

Def: Suppose 
$$
A_{1}, A_{2}, A_{3}, \ldots
$$

\nare an infinite number of sets.

\nThen,

\n
$$
\bigcap_{\tilde{\lambda}=1} A_{\tilde{\lambda}} = A_{1} \bigcap A_{2} \bigcap A_{3} \bigcap \cdots
$$
\n
$$
\overline{\lambda} = 1
$$
\n
$$
= \left\{ \times \mid \begin{array}{c} \times \text{is in every} \\ \text{one of the } A_{\tilde{\lambda}} \end{array} \right\}
$$

 $\bigcup_{i=1}^{\infty} A_{i} = A_{1} \cup A_{2} \cup A_{3} \cup \cdots$ <br>  $= \left\{ x \mid \begin{matrix} x & \text{is in at least} \\ \text{one of the } A_{i} \end{matrix} \right\}$ 

Lef: Let <sup>A</sup> and <sup>B</sup> be sets. A <u>function</u> f from  $A$  to  $B$ , Vnctiun<br>notated f: A-B B, is <sup>a</sup> role that assigns to each element of <sup>A</sup> a unique element Def: Let A and B be<br>Sets. A function f from<br>A to B, notated f: A >B<br>is a rule that assigns<br>to each element of B<br>a vingue element of B<br>Ex: Let<br>S = {(H, H), (H, T), (T, T)}<br>regresent flipping a coin twice.<br>Let f: S > IR wher of B

Ex: Let  $S = \left\{ (H,H), (H,J), (T,H), (T,T) \right\}$  $EX: LET$ <br>  $S = \{(H,H), (H,T), (T,H), (T,T)\}$ <br>
represent flipping a coin twice.  $\frac{1}{2}$ reprisent flippi > IR where coin twice.<br>where f<br>heads occured. counts how many So<sub>J</sub> f(H,  $H = Z$   $f(T)$  $f(T, H) = 1$  $f(H,T) = 1$   $f(T,T) = 0$ 



Later in the class, f will be called a class, f will<br>random variable.

Example of making a probability space  $\frac{1}{5}$  X W Suppose we want to model the suppose voe in slling a 4-sided die Let  $S = \{1, 2, 3, \}$ ant to model inc<br>rolling a 4-sided<br>4 } <sup>S</sup> is called the sample space S is called the sample space of<br>S is called the sample space of  $Let \frac{0mega}{6}$  $r_s$  cancor<br>all possible outcome<br>Let  $(mega)$ <br> $\Omega = 54, 813,$  $\{2\}, 23,$ 245<br>2<br>3<br>...  $\{1,2\},$   $\{1,3\},$  $\{ |y| \}$  $\{2,3\}$  ,  $\{2,4\}$  ,  ${23,45}$  $\left\{ \begin{array}{c} 1 \\ 2 \end{array} \right\}$ 33, {1,2,4}, {1,3,4}<br>2 x = 2 422 ,  $\{2,3,4\}$ ,  $\{1,2,3,4\}$ -2 is the set that contains all the subsets of S.

IL is called the set of events. I contains all the events that we want to be able to measure the probability of. When S is finite (like now) 2 contains all the events t<br>We want to be able to<br>measure the probability of<br>we usually make R consist<br>we usually make R consist<br>of all subsets of S. weasure ine lie<br>we usually make  $\Omega_c$ <br>of all subsets of S. do these events mean? What  $f_S$  no number  $\phi \leftarrow \frac{represents^{nts}}{appeared}$  on the die appeared on the die<br>appeared on the die when you 2 represents <sup>a</sup>  $\{23\}$  occured when you rolled the die represents either  ${5,4} <$  $43 <$  represents clinced

When you roll the die  $t_1$  that  $\{2,3,4\} \leftarrow$  represen either 2 or <sup>3</sup> or <sup>4</sup> occured represents that  $\{1, 2, 3, 4\} \leftarrow \begin{matrix} \text{represen} \ \text{pither} \end{matrix}$  $3,43 \leftarrow$  represents<br>either los 2 or <sup>4</sup> occured 3 or Now we make a probability Now we make a proba<br>function P:  $\Omega \rightarrow \mathbb{R}$ Let's assume each side of the die is equally likely. the die is equally likely.<br>First assign  $P(\phi) = 0$ 

