Math 4740

9/11/24

Ex: Suppose we have 3 math books and 2 history books. How many ways can we put the 5 books on a shelf where the math books are next each other?

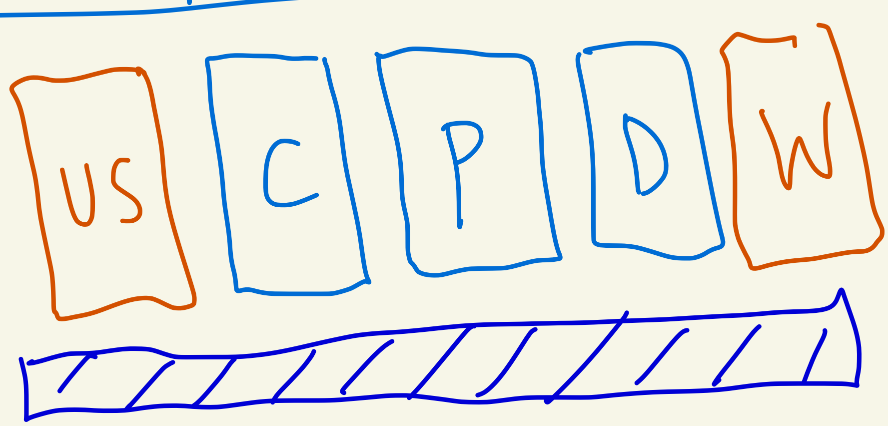
Math

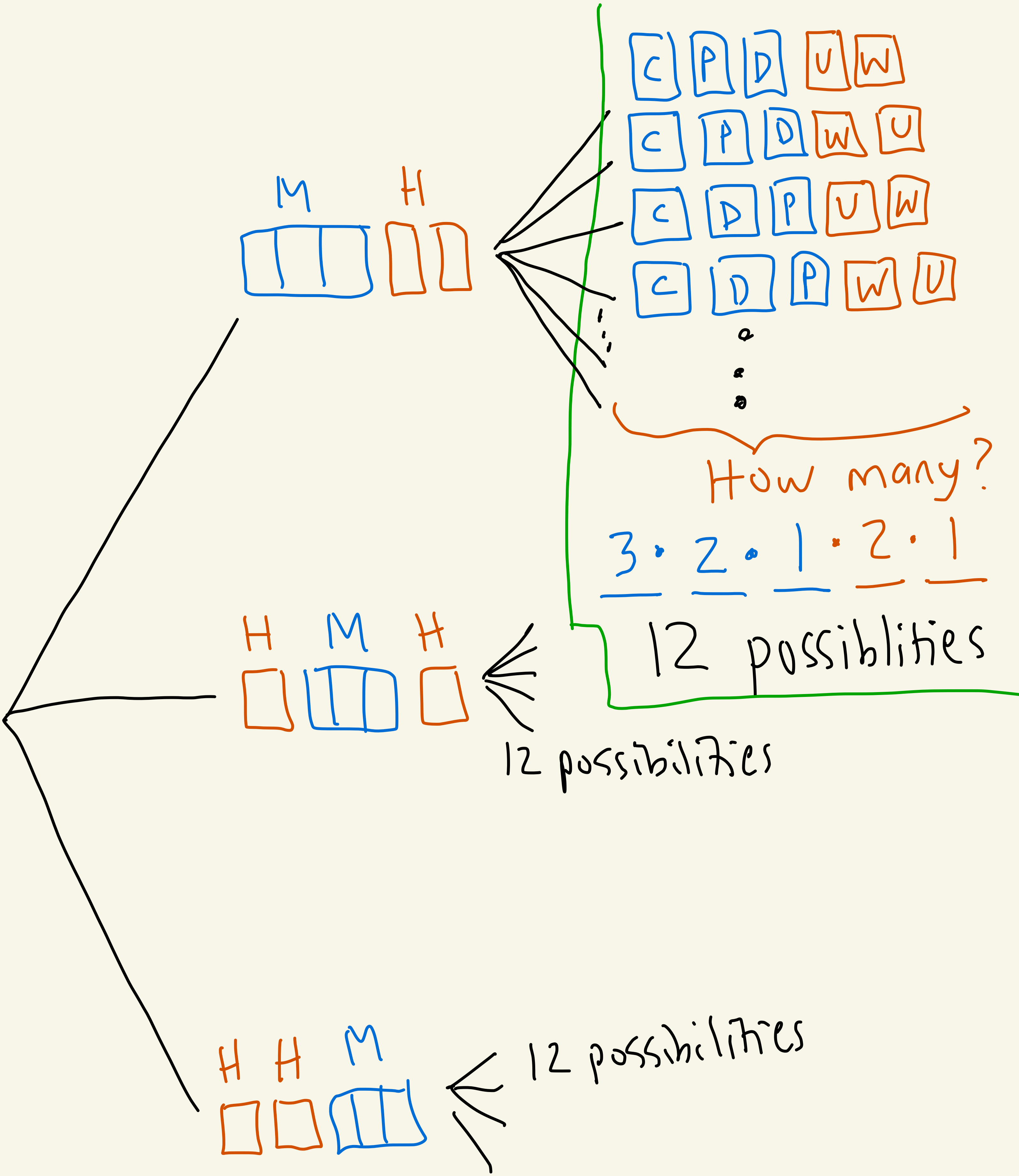
Probability
Calculus
Diff. Eqns

History

US History
World History

Ex:



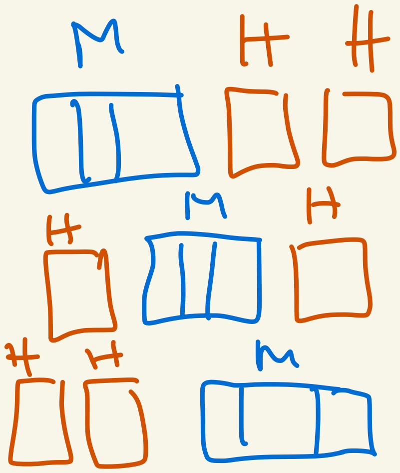


