Math 4740
9/18/24

Ex of 5-card poker hands

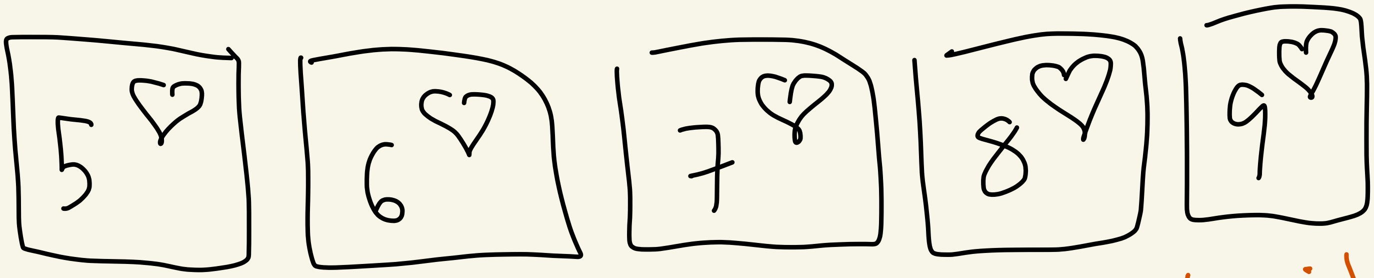
J♦ 2♣ J♥ 3♠ J♠ ← 3 of a kind

2♣ 5♦ A♣ J♥ 2♦ ← pair

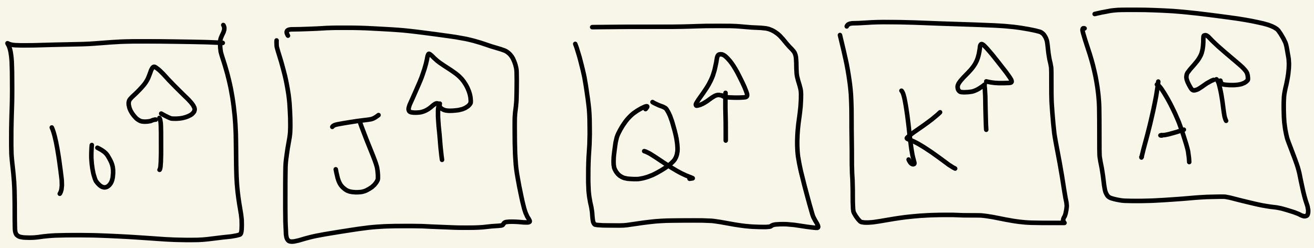
2♣ A♦ A♣ J♥ 2♦ ← two pair

2♥ 2♣ K♦ K♥ 2♦ ← full house

(pair + 3-of-a-kind)



Straight-flush



Royal flush

Ex: Suppose you are dealt 5 cards from a standard 52 card deck. What is the probability that you get a royal flush?

The sample space size is

$$\binom{52}{5} = 2{,}598{,}960$$

possible 5-card poker hands.

How many of these are royal flushes?

There are four royal flushes:

#1: $\boxed{10\heartsuit}$ $\boxed{J\heartsuit}$ $\boxed{Q\heartsuit}$ $\boxed{K\heartsuit}$ $\boxed{A\heartsuit}$

#2: 10♦ J♦ Q♦ K♦ A♦

#3: 10♣ J♣ Q♣ K♣ A♣

#4: 10♠ J♠ Q♠ K♠ A♠

Probability of royal flush is

$$\frac{4}{2{,}598{,}960} = \frac{1}{649{,}740}$$

$$\approx 0{,}0001539\ \%$$

Ex: What's the probability you get a pair and nothing better?

Let's count how many pairs there are.

Step 1: Pick a face value for the pair:

2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

There are

$$\binom{13}{1} = 13$$

ways to do this step

Ex: $\boxed{A}$ $\boxed{A}$ $\boxed{}$ $\boxed{}$ $\boxed{}$

Step 2: Pick 2 suits from

♡, ♢, ♤, ♧

for the pair.

There are

$$\binom{4}{2} = \frac{4!}{2!\,2!} = 6$$

ways to do this

Ex: $\boxed{A^{\diamondsuit}}$ $\boxed{A^{\spadesuit}}$ $\boxed{}$ $\boxed{}$ $\boxed{}$

Step 3: Pick the other 3 face values. They can't be the same as step 1 and they all have to be different.

2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, ~~A~~

There are

$$\binom{12}{3} = \frac{12!}{3!(12-3)!} = \frac{12!}{3!\,9!} = 220$$

ways to do this.

