Math 4740

HW 2 Solutions

1 (a)

$1-9$	$0-9$	$0-9$	$0-9$	$0-9$	$0-9$
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
9 possibilities	10 possibilities	10	10	10	10

There are

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = \boxed{900{,}000}$$

six digit numbers

1 (b) Instead we calculate all the six digit numbers <u>without</u> a 5 and subtract this from 1(a).

1–9 not 5	0–9 not 5	0–9 not 5	0–9 not 5	0–9 not 5	0–9 not 5
8 possibilities	9 possibilities	9	9	9	9

There are

$$\underbrace{900{,}000}_{\#\text{ six digit numbers}} - \underbrace{8\cdot 9\cdot 9\cdot 9\cdot 9\cdot 9}_{\#\text{ six digit numbers without a } 5}$$

$$= 900{,}000 - 472{,}392$$

$$= \boxed{427{,}608}$$

six digit numbers without a 5.

2

A - Z	A - Z not same as spot 1	A - Z not same as spot 1 or 2	0 - 9	0 - 9 not same as spot 4	0 - 9 not same as spot 4 or 5
___	___	___	___	___	___
↑	↑	↑	↑	↑	↑
26 possibilities	25 possibilities	24 possibilities	10 possibilities	9 possibilities	8 possibilities

\# of license plates is

$$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = \boxed{11,232,000}$$

(3) (a) There are 5 letters so there are $5! = 120$ permutations.

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(3) (b)

a	b/d/e	b/d/e not same as spot 2	b/d/e not same as spot 3 2 or 3	c
	3 possibilities	2 possibilities	1 possibility	

There are $\boxed{3 \cdot 2 \cdot 1 = 6}$ possible permutations.

Here is the choice tree:

- a b _ _ c
  - a b d _ c — a b d e c
  - a b e _ c — a b e d c
- a d _ _ c
  - a d b _ c — a d b e c
  - a d e _ c — a d e b c
- a e _ _ c
  - a e b _ c — a e b d c
  - a e d _ c — a e d b c

4 (a) Some examples are

01121120

11111111

12000121

Let's count!

$$\begin{array}{cccccccc} 0/1/2 & 0/1/2 & 0/1/2 & 0/1/2 & 0/1/2 & 0/1/2 & 0/1/2 & 0/1/2 \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \end{array}$$

possibilities

There are $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^8$

$= \boxed{6561 \text{ possible sequences}}$

④ (b)

Step 1:

Pick 4 spots from the 8 total spots where the 0's go. This can be done in $\binom{8}{4} = \frac{8!}{4!4!}$

$= 70$ ways

One of the 70 ways:

$\underline{0}\ \ \underline{0}\ \ \underline{\quad}\ \ \underline{0}\ \ \underline{\quad}\ \ \underline{0}\ \ \underline{\quad}\ \ \underline{\quad}$

Step 2:

Now there is no choice at this point, you must fill the remaining 4 spots with 1's. Thus, only 1 possibility at this step.

The above example becomes

$\underline{0}\ \ \underline{0}\ \ \underline{1}\ \ \underline{0}\ \ \underline{1}\ \ \underline{0}\ \ \underline{1}\ \ \underline{1}$

Answer:

Total # of sequences is $70 \cdot 1 = \boxed{70}$

4 (c)

Step 1:

Pick 3 spots from the 8 total spots to put the 0's. There are

$$\binom{8}{3} = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{5!}\,\cancel{6}} = 56$$

ways to do this

Example possibility

0 _ 0 0 _ _ _ _

Step 2:

Pick 3 spots from the remaining 5 spots to put the 1's in. There are

$$\binom{5}{3} = \frac{5!}{2!\,3!} = \frac{5\cdot 4\cdot \cancel{3!}}{2\cdot \cancel{3!}} = 10$$

ways to do this

Example possibility

0 _ 0 0 1 1 _ 1

Step 3: Only 1 choice to make now: Fill the remaining spots with 2's

Example possibility

0 2 0 0 1 1 2 1

Answer: Total number of sequences is $56 \cdot 10 \cdot 1 = \boxed{560}$

(5) The sample space size is $|S| = 6^6 = 46{,}656$.

Let E be the event that at least two of the dice have the same number. We want $P(E)$.

