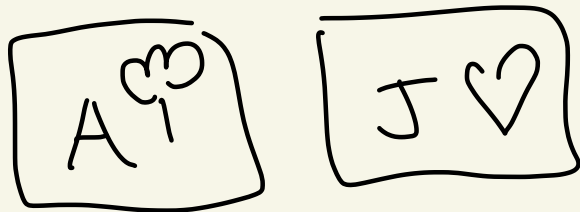
?

Step 2: Pick the next card.

10♥, 10♣, 10♠, 10♦
J♥, J♠, J♠, J♦
Q♥, Q♣, Q♠, Q♠
K♥, K♣, K♠, K♦

pick
of 1
of these

$$\# \text{ ways} = \binom{16}{1} = 16$$



$$\# \text{ of blackjack} = \underbrace{4}_{\text{step 1}} \cdot \underbrace{16}_{\text{step 2}} = 64$$

$$\text{probability} = \frac{64}{1326} \approx 0.048265... \\ \approx 4.8265... \%$$