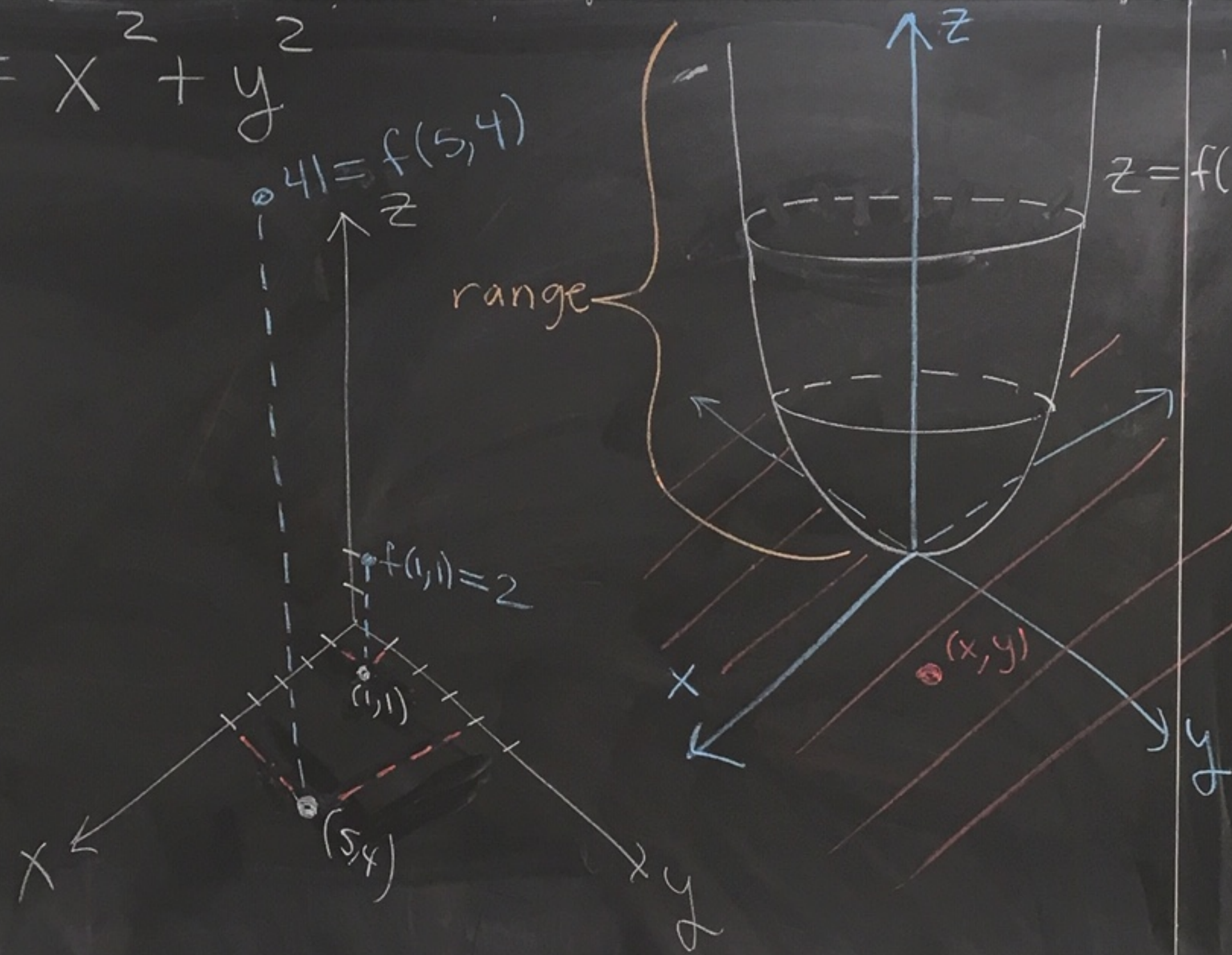


Ex: $f(x,y) = x^2 + y^2$

| (x,y) | $z = f(x,y) = x^2 + y^2$ |
|----------|--------------------------|
| $(1,1)$ | $1^2 + 1^2 = 2$ |
| $(2,-2)$ | $2^2 + (-2)^2 = 8$ |
| $(4,5)$ | $4^2 + 5^2 = 41$ |

