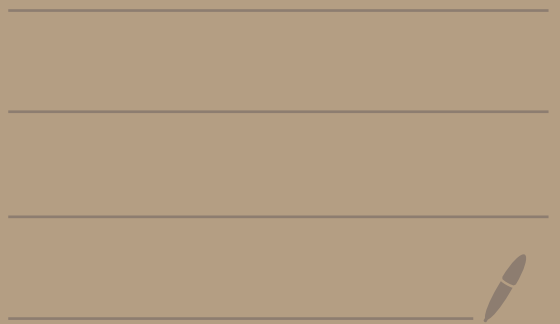


Math 5800

8/23/21



- I'll use your calstate la.edu email to mass email the class announcements. If you want me to use a different email just let me know.

- Tests are done on canvas. No class on the test day. The test will appear on canvas at 6am on the test day and disappear at noon the following day.

[Ex: Mon 6am - Tues noon]

During that window you pick a 2.5 hour time to take the test. When you open the test canvas will time you. When done scan and upload as a pdf.

Topic 1 - Countable and Uncountable sets

pg
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Def: We say that a set S is countably infinite if S is infinite and the elements of S can be arranged in a sequence without repeats, that is S is of the form

$$S = \{s_1, s_2, s_3, s_4, \dots\}$$

Another def is S is countably infinite if there exists a bijection $f: \mathbb{N} \rightarrow S$. Then,

$$S = \{f(1), f(2), f(3), f(4), \dots\}$$

Def: We say that S is countable if S is finite or S is countably infinite.

Ex: Some countable sets:

$$S = \{5, 10, \pi, 3\}$$

← finite

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$$

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, 4, -4, \dots\}$$

← Countably infinite

Theorem: Let A and B
be countable. Then $A \cup B$
is countable.

proof:

Case 1: If A and B are both
finite then $A \cup B$ is finite and
so its countable.

Case 2: Suppose $A = \{a_1, a_2, \dots, a_n\}$
is finite and
 $B = \{b_1, b_2, b_3, b_4, \dots\}$
is countably infinite.

Then
 $A \cup B = \{a_1, a_2, \dots, a_n, b_1, b_2, b_3, b_4, \dots\}$
is an enumeration of $A \cup B$. Skip
any duplicates when you enumerate.

case 3: If B is finite and A is countably infinite do the same thing as case 2.

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case 4: Suppose

$$A = \{a_1, a_2, a_3, a_4, \dots\}$$

and

$$B = \{b_1, b_2, b_3, b_4, \dots\}$$

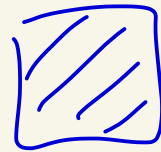
are both countably infinite and the above are enumerations without duplicates.

Then

$$A \cup B = \{a_1, b_1, a_2, b_2, a_3, b_3, \dots\}.$$

Skip any repeats.

So, $A \cup B$ is countable.