Instead we will calculate $P(E) = 1 - P(\overline{E})$ where $\overline{E}$ is the event that none of the dice have the same number.

die 1	die 2	die 3	die 4	die 5	die 6
6 choices	5 choices since can't be the same as die 1	4 choices since can't be the same as die 1 or die 2	3 choices since can't be the same as die 1 or die 2 or die 3	2 choices since can't be the same as die 1 or die 2 or die 3 or die 4	1 choice since can't be the same as die 1 or die 2 or die 3 or die 4 or die 5

So, $P(\overline{E}) = \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6^6} = \dfrac{720}{46{,}656}$

Thus, $P(E) = 1 - P(\overline{E}) = \dfrac{46{,}656 - 720}{46{,}656} = \boxed{\dfrac{45{,}936}{46{,}656}}$

$\approx \boxed{0.9845679\ldots}$

$\approx \boxed{98.46\%}$

⑥ The sample space has size

$|S| = 8 \cdot 8 \cdot 8 \cdot 8 = 8^4 = 4{,}096.$

(a) First choose where the two 3's can go by picking two of the four spots.

$$\boxed{\binom{4}{2} = \frac{4!}{2!\,2!} = 6}$$

3	3	_	_
3	_	3	_
3	_	_	3
_	3	3	_
_	3	_	3
_	_	3	3

these are the 6 poss-ibilities we are counting

Then fill in the remaining two spots with two numbers that aren't 3's.

ex: 3 3 _ _

7 choices 7 choices

$$\boxed{7 \cdot 7 = 49}$$

There are $6 \cdot 49 = 294$ possibilities.

So, the probability is $\boxed{\dfrac{294}{4096}} \approx \boxed{0.07178\ldots}$

$\approx \boxed{7.18\ \%}$

(b)

$$P(\text{at most two 8's}) = P(\text{no 8's}) + P(\text{exactly one 8}) + P(\text{exactly two 8's})$$

$$= \frac{\overbrace{7\cdot 7\cdot 7\cdot 7}^{\text{fill in the four spots with \#s that aren't 8's}}}{4096}$$

$$+ \frac{\overbrace{\binom{4}{1}\cdot 7\cdot 7\cdot 7}^{\text{choose where the one 8 goes then fill in the remaining three spots with \#s that aren't 8's}}}{4096}$$

$$+ \frac{\overbrace{\binom{4}{2}\cdot 7\cdot 7}^{\text{choose where the two 8's go and then fill in the remaining two spots with \#s that aren't 8's}}}{4096}$$

$$= \frac{2401+1372+294}{4096} = \boxed{\frac{4067}{4096}} \approx \boxed{0.9929\ldots} \approx \boxed{99.3\%}$$

note: $\binom{4}{1}=4$ is counting these 4 possibilities

8 _ _ _

_ 8 _ _

_ _ 8 _

_ _ _ 8

Note

$\binom{4}{1}=4$

$\binom{4}{2}=6$

(c)

$$P(\text{at least three 1's}) = P(\text{exactly three 1's}) + P(\text{exactly four 1's})$$

pick 3 spots out of the 4 spots for the 1's. Then fill the remaining spot with #s that aren't 1.

fill in all 4 spots with 1's there is only one way to do this

$$= \frac{\binom{4}{3}\cdot 7}{4096} + \frac{\binom{4}{4}}{4096}$$

$$= \frac{4\cdot 7}{4096} + \frac{1}{4096} = \boxed{\frac{29}{4096}} \approx \boxed{0.00708}$$

$$\approx \boxed{0.7\%}$$

⑦

The sample space has size $6^{10} = 60{,}466{,}176$

Now count possibilities

example possibility at this step

Step 1: Pick where the one 4 goes.

$\binom{10}{1} = 10$ possibilities

_ _ _ 4 _ _ _ _ _ _

example possibility at this step

Step 2: Pick where the six 5's go.

$$\binom{9}{6} = \frac{9!}{6!\,3!} = \frac{9\cdot 8\cdot 7\cdot 6!}{6!\,3!} = \frac{9\cdot 8\cdot 7}{3!} = 84 \text{ possibilities}$$

_ 5 5 4 5 _ 5 _ 5 5

example possibility at this step

Step 3: Fill in the other three spots with numbers that aren't 4 or 5.

$4\cdot 4\cdot 4 = 64$ possibilities

1 5 5 4 5 1 5 2 5 5

4 choices 4 choices 4 choice

The probability is thus

$$\frac{(10)(84)(64)}{60{,}466{,}176} = \boxed{\frac{53{,}760}{60{,}466{,}176}}$$

$$\approx \boxed{0.000889\ldots}$$

$$\approx \boxed{0.0889\,\%}$$

8. The sample space has size $|S| = 2^5 = 32$

(a) Pick where the one head goes: $\binom{5}{1} = 5$

Fill in the remaining 4 spots with tails: $1 \cdot 1 \cdot 1 \cdot 1 = 1$

$$P(\text{exactly one head}) = \frac{5 \cdot 1}{32}$$

$$= \boxed{\frac{5}{32}} \approx \boxed{0.15625\ldots}$$

$$\approx \boxed{15.6\,\%}$$

We counted these possibilities:

