Math 4740
9/9/24

Topic 2 – Counting and probability

Basic counting principle

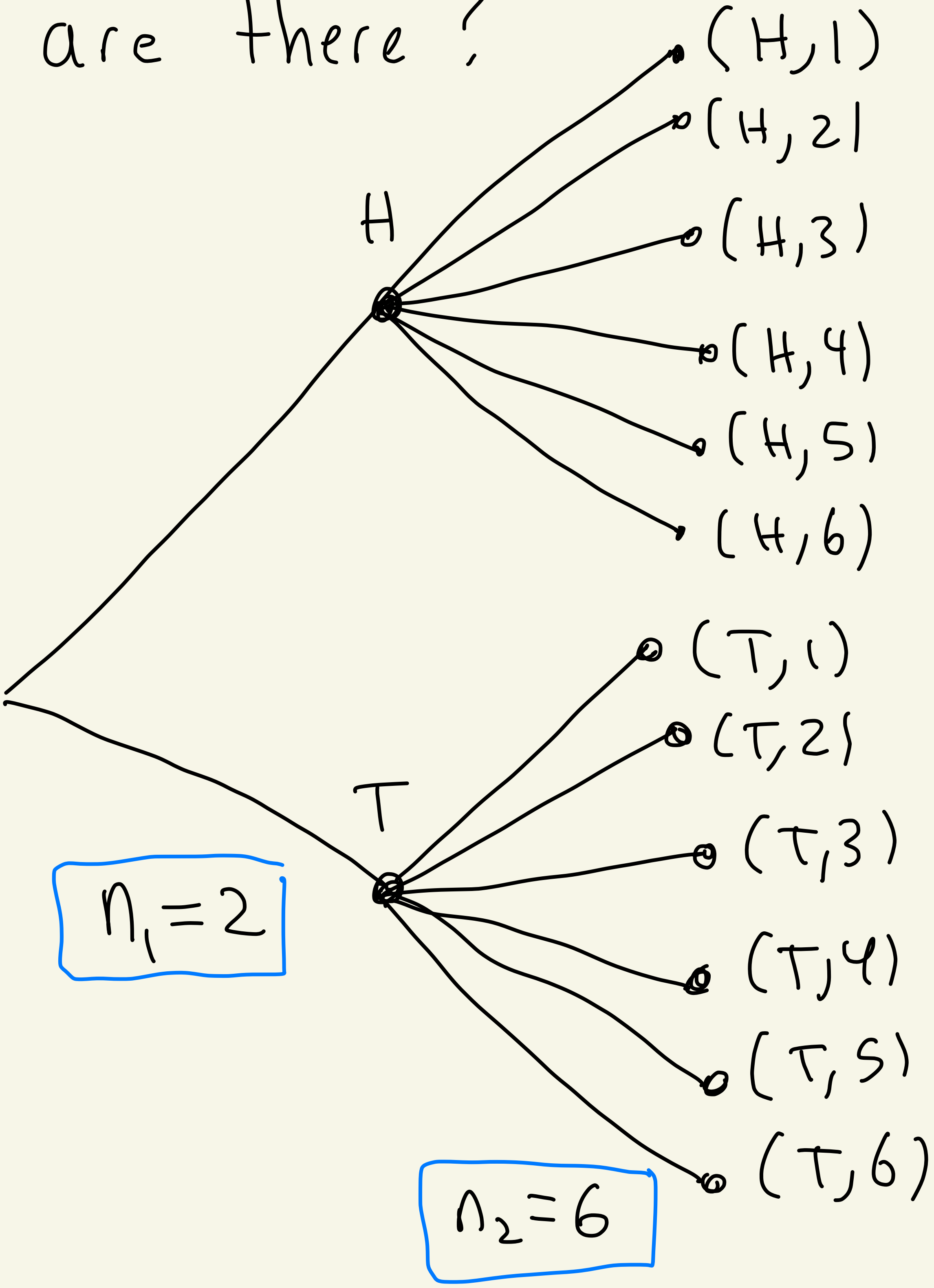
If r experiments are performed in a row such that the first experiment may result in n_1 possible outcomes; and if for each of these n_1 possible outcomes there are n_2 possible outcomes for the second experiment; and if for each of the possible outcomes of the first two experiments there are n_3 possible outcomes for the third experiment; and if, …,

then there are

$$n_1 \cdot n_2 \cdot n_3 \cdots n_r$$

Possible outcomes there are for the r experiments.