Total = 3 · 12 = 36 possibilities

Could say it this way

Step 1

Pick one of these →



3 ways

Step 2

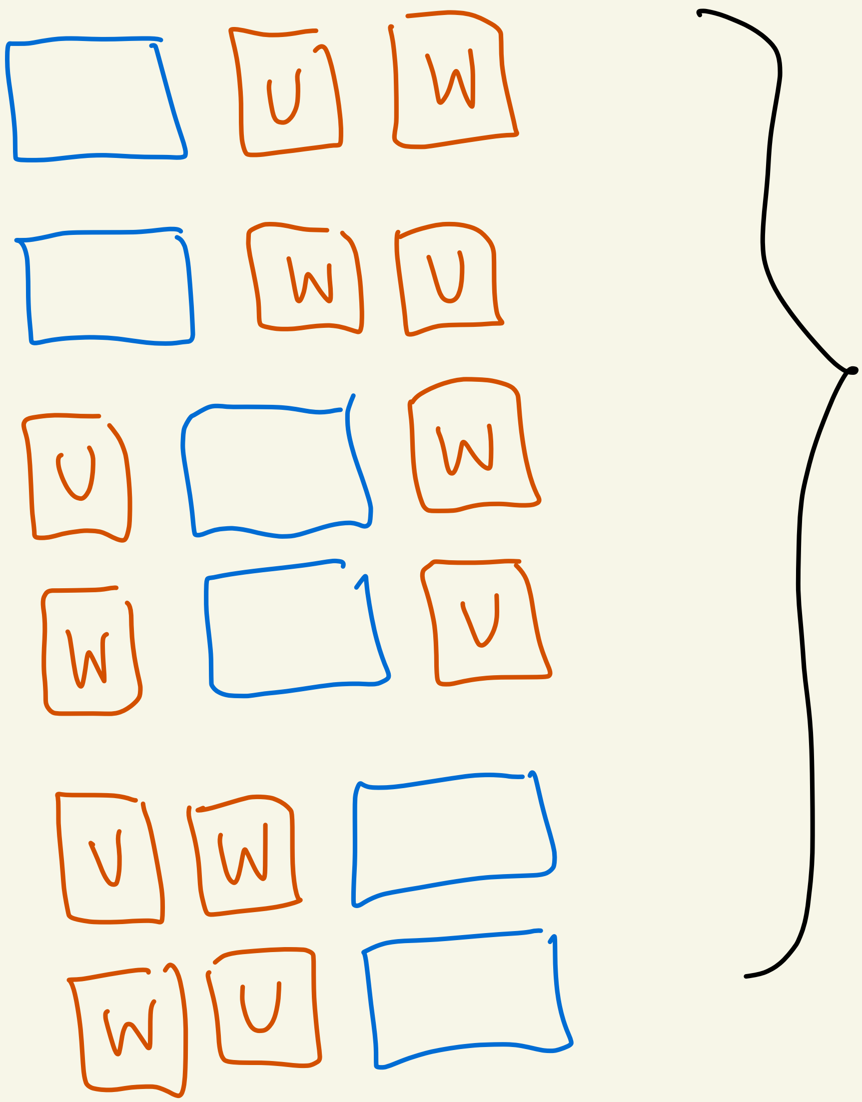
Fill in the books

$$\underset{M}{\underline{3}} \cdot \underset{M}{\underline{2}} \cdot \underset{M}{\underline{1}} \cdot \underset{H}{\underline{2}} \cdot \underset{H}{\underline{1}} = 12 \text{ ways}$$

