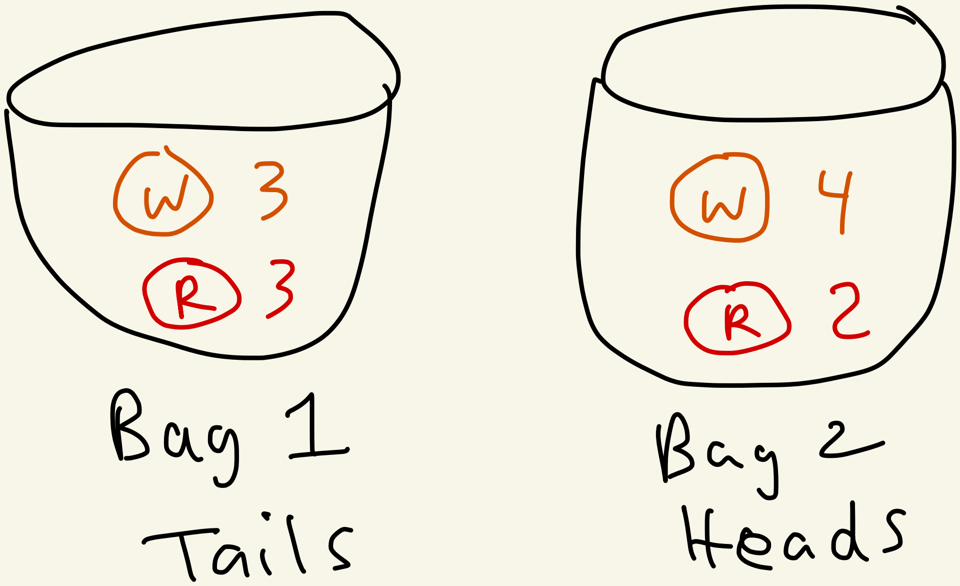
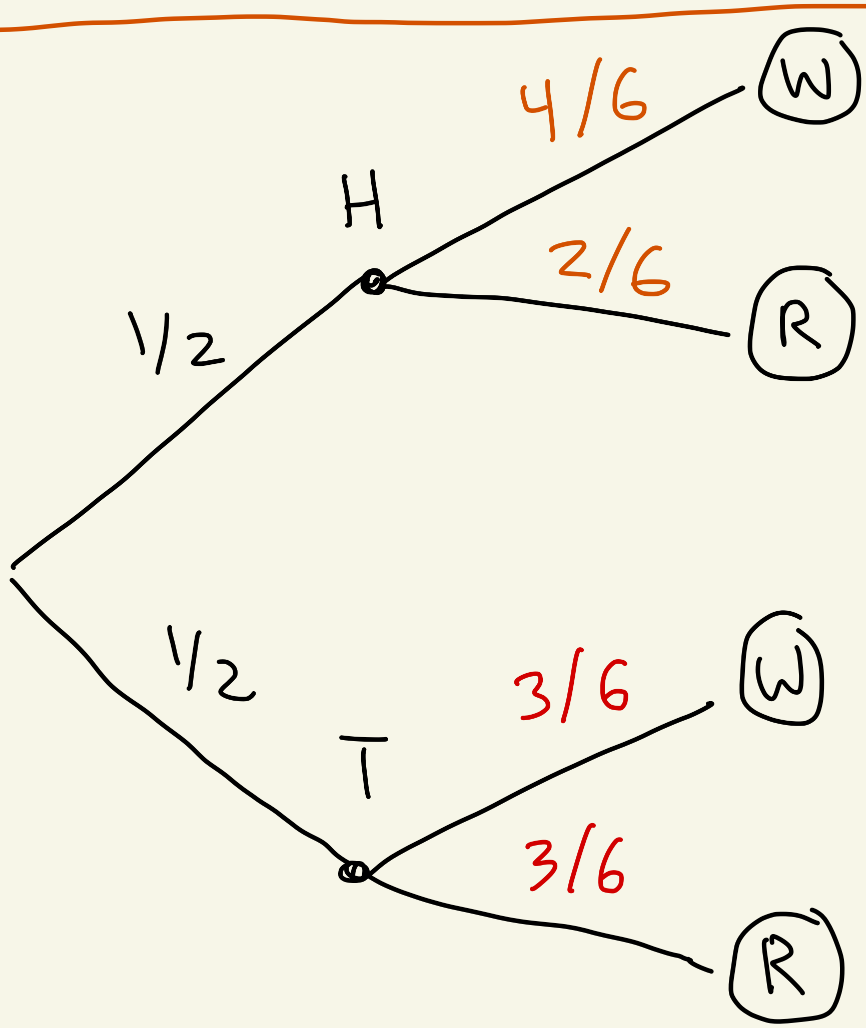
4740
3/17/25