H	T	T	T	T
T	H	T	T	T
T	T	H	T	T
T	T	T	H	T
T	T	T	T	H

(b) Pick where the three heads go: $\binom{5}{3} = \frac{5!}{3!2!} = 10$

Fill in the remaining 2 spots with tails: $1 \cdot 1 = 1$

$$P(\text{exactly three heads}) = \boxed{\frac{10}{32}} \approx \boxed{0.3125\ldots} \approx \boxed{31.25\ \%}$$

Note: The count of 10 above counted these:

H H H T T
H H T H T
H H T T H
H T H H T
H T H T H
H T T H H
T H H H T
T H H T H
T H T H H
T T H H H

(c) There is only 1 way to get all tails. It is $\underline{T}\ \underline{T}\ \underline{T}\ \underline{T}\ \underline{T}$

So,

$$p(\text{all tails}) = \boxed{\frac{1}{32}} \approx \boxed{0.03125} \approx \boxed{3.125\%}$$

(9) The sample space has size

$$|S| = 2^{20} = 1{,}048{,}576$$

(a)

$$P(\text{at least 2 heads}) = 1 - P(\text{less than 2 heads})$$

$$= 1 - P(\text{exactly 0 heads}) - P(\text{exactly 1 head})$$

$$= 1 - \frac{\overbrace{1}^{\substack{\text{only 1 way to} \\ \text{have 0 heads.} \\ \text{Fill all 20 spots} \\ \text{with tails}}}}{1{,}048{,}576} - \frac{\overbrace{\binom{20}{1}}^{\substack{\text{exactly 1 head.} \\ \text{Pick the spot where the} \\ \text{head goes in } \binom{20}{1} = 20 \text{ ways.} \\ \text{Then fill the remaining spots} \\ \text{with tails in 1 way.}}}}{1{,}048{,}576}$$

$$= \frac{1{,}048{,}576 - 1 - 20}{1{,}048{,}576} = \boxed{\frac{1{,}048{,}555}{1{,}048{,}576}}$$

$$\approx \boxed{0.99997997\ldots}$$

$$\approx \boxed{99.998\,\%}$$

(b)

$$P(\text{at most 3 heads}) = P(0 \text{ heads}) + P(\text{exactly 1 head}) + P(\text{exactly 2 heads}) + P(\text{exactly 3 heads})$$

$$= \frac{1}{2^{20}} + \frac{\overbrace{\binom{20}{1}}^{\substack{\text{pick 1 spot} \\ \text{out of 20} \\ \text{for the head} \\ \text{then fill the} \\ \text{rest with tails} \\ \text{in 1 way}}}}{2^{20}} + \frac{\overbrace{\binom{20}{2}}^{\substack{\text{pick 2 spots} \\ \text{out of 20} \\ \text{for the heads} \\ \text{then fill the} \\ \text{rest with tails} \\ \text{in 1 way}}}}{2^{20}} + \frac{\overbrace{\binom{20}{3}}^{\substack{\text{pick 3 spots} \\ \text{out of 20} \\ \text{for the heads} \\ \text{then fill the} \\ \text{rest with tails} \\ \text{in 1 way}}}}{2^{20}}$$

$$= \frac{1+20+190+1140}{1{,}048{,}576} = \boxed{\frac{1{,}351}{1{,}048{,}576}}$$

$$\approx \boxed{0.00128841} \approx \boxed{0.1288\ \%}$$

(10) There are $6^4 = 1296$ ways to roll a 6-sided die four times in a row.

Let E be the event that a 3 occurs at least once in the four rolls. Then $\overline{E}$ is the event that no 3's occur in the four rolls.

$$\underbrace{\frac{1,2,4,\ 5,\text{ or } 6}{5}}_{\text{possibilities}} \cdot \underbrace{\frac{1,2,4,\ 5,\text{ or } 6}{5}}_{\text{possibilities}} \cdot \underbrace{\frac{1,2,4,\ 5,\text{ or } 6}{5}}_{\text{possibilities}} \cdot \underbrace{\frac{1,2,4,\ 5,\text{ or } 6}{5}}_{\text{possibilities}} = 5^4 = 625$$

counting $\overline{E}$

Thus, $P(\overline{E}) = \dfrac{625}{1296} \approx 0.48$.