$$\begin{array}{r} 16 \\ 25 \\ \hline 41 \end{array}$$



$D = \text{domain is all } (x,y).$
 range = possible z -values
 $= [0, \infty)$

Ex: $f(x,y) = \sqrt{x+y+1}$

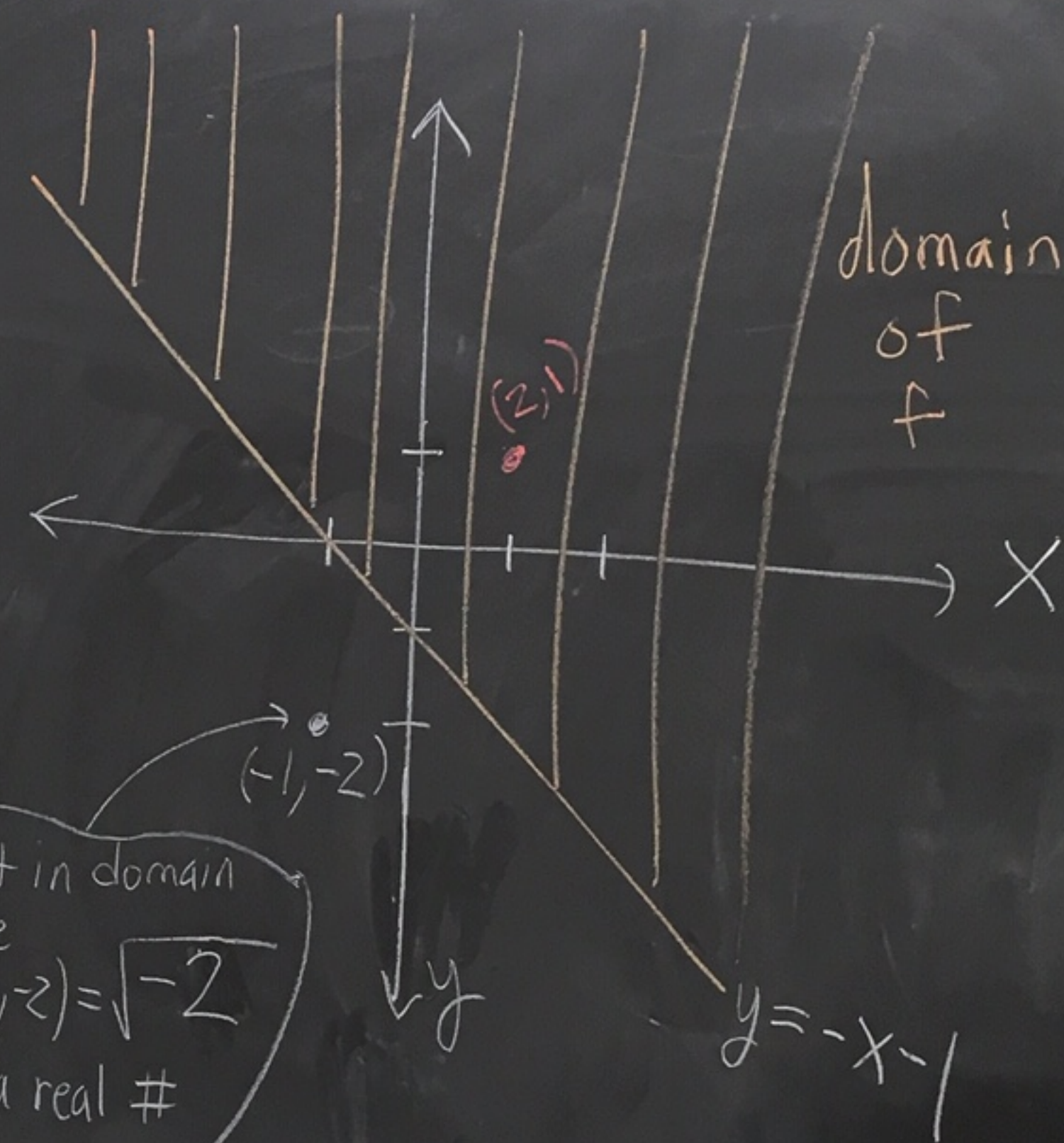
$$f(2,1) = \sqrt{2+1+1} = \sqrt{4} = 2$$

What is the domain of f ?
I.e. what (x,y) can we plug into f ?