HW 3

4





(a) $P(\text{red ball}) = \frac{1}{2} \cdot \frac{2}{6} + \frac{1}{2} \cdot \frac{3}{6} = 5/12$

(b) $P(\text{white ball}) = \frac{1}{2} \cdot \frac{4}{6} + \frac{1}{2} \cdot \frac{3}{6} = 7/12$

HW 3

(8) coin flipped 3 times

A = at least one head

B = at least two heads

Want $P(B|A)$

Method 1

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$= \frac{4/8}{7/8} = 4/7$$

$A \cap B = \{(H,H,T), (H,T,H), (T,H,H), (H,H,H)\}$

$A = \{(H,T,T), (T,H,T), (T,T,H),$
$(H,H,T), (H,T,H), (T,H,H)$
$(H,H,H)\}$

$\boxed{\text{Method 2}}$

Given A has happened, It's the new sample space,

So you just do $\dfrac{|A \cap B|}{|A|} = \dfrac{4}{7}$

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9 (REDO - Method 2)

Dealt 2 cards

B = both cards aces

A = at least one ace

A_s = one is A♠

(a) $P(B \mid A_s)$

(b) $P(B|A)$

Last time we used

$$P(E|F) = \frac{P(E \wedge F)}{P(F)}.$$

Let's try another way.

(a) Given: $\boxed{A\spadesuit}$ $\boxed{?}$

51 possibilities

3 with ace here

new sample space size 51.

$$\{ \boxed{A\spadesuit}\,\boxed{2\clubsuit},\ \boxed{A\spadesuit}\,\boxed{A\diamondsuit},\ \boxed{A\spadesuit}\,\boxed{K\heartsuit},\ \ldots \}$$

3 have another ace.

Answer = 3/51

(b) Use original way

HW 2

15 Dealt 2 cards from 52-card deck.

(a) P(both aces)

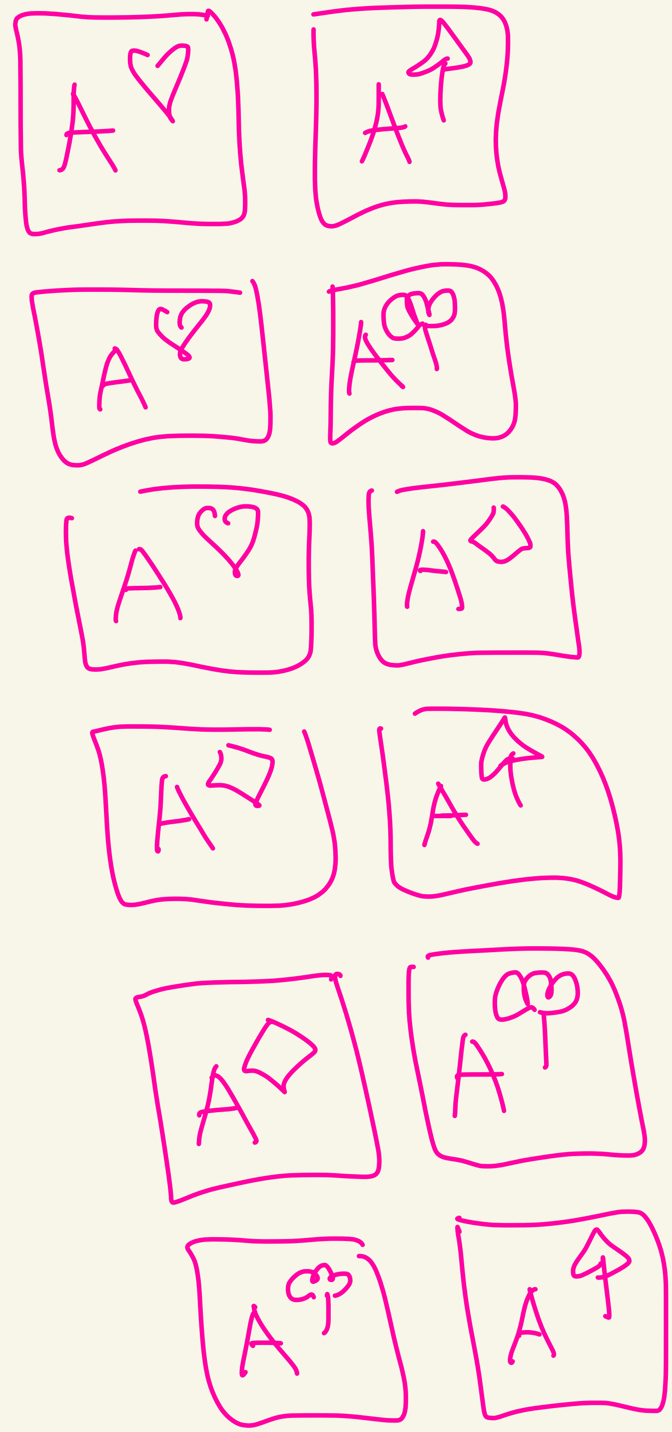
(b) P(both same face value)