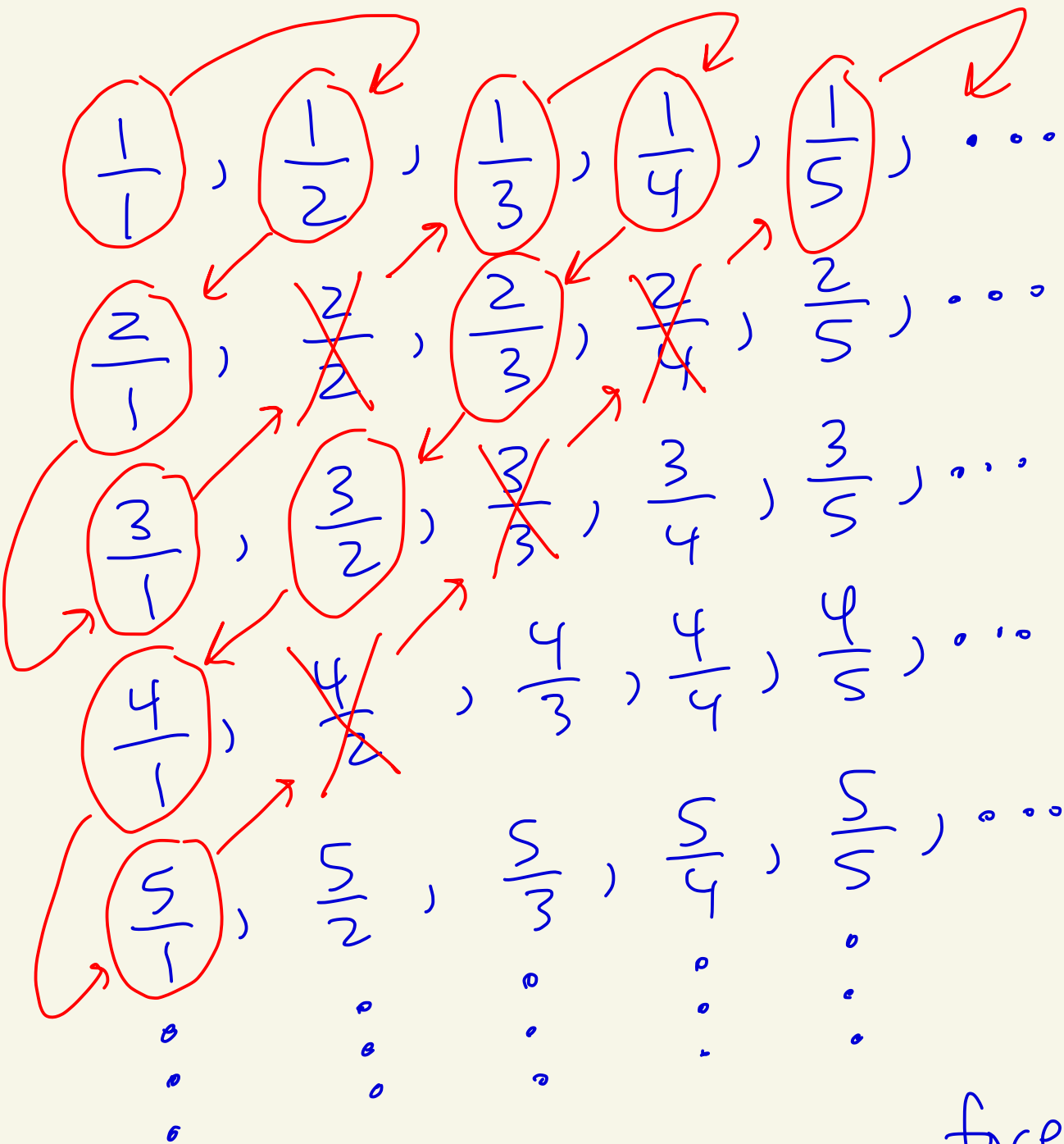
Theorem: The rational numbers \mathbb{Q} is countable. pg
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proof: Let $\mathbb{Q}_+ = \{r \mid r > 0, r \in \mathbb{Q}\}$
and $\mathbb{Q}_- = \{r \mid r < 0, r \in \mathbb{Q}\}$.

Then, $\mathbb{Q} = \mathbb{Q}_+ \cup \mathbb{Q}_- \cup \{0\}$.
Let's show that \mathbb{Q}_+ is countable.

Consider the following way to arrange \mathbb{Q}_+ where we cross out any elements not in lowest form and circle the elements of \mathbb{Q}_+ .





Continuing in this way forever yields a sequence which enumerates \mathbb{Q}_+ without repeats. That is,

$$\mathbb{Q}_+ = \left\{ 1, \frac{1}{2}, 2, 3, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{2}, 4, 5, \frac{1}{5}, \dots \right\}$$

So, \mathbb{Q}_+ is countable.

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8)

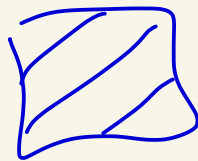
Similarly \mathbb{Q}_- is countable.

Also, $\{0\}$ is countable.

Thus, by the previous theorem

$$\mathbb{Q} = \mathbb{Q}_+ \cup \mathbb{Q}_- \cup \{0\}$$

is countable.



Theorem: The union of a countable number of countable sets is countable.

proof:

Case 1: Suppose we have a finite number of countable sets $S_1, S_2, S_3, \dots, S_n$.