So, $P(E) = 1 - P(\overline{E}) = 1 - \dfrac{625}{1296}$

$$= \frac{671}{1296} \approx \boxed{0.52}$$

11

The sample space has size

$$|S| = \binom{20}{5} = \frac{20!}{5!\,15!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot \cancel{15!}}{5!\,\cancel{15!}}$$

$$= \frac{1{,}860{,}480}{120} = 15{,}504$$

To count how many ways we can pick 5 numbers so the smallest number is larger than 6 we must pick 5 numbers from the 14 circled below.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

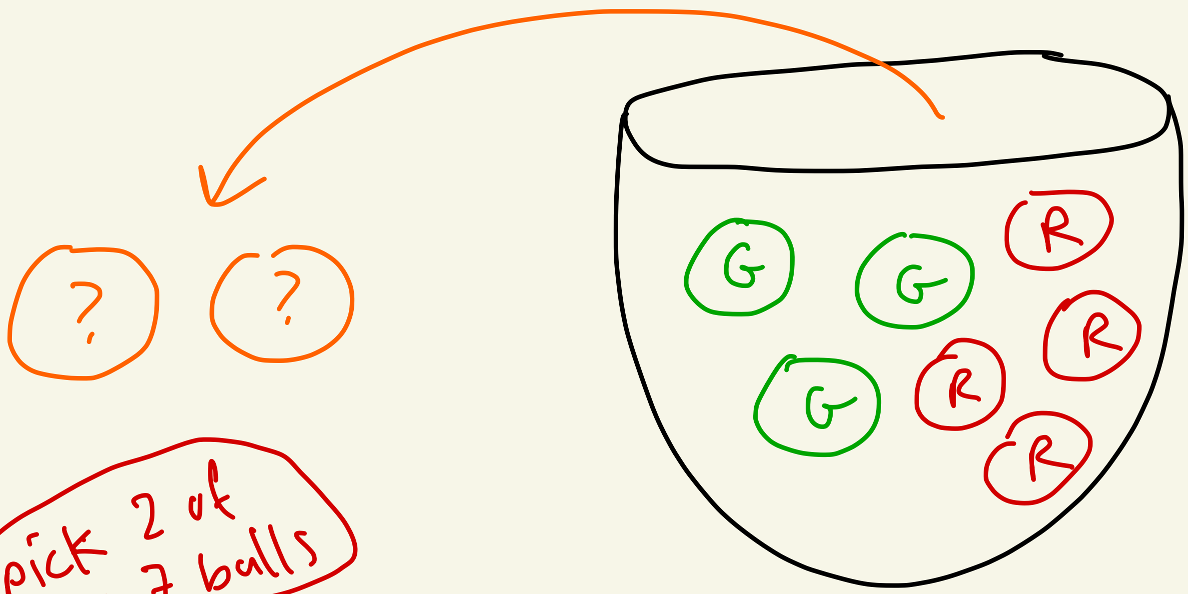
This can be done in $\binom{14}{5} = \frac{14!}{5!\,9!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{5!\,\cancel{9!}}$

$$= \frac{240{,}240}{120} = 2{,}002$$

Thus the probability is

$$\boxed{\frac{2{,}002}{15{,}504}} \approx \boxed{0.129} \approx \boxed{12.9\%}$$

12



pick 2 of the 7 balls

$$|S| = \binom{7}{2} = \frac{7!}{2!\,5!} = \frac{7\cdot 6\cdot \cancel{5!}}{2\cdot \cancel{5!}} = 21$$

(a) How many ways can two red balls be selected?

$$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{24}{4} = 6$$

pick 2 of the 4 red balls

So, the probability of selecting two red balls is $\boxed{\frac{6}{21} \approx 0.286 \approx 28.6\%}$

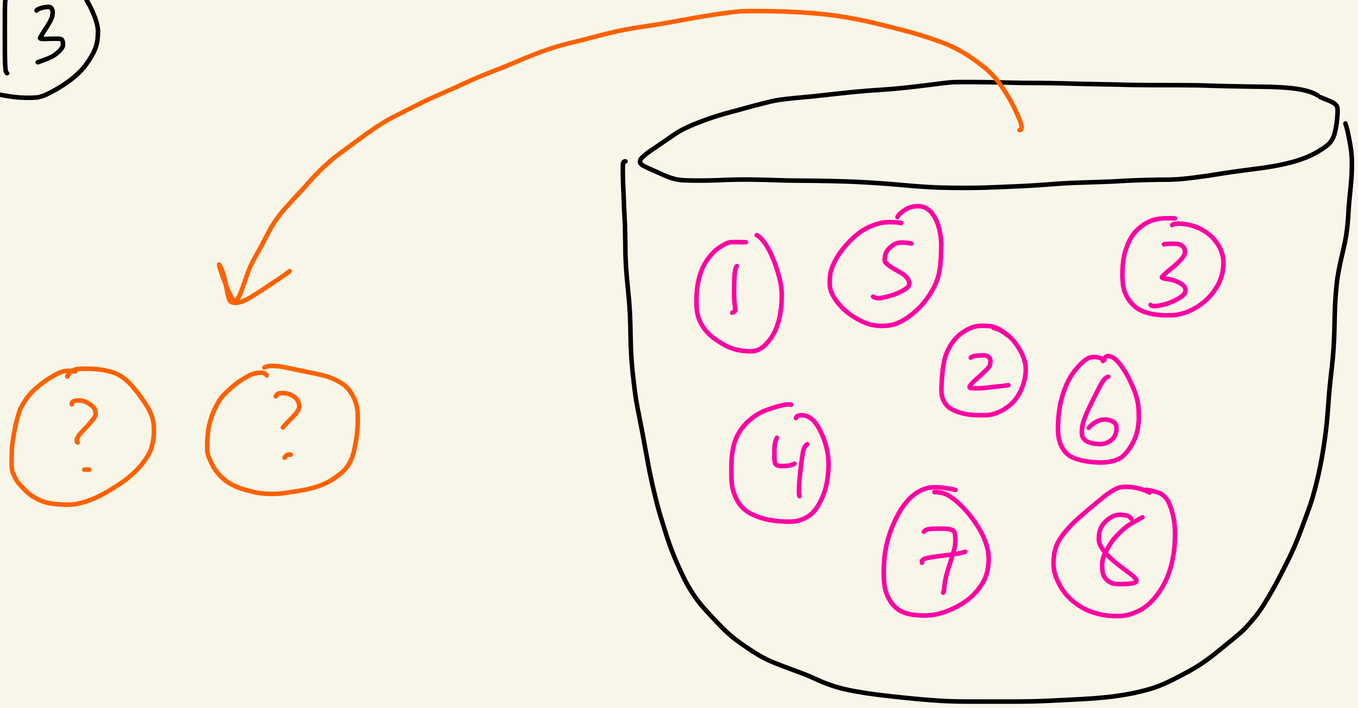
(b) How many ways can one red ball and one green ball be selected?