~~(c) P(blackjack)~~

$$|S| = \binom{52}{2} = \frac{52!}{2!\,50!} = \frac{52 \cdot 51 \cdot \cancel{50!}}{2 \cdot \cancel{50!}}$$

$$= 1326$$

(a) $\boxed{A^?}$ $\boxed{A^?}$ & how many $= \binom{4}{2} = 6$



Answer

6/1326

(b) P(both same face value)

13 face values

A? A? ← $\boxed{\binom{4}{2} = 6}$

2? 2? ← 6

3? 3? ← 6

⋮

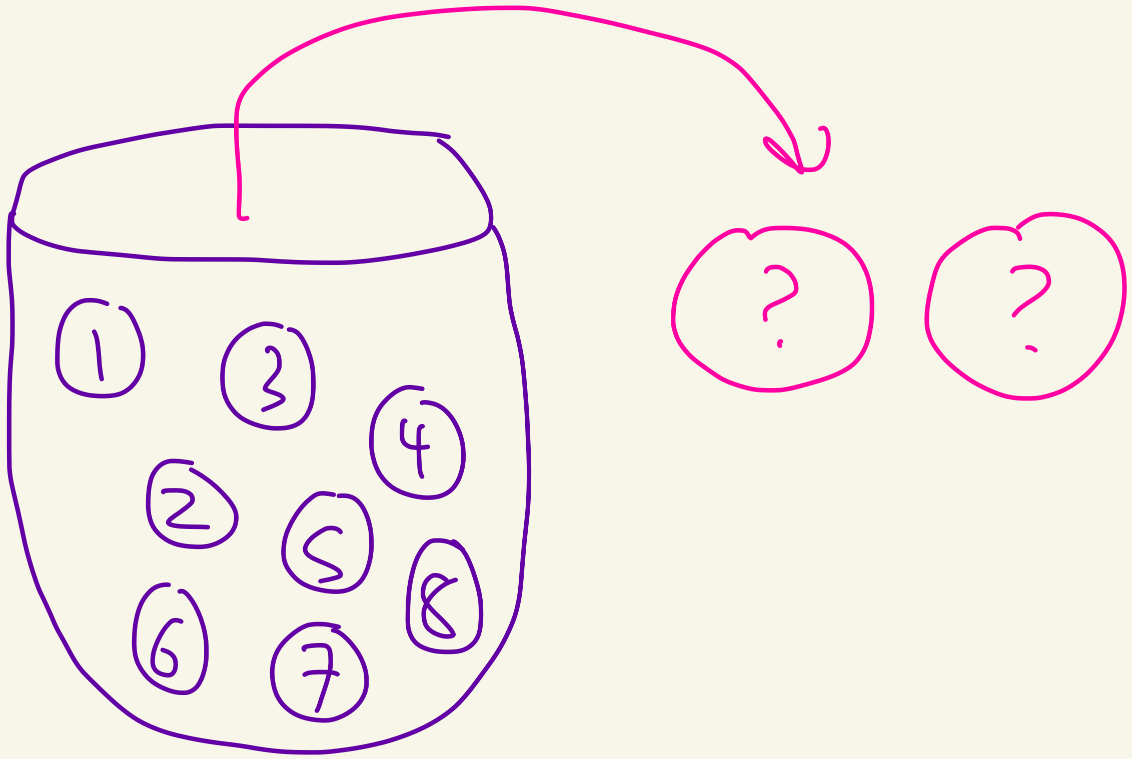
Q? Q? ← 6

K? K? ← 6

<u>Answer</u> = $\cancel{\dfrac{13 \cdot 6}{1326}} = \dfrac{78}{1326}$

HW 2

13



$$P(\text{both even}) = \frac{\binom{4}{2}}{\binom{8}{2}} = \frac{6}{28}$$

$$\binom{8}{2} = \frac{8!}{2!\,6!} = \frac{8 \cdot 7 \cdot \cancel{6!}}{2!\,\cancel{6!}} = 28$$

both even

2 4 | 2 8 | 4 8

2 6 | 4 6 | 6 8

first even picked
second even

$$\text{order matters} = \frac{4 \cdot 3}{8 \cdot 7} = \frac{12}{56} = 6/28$$

first ball picked
second picked

HW 2

16(a) 5 card poker hand

$P(\text{flush})$

flush means all same suit

ex: 2♦ J♦ Q♦ 3♦ 10♦