Total $= 3 \cdot 12 = 36$ ways

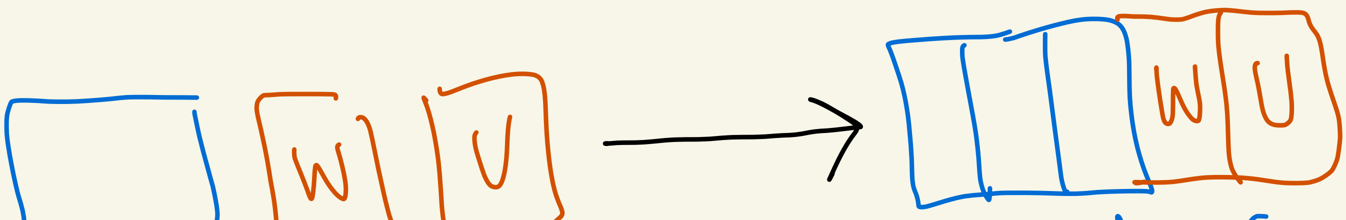
Another way: Think of math as a unit

Step 1: Pick one of these.



6 ways

Step 2: Fill in math books.



$3 \cdot 2 \cdot 1 = 6$ ways

Total $= 6 \cdot 6 = 36$ ways

Combinations:

Consider a set of n objects. The number of subsets of size k where $0 \le k \le n$ is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

proof in notes online

read: "n choose k"

This is the same as the # of ways to choose k objects from n objects where the order doesn't matter.

Ex: $S = \{1, 2, 3\}$ ← $\boxed{n = 3}$

How many subsets of size $k=2$ are there?

Subsets of size 2

$\{1,2\}$
$\{1,3\}$
$\{2,3\}$

3 subsets

EQUAL

calculation

$$\binom{3}{2} = \frac{3!}{2!(3-2)!}$$

$$= \frac{3!}{2!\,1!}$$

$$= \frac{3\cdot 2\cdot 1}{2\cdot 1}$$

$$= 3$$

Why does this work?

ways to pick where order matters

12
13
21
23
31
32

$\underset{\text{poss.}}{3}$ $\underset{\text{poss.}}{2}$ = 6 possibilities

$$3\cdot 2 = \frac{3\cdot 2\cdot 1!}{1!}$$

$$= \frac{3!}{(3-2)!} = \frac{n!}{(n-k)!}$$

Divide out double counting so order doesn't matter.

Divide by $2! = k!$

So we get: $\dfrac{3!}{(3-2)!\,2!} = \binom{3}{2}$

Ex: Suppose a dealer has the following cards:

2♣ Q♦ J♥ A♠

How many ways can the dealer deal you two cards from these four? Order doesn't matter.

Ex: Q♦ A♠ ← same as A♠ Q♦

Answer: 6 ways

2♣ Q♦ | Q♦ J♥

2♣ J♥ | Q♦ A♠

2♣ A♠ | J♥ A♠

Note: $\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!}$

$$= \frac{\cancel{4}^{2} \cdot 3 \cdot \cancel{(2!)}}{\cancel{2!}\,\cancel{2!}} = 6$$

Ex: A dealer has a standard 52-card deck. They deal you 5 cards. How many possible hands are there that you can get? Order doesn't matter.

Ex hand: K♠ Q♠ J♠ 10♠ A♠

(called a royal flush)

possible # hands

$$= \binom{52}{5} = \frac{52!}{5!(52-5)!}$$

$$= \frac{52!}{5!\,47!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \cancel{(47!)}}{5!\,\cancel{47!}}$$

$$= \frac{\overset{26}{\cancel{52}} \cdot \overset{17}{\cancel{51}} \cdot \overset{10}{\cancel{50}} \cdot 49 \cdot \overset{12}{\cancel{48}}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 26 \cdot 17 \cdot 10 \cdot 49 \cdot 12$$

$$= \boxed{2{,}598{,}960}$$