$$\underbrace{\binom{4}{1}}_{\text{pick one red ball}}\underbrace{\binom{3}{1}}_{\text{pick one green ball}} = 4 \cdot 3 = 12$$

So, the probability of selecting one red ball and one green ball is

$$\boxed{\frac{12}{21} \approx 0.571 \approx 57.1\%}$$

13



$$|S| = \binom{8}{2} = \frac{8!}{2!\,6!} = \frac{8\cdot 7\cdot \cancel{6!}}{2\cdot \cancel{6!}} = 28$$

How many ways can you select two even numbered balls?

$$\underbrace{\binom{4}{2}}_{\text{pick 2 of the 4 even numbered balls}} = \frac{4!}{2!\,2!} = 6.$$

Thus the probability of selecting two even numbered balls is

$$\boxed{\frac{6}{28} \approx 0.214 \approx 21.4\%}$$

(14) Recall there are

$\binom{47}{5} \cdot 27 = 41{,}416{,}353$ possible tickets

(a) The number of tickets that get 2 of the 5 lucky #s correct and the mega number is

$$\frac{\underbrace{\binom{5}{2}}_{\text{pick 2 of the 5 winning lucky numbers}} \cdot \underbrace{\binom{42}{3}}_{\text{pick 3 non-winning lucky numbers}} \cdot \underbrace{\binom{1}{1}}_{\text{pick the winning mega number}}}{41{,}416{,}353} = \frac{(10)(11{,}480)}{41{,}416{,}353} = \boxed{\frac{114{,}800}{41{,}416{,}353}}$$

$$\approx \boxed{0.00277\ldots}$$

$$\approx \boxed{0.277\ \%}$$

(b) The number of tickets that get 4 of the 5 lucky #s correct and the mega number is

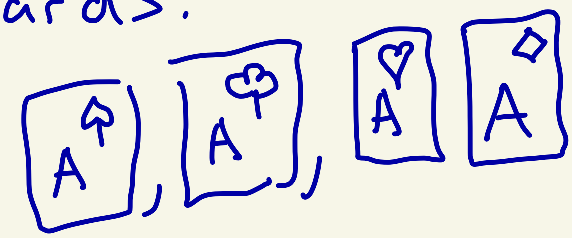
$$\frac{\underbrace{\binom{5}{4}}_{\text{pick 4 of the 5 winning lucky numbers}} \cdot \underbrace{\binom{42}{1}}_{\text{pick 1 non-winning luck number}} \cdot \underbrace{\binom{1}{1}}_{\text{pick the winning mega number}}}{41,416,353} = \frac{(5)(42)}{41,416,353} = \boxed{\frac{210}{41,416,353}}$$

$$\approx \boxed{0.00000507\ldots}$$

$$\approx \boxed{0.000507\%}$$

15

There are $\binom{52}{2} = \frac{52!}{2!\,50!} = \frac{52 \cdot 51}{2} = 1326$ ways to be dealt two cards.

(a) There are four aces: 

Thus there are $\binom{4}{2}$ possible ways to be dealt two aces.

$$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$$

Thus the probability of such an event is

$$\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} \approx \boxed{0.00452489\ldots}$$

$$\boxed{\text{or } \approx 0.45\%}$$

(b)

There are 13 possible face values:

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Each face value has 4 suits.

