Math 4740

8/21/24

Topic 1 - Sets and probability spaces

Def: A **set** is a collection of objects or elements.

If x is an element of a set S, then we write $x \in S$.

read: "x is in S"

If x is not in S, then we write $x \notin S$.

If S has a finite number

of elements, then we write $|S|$ to denote the number of elements.

Ex: Let's make a set to model rolling a six-sided die.

Let

$$S = \{1, 2, 3, 4, 5, 6\}$$

outcomes of rolling a 6-sided die

We have

$$|S| = 6$$

$$2 \in S$$

$$7 \notin S$$

Later we will call S the sample space

Note: In a set order doesn't matter. Ex:

$\{1,2,3,4,5,6\} = \{2,3,1,5,4,6\}$

Also sets can't have duplicates.

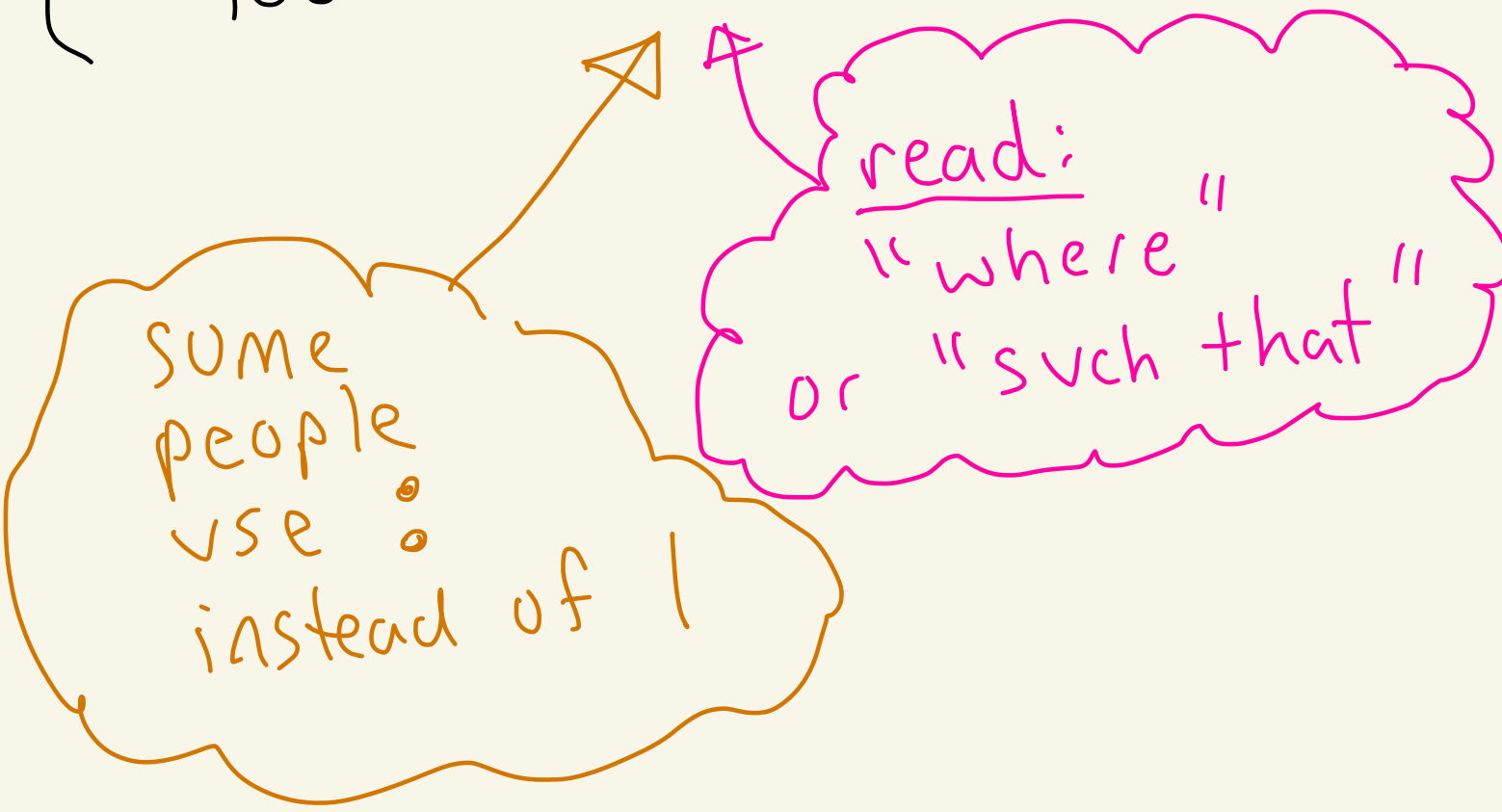
For example, $\{1,2,1\}$

is not a set because

1 is duplicated.