Then assign the probability of each outcome.  $P(\{1\}) = \frac{1}{4}$  ( these add  $P(\{23\}) = \frac{14}{14}$  $P(\{33\}) = \frac{14}{10}$ UP<br>To<br>1  $p(\xi 43) = 74$ 

Now we extend <sup>P</sup> across all of 12 by doing disjoint sums. For example ng<br>e)

 $P(\{2,4\}) = P(\{2\}) + P(\{4\})$  $=$   $\frac{1}{4} + \frac{1}{4}$ =  $\frac{1}{2}$ 

$$
P(\xi_{1,2,3}) = P(\xi_{1,3}) + P(\xi_{2,3}) + P(\xi_{3,3})
$$
  
=  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$   
=  $\frac{3}{4}$   

$$
P(\xi_{1,2,3,4\zeta}) = P(\xi_{1,3}) + P(\xi_{2,3}) + P(\xi_{1,3,3}) + P(\xi_{2,3,3}) + P(\xi_{2,3,3}) + P(\xi_{1,3,3})
$$
  
=  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$   
= 1

Def: ef: A probability space<br>insists of two sets and<br>unction (S, S, P). probability space consists of two sets and <sup>a</sup> function (S, D, P). Def: A probability space<br>Consists of two sets and a<br>function (S, Ω, P).<br>S is called the sample space<br>of our experiment. The elen<br>of S are called the <u>outcom</u> sample space of our experiment. The elements runction<br>S is called the sar<br>of our experiment of S are obability space<br>f two sets and a<br>(S, 1, P).<br>the sample space<br>periment. The elements<br>periment. of our experiment.<br>of S are called the <u>outcomes</u><br>of the experiment. of the experiments<br>I is a set whose elements are a suites The elements of I are called events experin<br>
sets of<br>
bsets of<br>
ments d<br>
P IR . is a function  $P : \Omega \longrightarrow \mathbb{R}$ called events.<br> $P: \Omega \rightarrow \mathbb{R}$  is a function<br>where for each  $E$  in  $\Omega$ where for each  $E$  in  $\Omega$ <br>we get a probability  $P(E)$ . Furthermore, the following

Properties must hold:
① S is an element of $\Omega$ .
① Now want the table to measure $P(S)$
② If E is an event in $\Omega$ ,
② If E is an event in $\Omega$ .
③ If $E_{1}, E_{2}, E_{3}, ...$
③ If $E_{1}, E_{2}, E_{3}, ...$
③ If $E_{1}, E_{2}, E_{3}, ...$
③ If $E_{2}, E_{3}, ...$
③ If $E_{3}, E_{4}, ...$
③ If $E_{4}, ...$
③ If $E_{5}, ...$
② If $E_{6}, ...$
① If $E_{6}, ...$
④ If $E_{6}, ...$
① If $E_{7}, ...$
① If $E_{8}, ...$
④ If $E_{1}, E_{2}, ...$
③ If $E_{1}, ...$
③ If $E_{1}, ...$
④ If $E_{1}, ...$
④ If $E_{2}, ...$
③ If $E_{3}, ...$
④ If $E_{1}, ...$
① If $E_{1}, ...$
③ If $E_{2}, ...$
④ If $E_{1}, ...$
① If $E_{2}, ...$
① If $E_{1}, ...$

 $(5) P(S) = 1$  $(6)$  If  $E_1, E_2, E_3, \cdots$ is a finite or infinite sequence of events from  $\Omega$ that are pair-wise disjoint  $\frac{means}{i}$ :  $E_i$   $DE_j = \phi$  if  $\bar{\mu} \neq \bar{j}$ ie there is no overlap in  $\frac{1}{2}$   $+hen$  $P(\bigcup_{i} E_{i}) = \sum_{i} P(E_{i})$  $P(E_{1}UE_{2}UE_{3}U...)=P(E_{1})+P(E_{2})+P(E_{3})+...$ end of deff