Thus, there are

$$\underbrace{13}_{\text{choose the face value}} \cdot \underbrace{\binom{4}{2}}_{\text{choose two of the 4 cards, ie choose two from } \heartsuit, \diamondsuit, \spadesuit, \clubsuit} = 13 \cdot \frac{4!}{2!2!} = 13 \cdot 6 = 78$$

ways to get two cards of the same face value.

Thus, the probability of such an event is

$$\frac{13 \cdot \binom{4}{2}}{\binom{52}{2}} = \frac{78}{1326} \approx \boxed{0.5882\ldots}$$

$$\boxed{\text{or } \approx 5.88\%}$$

(c) How many blackjacks are there?

There are 4 aces:

A♥, A♦, A♣, A♠

There are 16 tens, jacks, queens, kings:

10♥, 10♦, 10♣, 10♠

J♥, J♦, J♣, J♠

Q♥, Q♦, Q♣, Q♠

K♥, K♦, K♣, K♠

There are $\underbrace{\binom{4}{1}}_{\text{choose an ace}} \cdot \underbrace{\binom{16}{1}}_{\text{choose a 10, J, Q, or K}} = 4 \cdot 16 = 64$ blackjacks

Thus, the probability of being dealt a blackjack is $\dfrac{64}{\binom{52}{2}} = \boxed{\dfrac{64}{1326}} \approx \boxed{0.048\ldots}$

$\approx \boxed{4.8\%}$

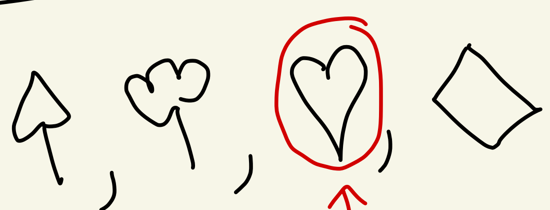
(16) Recall from class that there are

$$\binom{52}{5} = 2,598,960$$

possible 5-card poker hands.

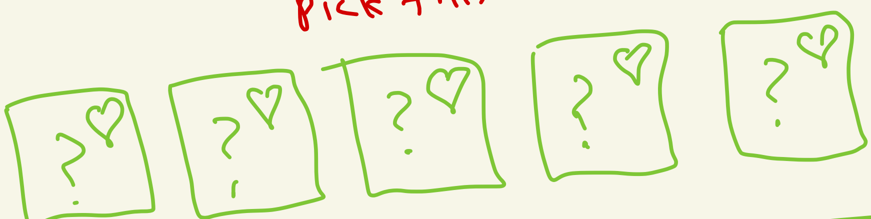
(a) We need to count the number of flushes.

Step 1: Pick the suit.



$\binom{4}{1} = 4$ possibilites

say we pick this one



after step 1

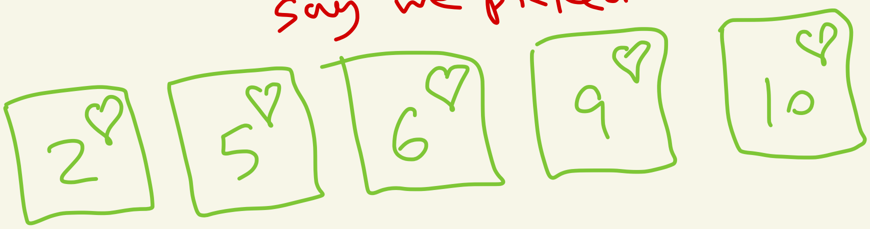
Step 2: Pick 5 face values

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

say we picked these 5

$$\binom{13}{5} = \frac{13!}{5!\,8!} = \frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot \cancel{8!}}{120\cdot \cancel{8!}} = 1287$$

possibilities



after step 2

So, # flushes is $4 \cdot 1287 = 5148$.

Thus, the probability of getting a flush is

$$\frac{5148}{2{,}598{,}960} \approx \boxed{0.00198\ldots} \approx \boxed{0.198\%}$$

(b) Let's count the # of three of a kinds.

<u>Step 1</u>: Pick the face value for the three of a kind.

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K $\leftarrow$ $\boxed{\binom{13}{1} = 13}$

say we picked 5

5 ? 5 ? 5 ? ? ? ? ? $\leftarrow$ after step 1

Step 2: Pick the suits for the three of a kind.