Then by iterating the theorem on page 4 we have that

$$S_1 \cup S_2 \cup S_3 \cup \dots \cup S_n$$

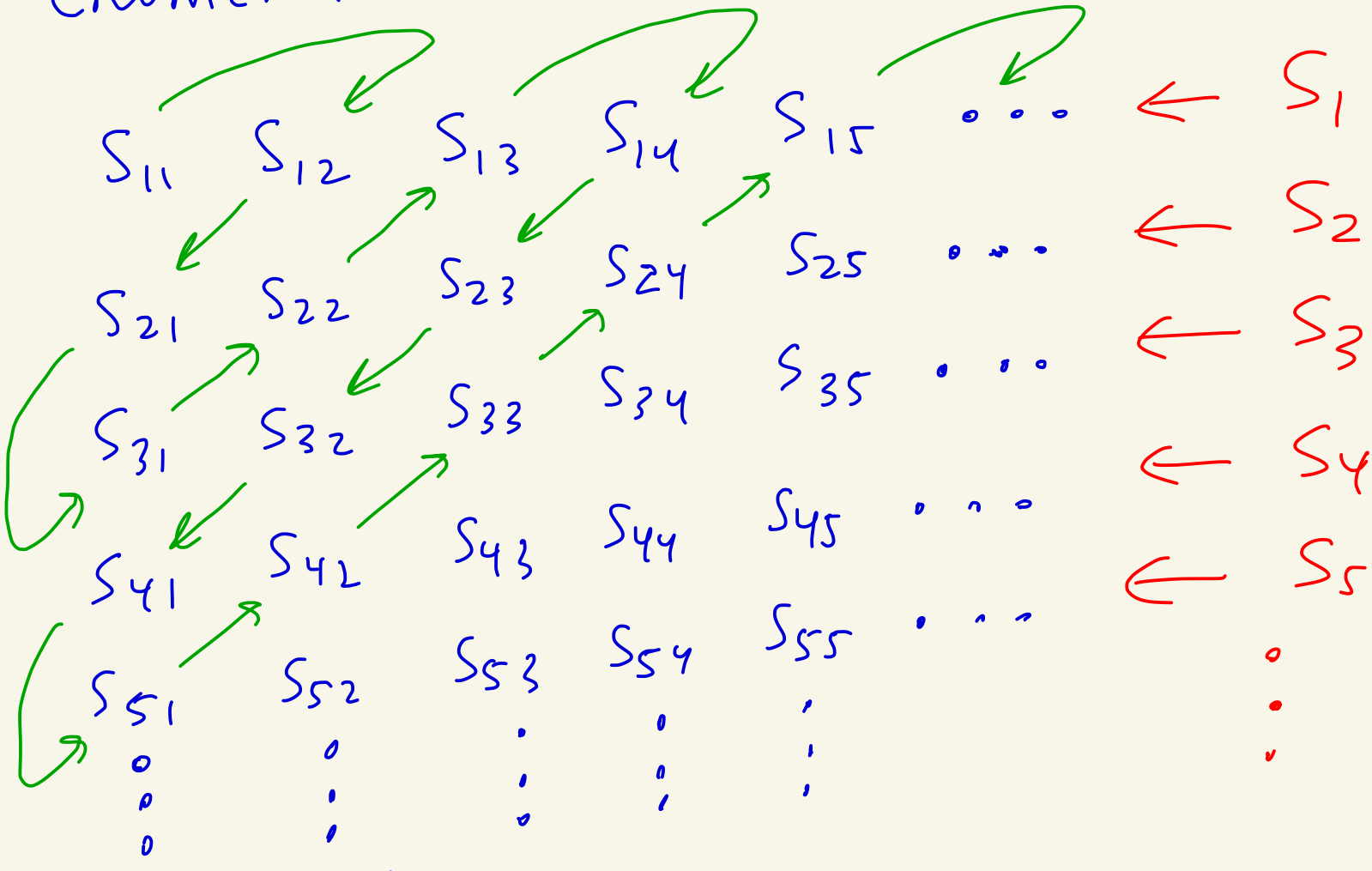
is countable.

Case 2: Suppose we have a countably infinite number of countable sets $S_1, S_2, S_3, S_4, \dots$

Suppose

$$S_i = \{s_{i1}, s_{i2}, s_{i3}, \dots\}$$

Arrange the sets as follows and enumerate like with \mathbb{Q}_+ :



List out

$$\bigcup_{i=1}^{\infty} S_i = \{s_{11}, s_{12}, s_{21}, s_{31}, s_{22}, s_{13}, \dots\}$$

as above, skipping any repeats,
to show that $\bigcup_{i=1}^{\infty} S_i$ is countable. ◻