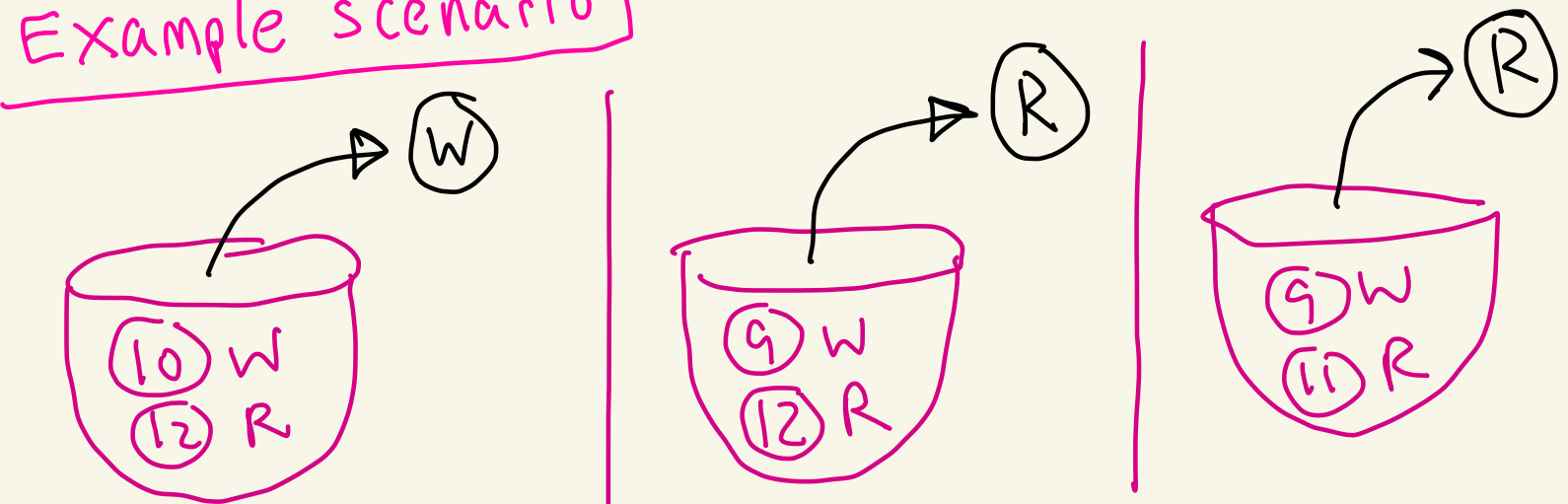
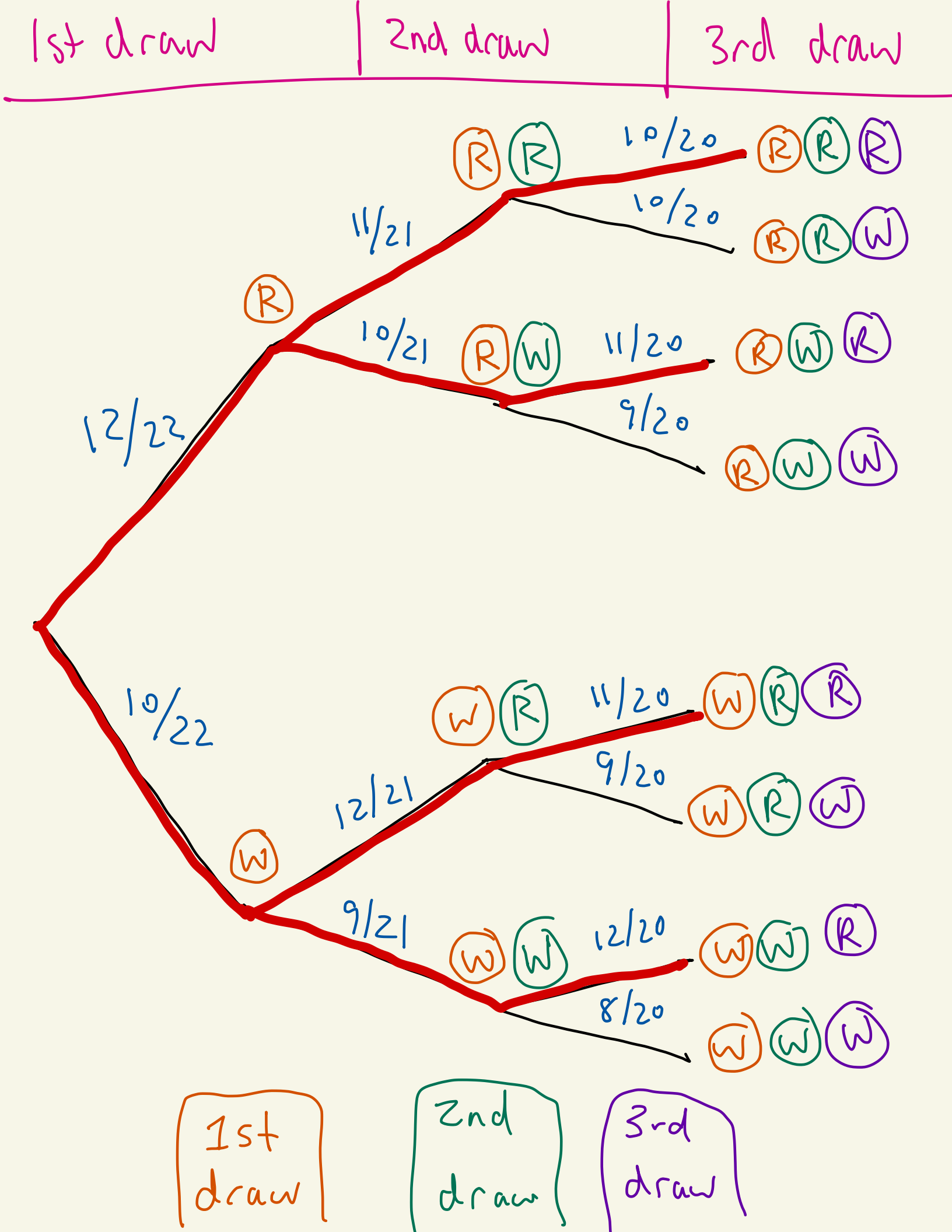
Math 4740
3/3/25