♥, ♦, ♠, ♣

$$\binom{4}{3} = \frac{4!}{3!\,1!} = 4$$

say we picked these three

5♥ | 5♠ | 5♣ | ? ? | ? ?

after step 2

Step 3: Pick the face values for the non-three of a kind part

A, 2, 3, 4, ~~5~~, 6, 7, 8, 9, 10, J, Q, K

$$\binom{12}{2} = 66$$

cant pick

say we picked these

5♥ | 5♠ | 5♣ | 2 ? | 8 ?

after step 3

Step 4: Pick the suits for the non-three of a kind part

♡, ◇, ♠, ♣ $\leftarrow \binom{4}{1} = 4$

♡, ◇, ♠, ♣ $\leftarrow \binom{4}{1} = 4$

(say we picked the circled above)

5♡ 5♠ 5♣ 2◇ 8♡ ← after step 4

Combining all 4 steps gives

$$13 \cdot 4 \cdot 66 \cdot 4 \cdot 4 = 54{,}912$$

three of a kinds.

Thus, the probability of getting a three of a kind is

$$\frac{54{,}912}{2{,}598{,}960} \approx \boxed{0.0211\ldots} \approx \boxed{2.11\%}$$

17. There are 49 remaining cards. Thus, there are $\binom{49}{2} = \frac{49!}{2!\,47!} = \frac{49 \cdot 48 \cdot 47!}{2!\,47!} = \frac{49 \cdot 48}{2} = 1{,}176$ possible two card combinations that you can get.

(a) There are $13 - 3 = 10$ remaining clubs. So, the odds of getting two clubs is

$$\frac{\binom{10}{2}}{\binom{49}{2}} = \boxed{\frac{45}{1{,}176}} \approx \boxed{0.038\ldots} \approx \boxed{3.8\ \%}$$

(b) The cards that give you a straight are

$$\underbrace{\boxed{A^{?}}\ \boxed{5^{?}}}_{\text{gives}} \quad \text{or} \quad \underbrace{\boxed{5^{?}}\ \boxed{6^{?}}}_{\text{gives}}$$

$\boxed{A^{?}}\ \boxed{2^{♣}}\ \boxed{3^{♣}}\ \boxed{4^{♣}}\ \boxed{5^{?}}$ $\qquad$ $\boxed{2^{♣}}\ \boxed{3^{♣}}\ \boxed{4^{♣}}\ \boxed{5^{?}}\ \boxed{6^{?}}$

where ? is any suit except you don't want to count $\boxed{A^{♣}}\ \boxed{5^{♣}}$ or $\boxed{5^{♣}}\ \boxed{6^{♣}}$ since those would give you a straight flush.

Thus, the number of hands that give you a straight but not a straight flush is

$$\underbrace{4 \cdot 4}_{\boxed{A^{?}}\ \boxed{5^{?}}} + \underbrace{4 \cdot 4}_{\boxed{5^{?}}\ \boxed{6^{?}}} - \underbrace{2}_{\substack{\boxed{A^{\clubsuit}}\ \boxed{5^{\clubsuit}} \\ \boxed{5^{\clubsuit}}\ \boxed{6^{\clubsuit}}}} = 30$$

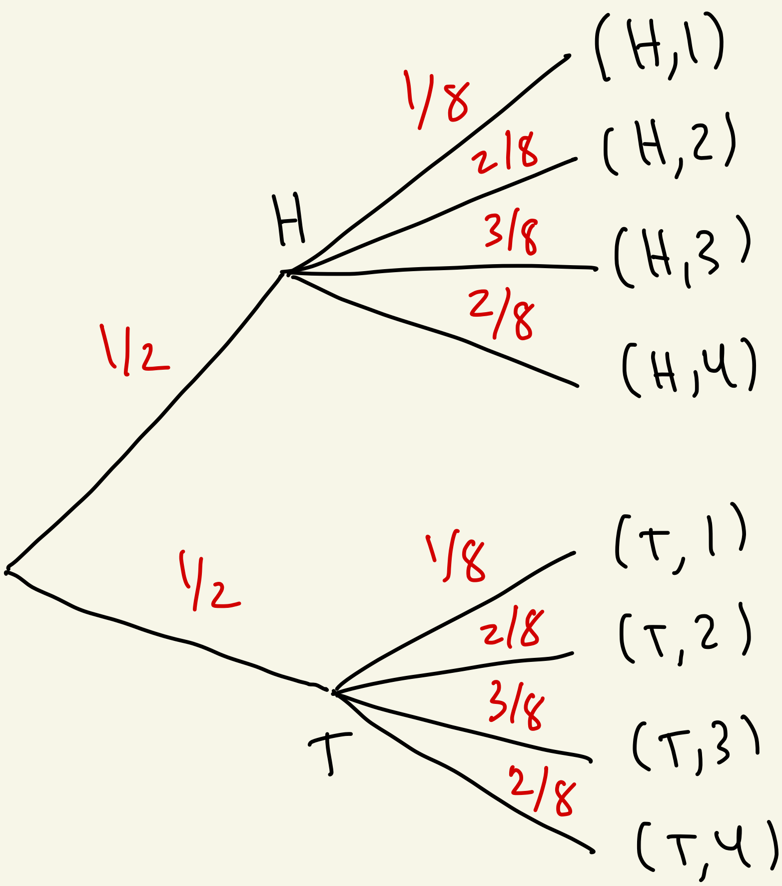
So, the probability is $\boxed{\dfrac{30}{1,176}} \approx \boxed{0,02551} \approx \boxed{2.551\%}$