$$|S| = \binom{52}{5} = \frac{52!}{5!\,47!} = \frac{52\cdot 51\cdot 50\cdot 49\cdot 48\cdot \cancel{47!}}{120\cdot \cancel{47!}}$$

$$= 2{,}598{,}960$$

How many flushes are there?

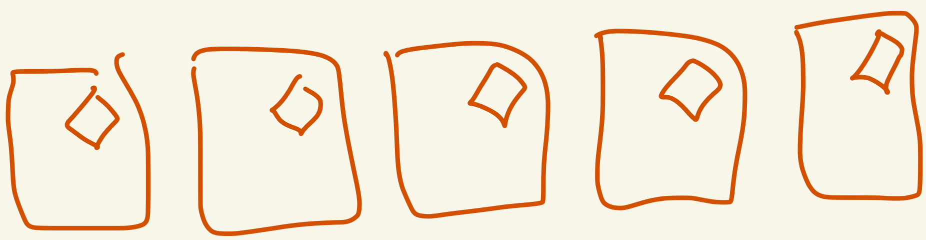
Step 1:

Pick suit.

$\binom{4}{1} = 4$

♥, ♦, ♣, ♠

Ex:



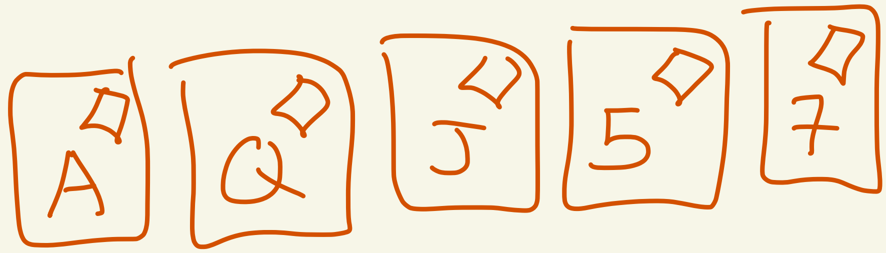
Step 2:

Fill in face values

$\binom{13}{5} = \frac{13!}{5!\,8!} = 1287$

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Ex:



Answer $= \dfrac{4 \cdot 1287}{2{,}598{,}960}$

$= \boxed{\dfrac{5148}{2{,}598{,}960}}$

HW 2

6 (c) Roll four 8-sided dice

P(at least three 1's)

1-8   1-8   1-8   1-8

$|S| = 8 \cdot 8 \cdot 8 \cdot 8 = 8^4$

$= 4096$

At least three 1's

1 1 1 1 ← four 1's, 1 way

exactly three ones

1 1 1 ?
- 1 1 1 2
- 1 1 1 3
- 1 1 1 4
- 1 1 1 5
- 1 1 1 6
- 1 1 1 7
- 1 1 1 8

1 1 ? 1

1 ? 1 1

? 1 1 1

$$\overbrace{\binom{4}{3}}^{\text{pick where 1's go}} \cdot \underbrace{7}_{\text{fill in other spot not 1}} = 4 \cdot 7 = \boxed{28 \text{ ways}}$$

Answer $= \dfrac{28+1}{4096} = \dfrac{29}{4096}$