(Topic 3 continued...)

HW 3 ⑥ A bag has 10 white coins and 12 red coins. One coin is randomly drawn from the bag and discarded (you don't put it back in the bag). Then a second coin is drawn and discarded. Then a third coin is drawn. What's the probability that the third coin is red?

Example scenario




1st draw
2nd draw
3rd draw
12/22
10/22
R
W
11/21
10/21
12/21
9/21
RR
RW
WR
WW
10/20
11/20
9/20
12/20
8/20
RRR
RRW
RWR
RWW
WRR
WRW
WWR
WWW
1st draw
2nd draw
3rd draw

Answer:

$$\frac{12}{22}\cdot\frac{11}{21}\cdot\frac{10}{20}+\frac{12}{22}\cdot\frac{10}{21}\cdot\frac{11}{20}$$

$$+\frac{10}{22}\cdot\frac{12}{21}\cdot\frac{11}{20}+\frac{10}{22}\cdot\frac{9}{21}\cdot\frac{12}{20}$$

$$=\boxed{\frac{6}{11}}$$

HW 3

(9) Suppose two cards are randomly dealt from a standard 52-card deck.

Let B be the event that both cards are aces.

Let A be the event that at

least one card is an ace.

Let A_s be the event that one of the cards is an ace of spades.

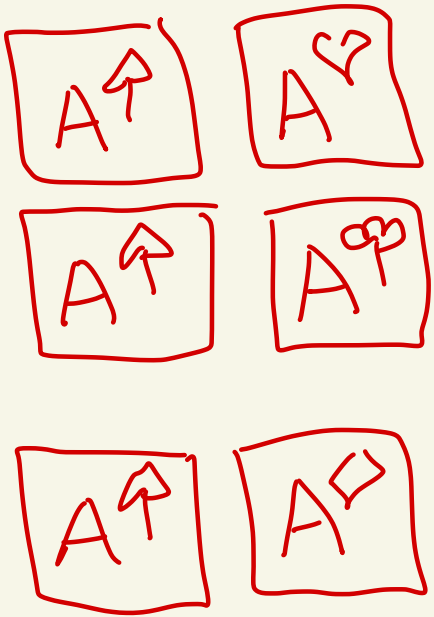
(a) Compute $P(B \mid A_s)$

(b) Compute $P(B \mid A)$

$|S| = \binom{52}{2} = 1326$