(c) The cards that give you a straight flush are $\boxed{A^{\clubsuit}}\ \boxed{5^{\clubsuit}}$ and $\boxed{5^{\clubsuit}}\ \boxed{6^{\clubsuit}}$. Thus, the probability is $\boxed{\dfrac{2}{1,176}} \approx \boxed{0.0017\ldots} \approx \boxed{0.17\%}$

18

$$S = \{ (H,1), (H,2), (H,3), (H,4), (T,1), (T,2), (T,3), (T,4) \}$$

Ω is the set of all subsets of S



$P(H,1) = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$

$P(H,2) = \frac{1}{2} \cdot \frac{2}{8} = \frac{2}{16}$

$P(H,3) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$

$P(H,4) = \frac{1}{2} \cdot \frac{2}{8} = \frac{2}{16}$

$P(T,1) = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$

$P(T,2) = \frac{1}{2} \cdot \frac{2}{8} = \frac{2}{16}$

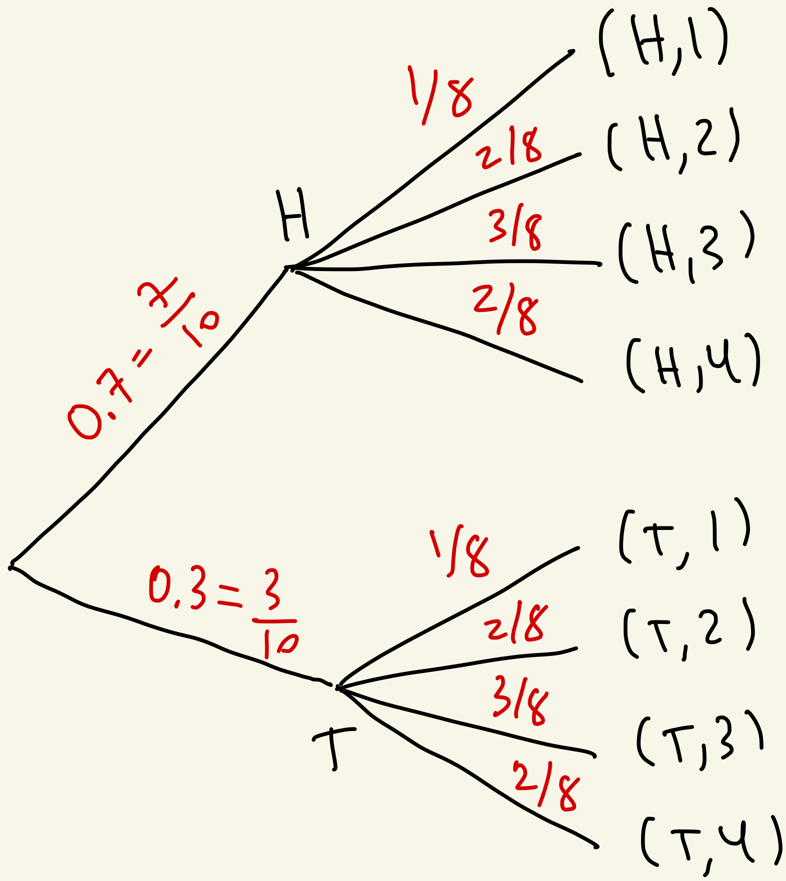
$P(T,3) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$

$P(T,4) = \frac{1}{2} \cdot \frac{2}{8} = \frac{2}{16}$

(19)

$S = \{(H,1), (H,2), (H,3), (H,4),$
$(T,1), (T,2), (T,3), (T,4)\}$

Ω is the set of all subsets of S



$P(H,1) = \frac{7}{10} \cdot \frac{1}{8} = \frac{7}{80}$

$P(H,2) = \frac{7}{10} \cdot \frac{2}{8} = \frac{14}{80}$

$P(H,3) = \frac{7}{10} \cdot \frac{3}{8} = \frac{21}{80}$

$P(H,4) = \frac{7}{10} \cdot \frac{2}{8} = \frac{14}{80}$

$P(T,1) = \frac{3}{10} \cdot \frac{1}{8} = \frac{3}{80}$

$P(T,2) = \frac{3}{10} \cdot \frac{2}{8} = \frac{6}{80}$

$P(T,3) = \frac{3}{10} \cdot \frac{3}{8} = \frac{9}{80}$

$P(T,4) = \frac{3}{10} \cdot \frac{2}{8} = \frac{6}{80}$