General way to describe a set

$$\{ \text{description of what the elements look like} \mid \text{conditions that elements must satisfy to be in the set} \}$$



Ex: Let's make a set to model the outcomes of rolling two 6-sided dice, one green and one red.

$$S = \{ (g,r) \mid \begin{matrix} g = 1,2,3,4,5,6 \\ r = 1,2,3,4,5,6 \end{matrix} \}$$

$$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$(2,5)$ means green die is 2, red die is 5

$(5,2)$ means green die is 5, red die is 2

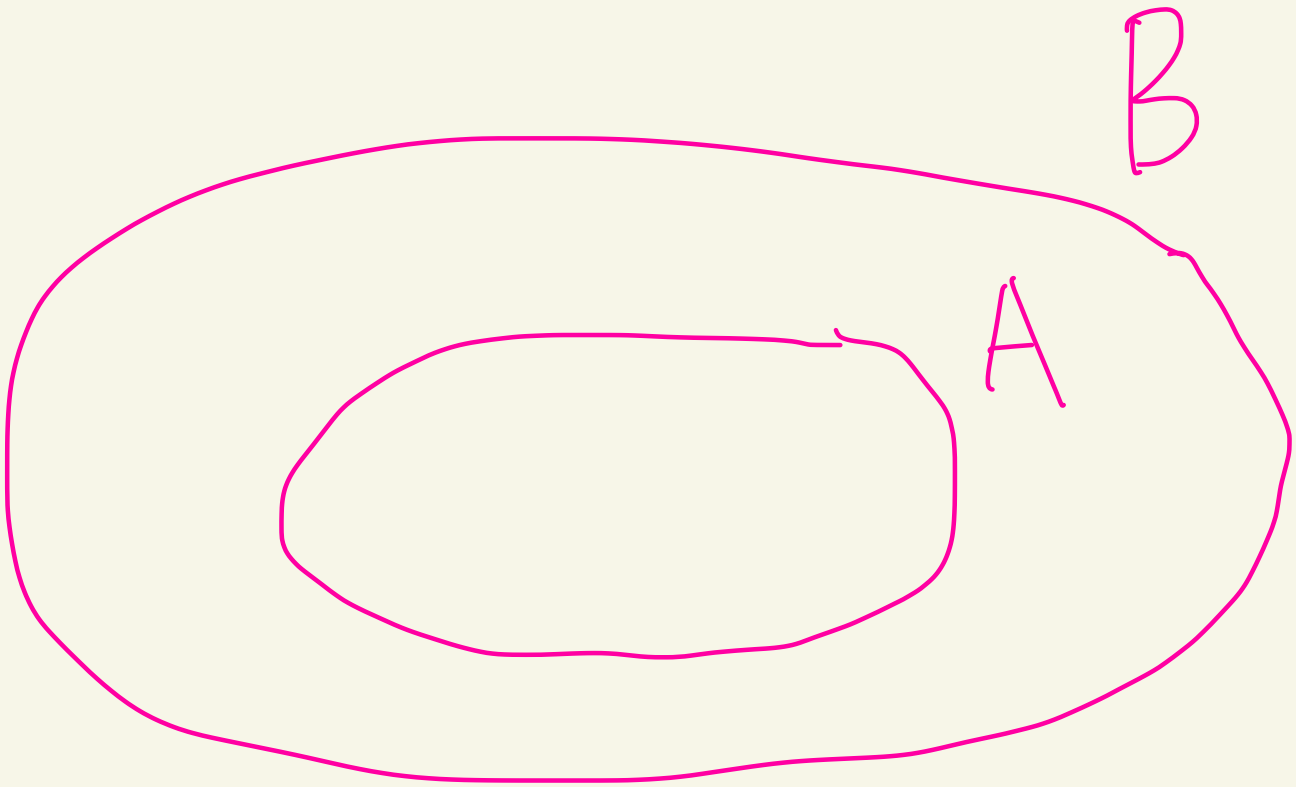
$|S| = 36$

We use ordered pairs (g,r) because order matters with the dice.