(a) Use $P(B \mid A_s) = \dfrac{P(B \cap A_s)}{P(A_s)}$

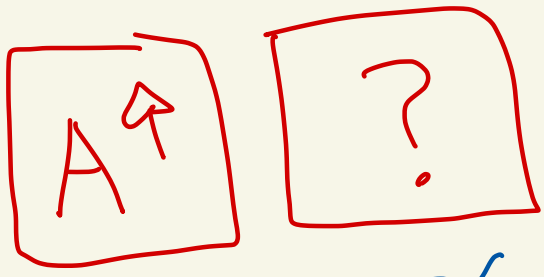
Elements in $B \cap A_s$



$|B \cap A_s| = 3$

$P(B \cap A_s) = \dfrac{3}{1326}$

Elements in A_s



$\leftarrow |A_s| = 51$

51 possibilities

$P(A_s) = \frac{51}{1326}$

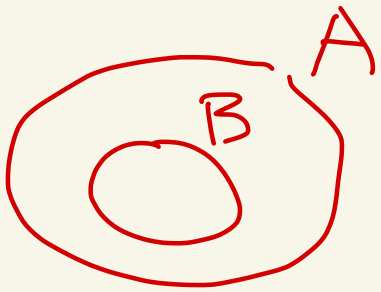
Answer:

$$P(B|A_s) = \frac{3/1326}{51/1326} = 3/51$$

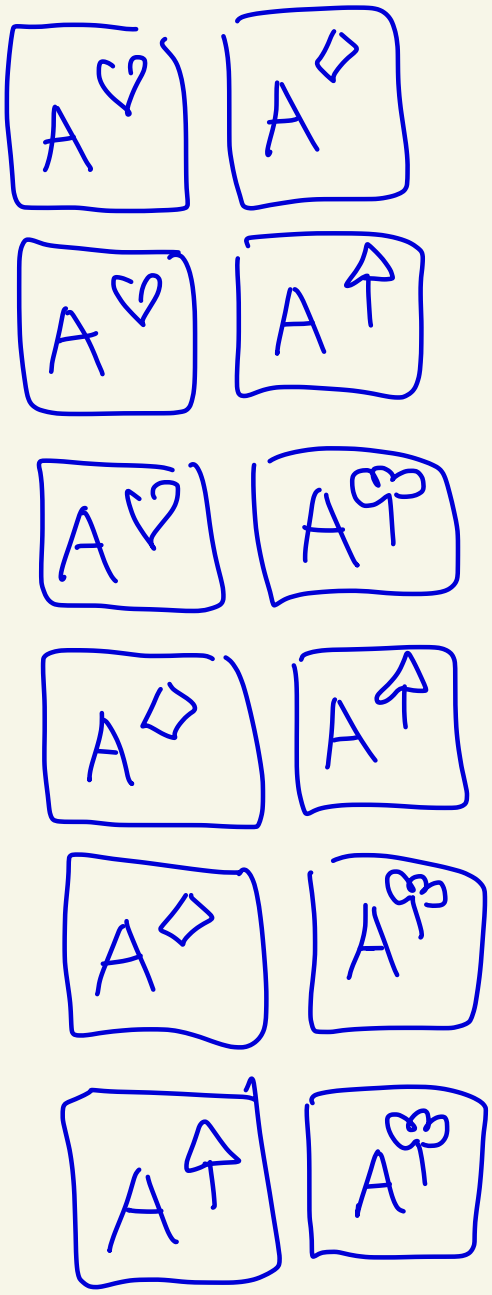
(b) Now let's compute

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Note that $B \cap A = B$



Elements of B



$|B| = 6$

or

$|B| = \binom{4}{2}$ ← pick 2 from 4 aces

$P(B \cap A) = P(B) = \frac{6}{1326}$

Now let's compute $P(A)$

A is at least one ace.

$\bar{A}$ is no aces

remove the 4 aces

$52-4$

$$|\bar{A}| = \underbrace{\binom{48}{2}}_{\text{pick 2 non-ace cards}} = \frac{48!}{2!46!} = \frac{48 \cdot 47}{2}$$

$$= 1128$$

So, $P(A) = 1 - P(\bar{A}) = 1 - \frac{1128}{1326}$

$$= \frac{198}{1326}$$

Answer:

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{6/1326}{198/1326} = \boxed{\frac{6}{198}}$$

$$\approx \boxed{0.03} \approx \boxed{3\%}$$