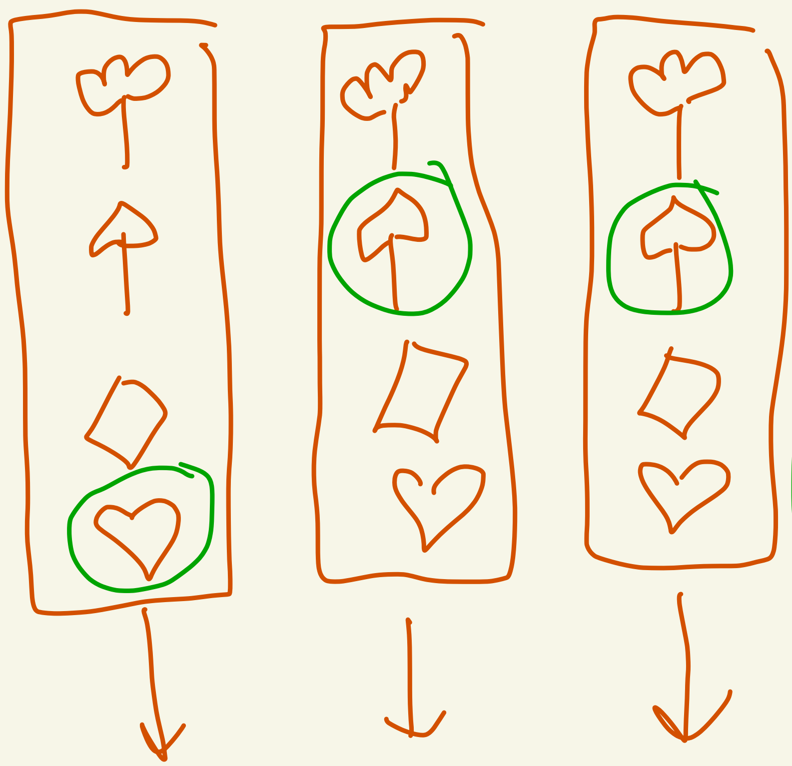
Ex: | A♢ | A♠ | 7 | 10 | 8 |

Step 4: Fill in the 3 remaining suits.

There are

$$\binom{4}{1}\binom{4}{1}\binom{4}{1} = 4 \cdot 4 \cdot 4 = 64$$

ways to do this.



Ex: | A♢ | A♠ | 7♡ | 10♢ | 8♢ |

Thus, the total # of hands that are a pair and no better are

$$\underbrace{13}_{\text{step } 1} \cdot \underbrace{6}_{\text{step } 2} \cdot \underbrace{220}_{\text{step } 3} \cdot \underbrace{64}_{\text{step } 4} = \boxed{1{,}098{,}240}$$

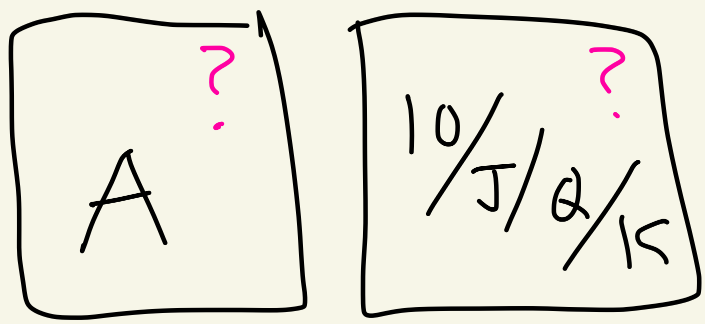
The probability of this occuring is

$$\frac{1{,}098{,}240}{2{,}598{,}960} \approx 0.422569\ldots$$

$$\approx \boxed{42.2569\,\%}$$

Ex: Suppose you are dealt 2 cards from a 52-card deck. What is the probability you get a blackjack?

Blackjack =



Total # of 2 card hands:

$$\binom{52}{2} = \frac{52!}{2!\,50!} = \frac{52 \cdot 51 \cdot \cancel{(50!)}}{2!\,\cancel{50!}}$$

$$= 1326$$

How many blackjacks?

Step 1: Pick the ace.

There are 4 ways to do this

A♥ A♦ A♠ A♣

Ex: A♥ $\boxed{}$

Step 2: Pick the other card from:

10♥ 10♠ 10♣ 10♦

J♥ J♠ J♣ J♦

Q♥ Q♠ Q♣ Q♦

K♥ K♠ K♣ K♦

16 ways to pick one of these

Ex: A♡ K◇

Total # blackjacks is

$$4 \cdot 16 = 64$$

probability of a blackjack is $\frac{64}{1326} \approx 0.048265\ldots$

$$\approx 4.8265\%$$