Ex: Suppose you toss a coin and then roll a 6-sided die. How many possible outcomes are there?



There are $n_1 \cdot n_2$ $= 2 \cdot 6$ $= 12$ possible outcomes

Another way to write:

$$\underline{\;H/T\;} \qquad \underline{\;1/2/3/4/5/6\;}$$

$$\underset{\text{possibilities}}{2} \cdot \underset{\text{possibilities}}{6} = \underset{\text{possibilities}}{12}$$

Ex: In CA, a license plate consists of one number (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) followed by three upper-case letters, followed by three numbers. The only exclusion is that the letters I, O, and Q cannot be used in spot 2 or spot 4.

Examples are:

$\underline{8}\ \underline{A}\ \underline{A}\ \underline{X}\ \underline{3}\ \underline{1}\ \underline{2}$

$\underline{0}\ \underline{B}\ \underline{Q}\ \underline{T}\ \underline{7}\ \underline{7}\ \underline{6}$

no I, O, Q

How many possible plates are there?

#	letter not I, O, Q	letter	letter not I, O, Q	#	#	#
↑	↑	↑	↑	↑	↑	↑
10 ·	23 ·	26 ·	23 ·	10 ·	10 ·	10

Answer = 137,540,000

Birthday Paradox

Suppose there are N people in a room. What is the probability that at least two people have the same birthday?

Same month/day not year
Such as two people both have 9/4

Assumptions:

① We will assume that no one has Feb 29 as their bday this is leap year.

② We will assume that each day is equally likely

③ Assume $N \leq 365$ because if $N > 365$ then the probability is 100%

For example if $N=3$ the sample space would be:

$$S = \left\{ (\text{date } 1, \text{date } 2, \text{date } 3) \;\middle|\; \begin{array}{c}\text{date } i \\ \text{is a} \\ \text{calendar} \\ \text{day}\end{array} \right\}$$

$$= \left\{ (\underbrace{4/1}_{\substack{\text{person} \\ 1}}, \underbrace{1/12}_{\substack{\text{person} \\ 2}}, \underbrace{3/17}_{\substack{\text{person} \\ 3}}), \right.$$

$$\left. (\underbrace{\underbrace{9/9}, \underbrace{9/9}}_{\substack{\text{two have} \\ \text{same bday}}}, 11/28), \dots \right\}$$

Here $|S| = 365^3$

In general, the size of the sample space is $|S| = 365^N$.

Let E be the event that at least two people have the same birthday.

The probability of E occuring would be $\frac{|E|}{|S|}$ since each day is equally likely.