This def is based on the<br>Work of Andrey Kolmogorov<br>1930s<br>Remark: A set  $\Omega$  satisfying<br>0,0,3 above is called a This def is based on the Work of Andrey Kolmogorov This def is based on the<br>
Work of Andrey Kolmogorov<br>
1930s<br>
Remark: A set 12 satisfying<br>
1930s<br>
Commet: A set 12 satisfying<br>
Talgebra or 5-field<br>
Talgebra or 5-field<br>
Remark: If 12 is a 5-algebra 1930s Pemark: A set IL satisfying  $\bigcirc$  ,  $\frac{max\{k}{2}}$ above is called a  $\sigma$ -algebra This def is based on the<br>Work of Andrey Kolmogorov<br>(1930s<br>Cemart: A set 1 satisfying<br>(1,2).3 above is called a<br>T-algebra or T-field<br>(1,2). Then TEx<br>(1) If EyEz, E3, ware<br>in 1, then TEx<br>is in 1, then TEx or Enfield Remark:  $If \Omega$  is a  $\sigma$ -algebra cemair: +, =<br>une can show that  $\overline{e^{l(a)}}$  show<br>(a)  $\phi \in \Omega$  $(a)$   $\phi \in \Omega$ <br>(b) If EyEz,  $E_{2,5}E_{3,111}$  are<br>then  $DE_{x}$  $\frac{1}{i}n$   $\frac{1}{i}$  $\frac{1}{2}$  $\begin{array}{ccc}\n\Gamma & F & F \\
\Gamma & \Gamma & \Gamma\n\end{array}$ This def is based on the<br>
Work of Andrey Kolmogorov<br>
1930s<br>
<u>Remart:</u> A set  $\Omega$  satisfying<br>
1930s<br>
Concert: A set  $\Omega$  satisfying<br>
1930s<br>
Concert: A set  $\Omega$  satisfying<br>
1930s<br>
Concert: Experiment<br>
Concert: If  $\Omega$  is a Look at online notes for proof.

How to construct a probability<br>space when S is finite<br>space.<br>Suppose S is a finite sample<br>space.<br>Define  $\Omega$  to be the set<br>of all subsets of S.<br>For each outcome  $\omega$  in S<br>For each outcome  $\omega$  in S<br>pick a real number  $0 \le$ How to construct a probability space when <sup>S</sup> is finite Suppose <sup>S</sup> is <sup>a</sup> finite sample space . space.<br>Define 12 to be the set pose S is a timite sam:<br>Ree.<br>Gine \_D to be the se<br>of <u>all</u> subsets of S. w in <sup>S</sup> For each outcome pick <sup>a</sup>  $vert_0$ utcome come and define  $P(\{\omega\}) = n_{\omega}$ How to construct a probability<br>
space when S is finite<br>
space.<br>
Suppose S is a finite sample<br>
space.<br>
Define D to be the set<br>
of all subsets of S.<br>
For each outcome w in S<br>
For each outcome w in S<br>
For each outcome w in S

EX :  $\frac{EX}{S} = \{1\}$ 2, 3, 4 J  $P(\{1\}) = \frac{1}{4} = n_1$  $P(\{3\}) = \frac{1}{4} = n_3$  $P(\{z\}) = |\qquad| q = n_z$  $P(\Sigma 43) = V_4 = Ny$ 

At the same time pick the  
\nnumbers so that  
\n
$$
\sum_{w\in S} P(\overline{\{w\}}) = 1 \quad \boxed{\begin{array}{l} EX \text{ above} \\ D_1 + D_2 + D_3 + D_4 \\ D_1 + D_2 + D_3 + D_4 \\ D_2 + D_4 + D_5 + D_6 \end{array}} = \frac{1}{t} + \frac{1}{
$$

 $If E =$  $\phi$ , define  $P(\phi) = 0$ . If  $E = \phi_j$ , define  $P(\phi) = 0$ .<br>Theorem: The above construction<br>is a probability space.<br>See prout in notes. Theorem: The above construction heorem: The above a<br>is a probability space. If  $E = \phi_1$  define  $P(\phi) = 0$ .<br>Theorem: The above construction<br>is a probability space.<br>See prout in notes. ility spu<br>ility spu<br>in notes