$(2,5)$ ← order matters

$\{2,5\}$ ← order doesn't matter

Def: Let A and B be sets. We say that A is a subset of B, and write $A \subseteq B$, if every element of A is also an element of B.

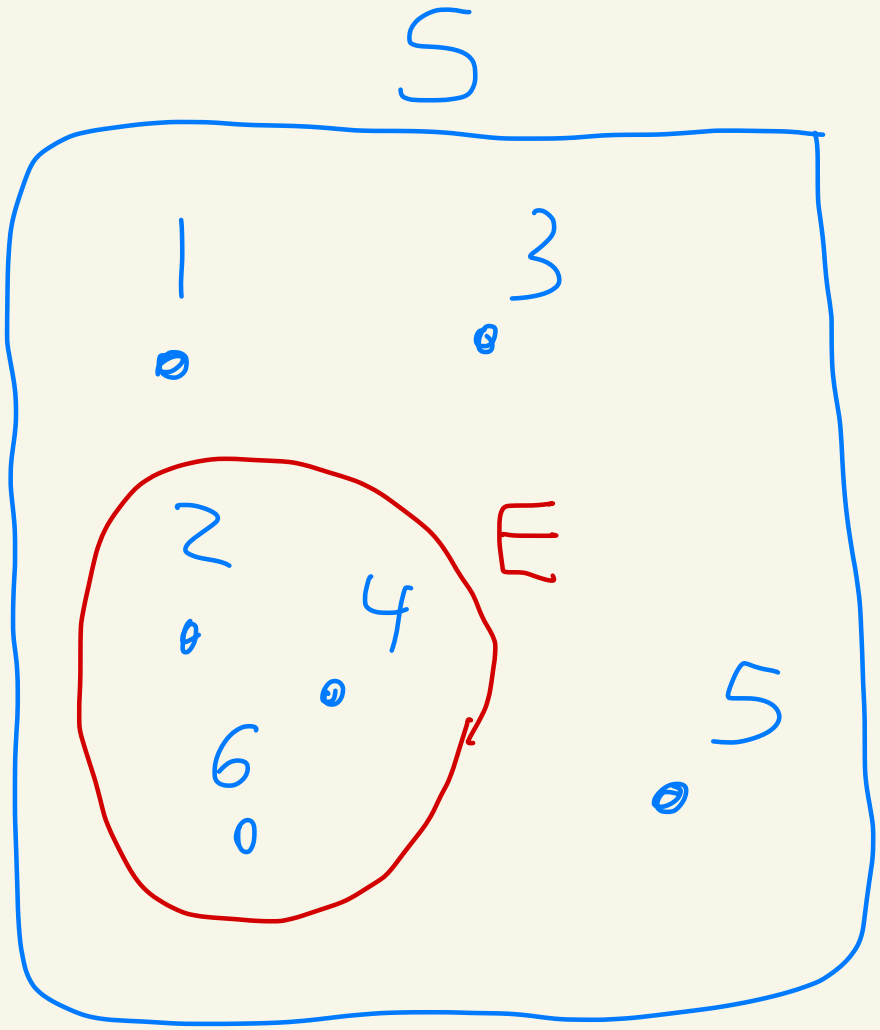


Ex:

$S = \{1, 2, 3, 4, 5, 6\}$ ← rolling a 6-sided die

$E = \{2, 4, 6\}$

Then $E \subseteq S$.



Later in the class E will represent the event that the die is an even number, ie either 2, 4, or 6 is rolled.

Ex: Suppose we roll two 6-sided dice, one green and one red.

$$S = \{(g,r) \mid \begin{matrix} g = 1,2,3,4,5,6 \\ r = 1,2,3,4,5,6 \end{matrix}\}$$

$$= \{(1,1), (1,2), \ldots, (5,6), (6,6)\}$$

32 more

Let's make a subset E of S that represents the outcomes where the sum of the dice is 7.

$$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

Note $|E| = 6$ and $|S| = 36$