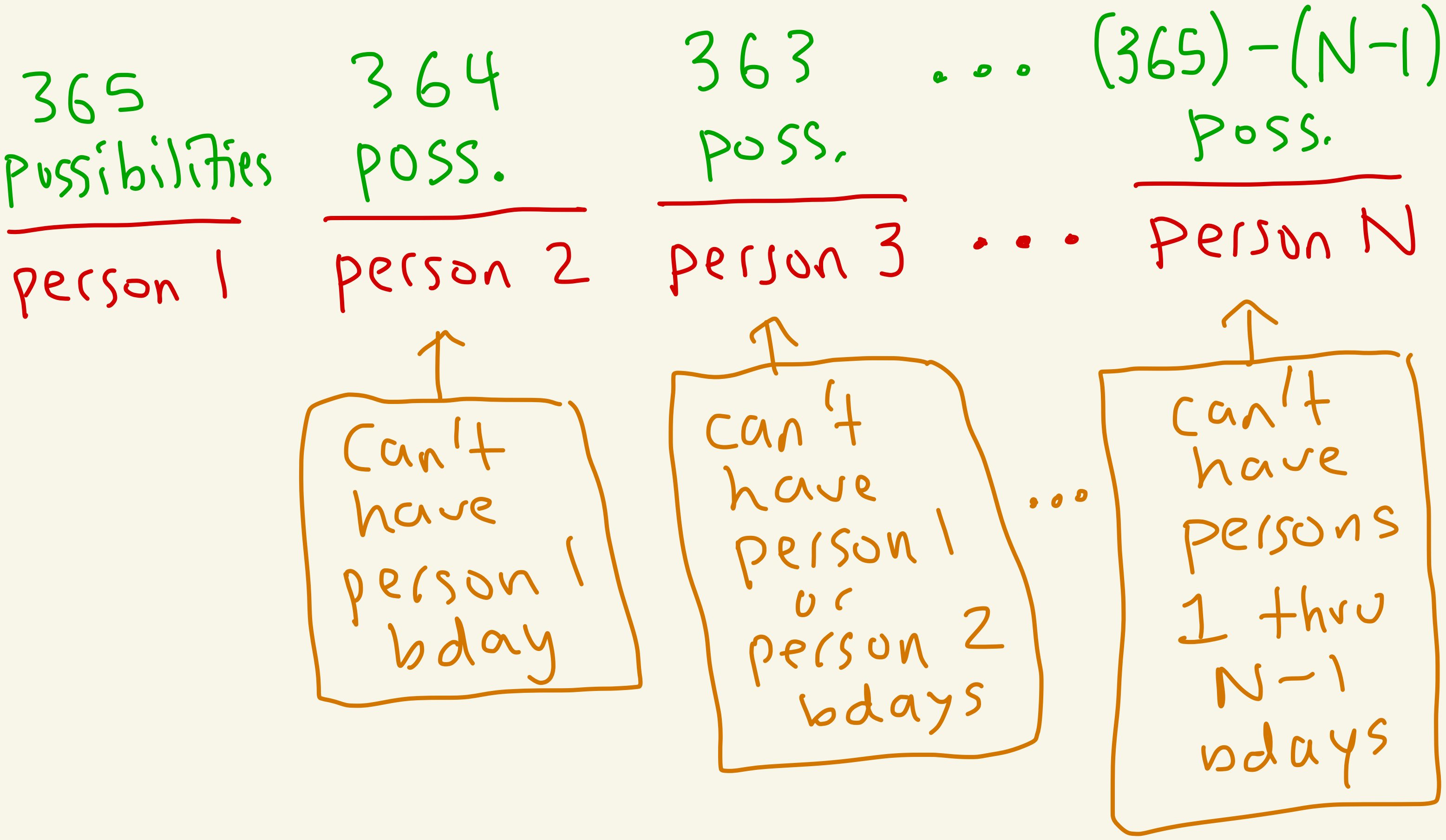
Instead we will calculate the size of $\overline{E}$ and use

$$\underbrace{\frac{|E|}{|S|}}_{P(E)} = 1 - \underbrace{\frac{|\overline{E}|}{|S|}}_{P(\overline{E})}$$

$$P(E) = 1 - P(\overline{E})$$

$\bar{E}$ is the event that no two people have the same bday.

Calculation of $|\bar{E}|$:



So,

$$|\bar{E}| = (365)(364)(363)\cdots(365-(N-1))$$

So,

$$P(E) = 1 - P(\bar{E}) = 1 - \frac{|\bar{E}|}{|S|}$$

$$= 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-(N-1))}{365^N}$$

If $N=3$,

$$P(E) = 1 - \frac{365 \cdot 364 \cdot 363}{365^3}$$

We looked at the table on the website now

Permutations

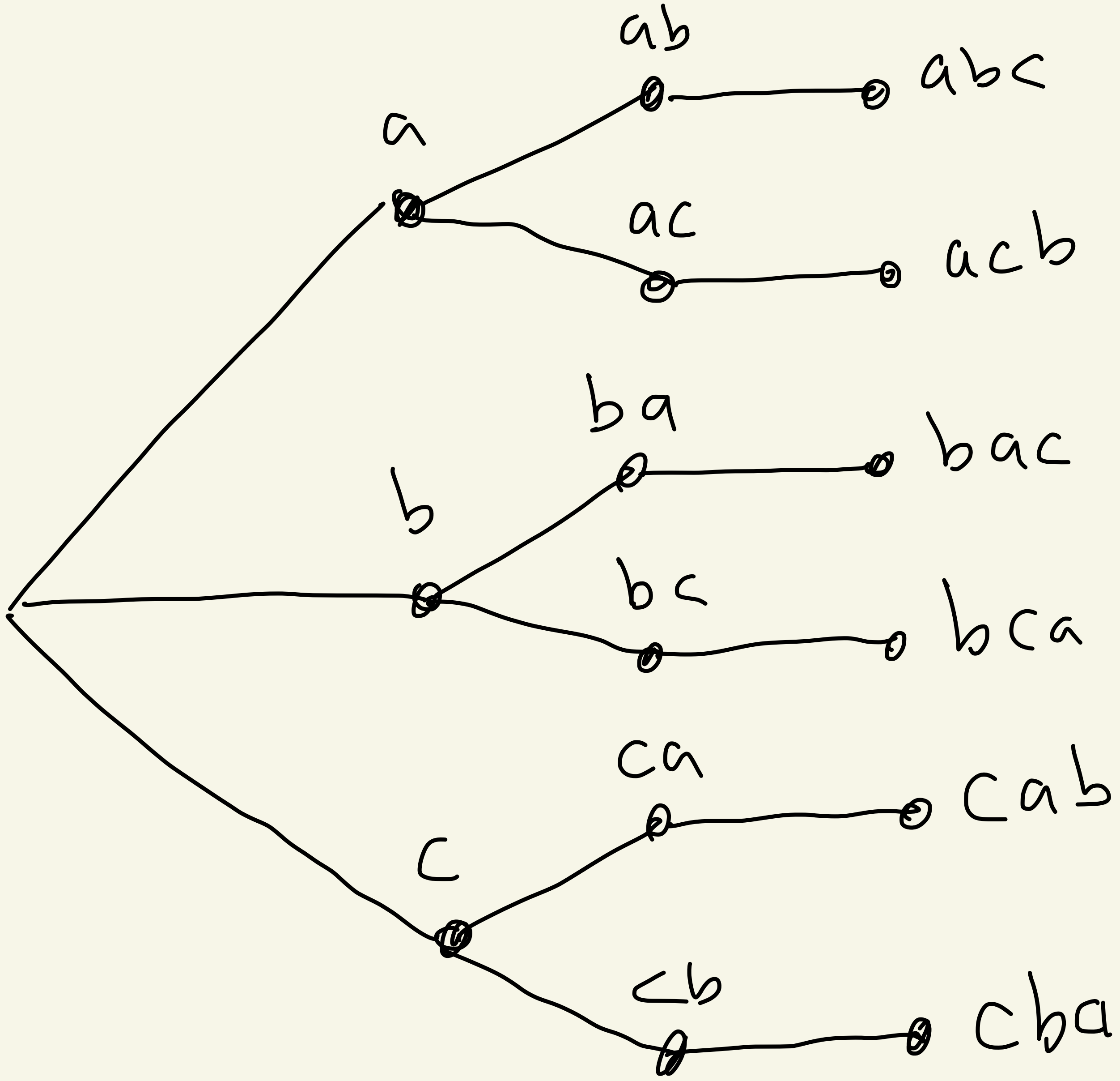
Suppose you have n objects. A **permutation** of these n objects is an ordered list of the n objects.

Ex: What are all the permutations of a, b, c?

permutations	math way
abc	(a,b,c)
acb	(a,c,b)
bac	(b,a,c)
bca	(b,c,a)
cab	(c,a,b)
cba	(c,b,a)

There are 6 permutations.

math people use parentheses when order matters



3 possibilites

2 possibilities

1 possibilites

$3 \cdot 2 \cdot 1 = 6 = 3!$

In general, there are

$$n! = n \cdot (n-1)(n-2)\cdots(2)(1)$$

permutations of n objects