To plug (x,y) into f we need

$$x+y+1 \geq 0$$

$$y \geq -x-1$$



not in domain
since
 $f(-1,-2) = \sqrt{-2}$
not a real #

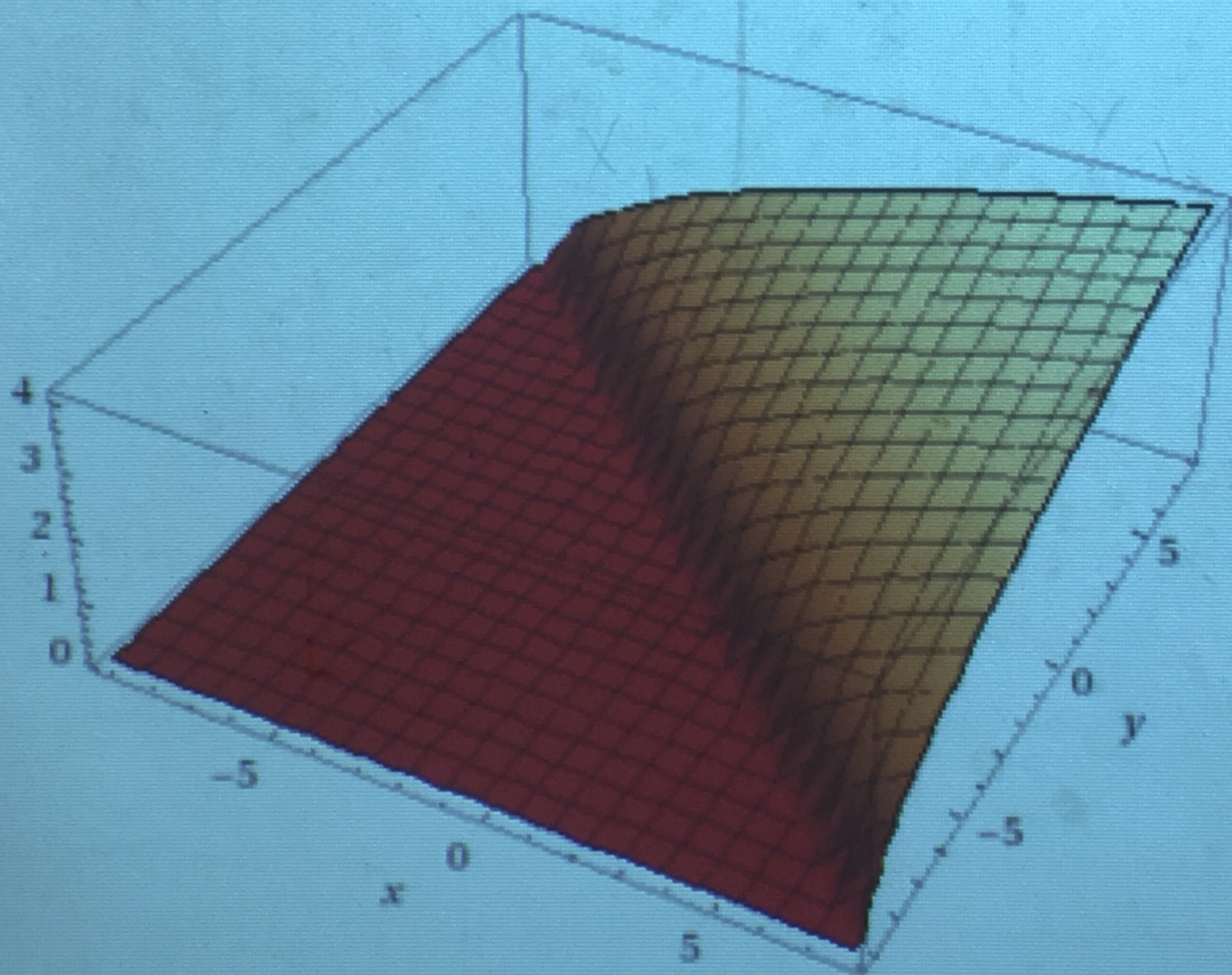
ible z-values
(∞)

Input interpretation:

| | |
|------|--------------------|
| plot | $z = \sqrt{x+y+1}$ |
|------|--------------------|

3D plots:

Real part



The level curves are on the xy -plane

Def: The level curves of a function f of two variables are the curves with equations $k = f(x, y)$ where k is any constant in the range of f .

Ex: Find and draw some level curves of $f(x, y) = x^2 + y^2$.

range of f is $[0, \infty)$

possible k values

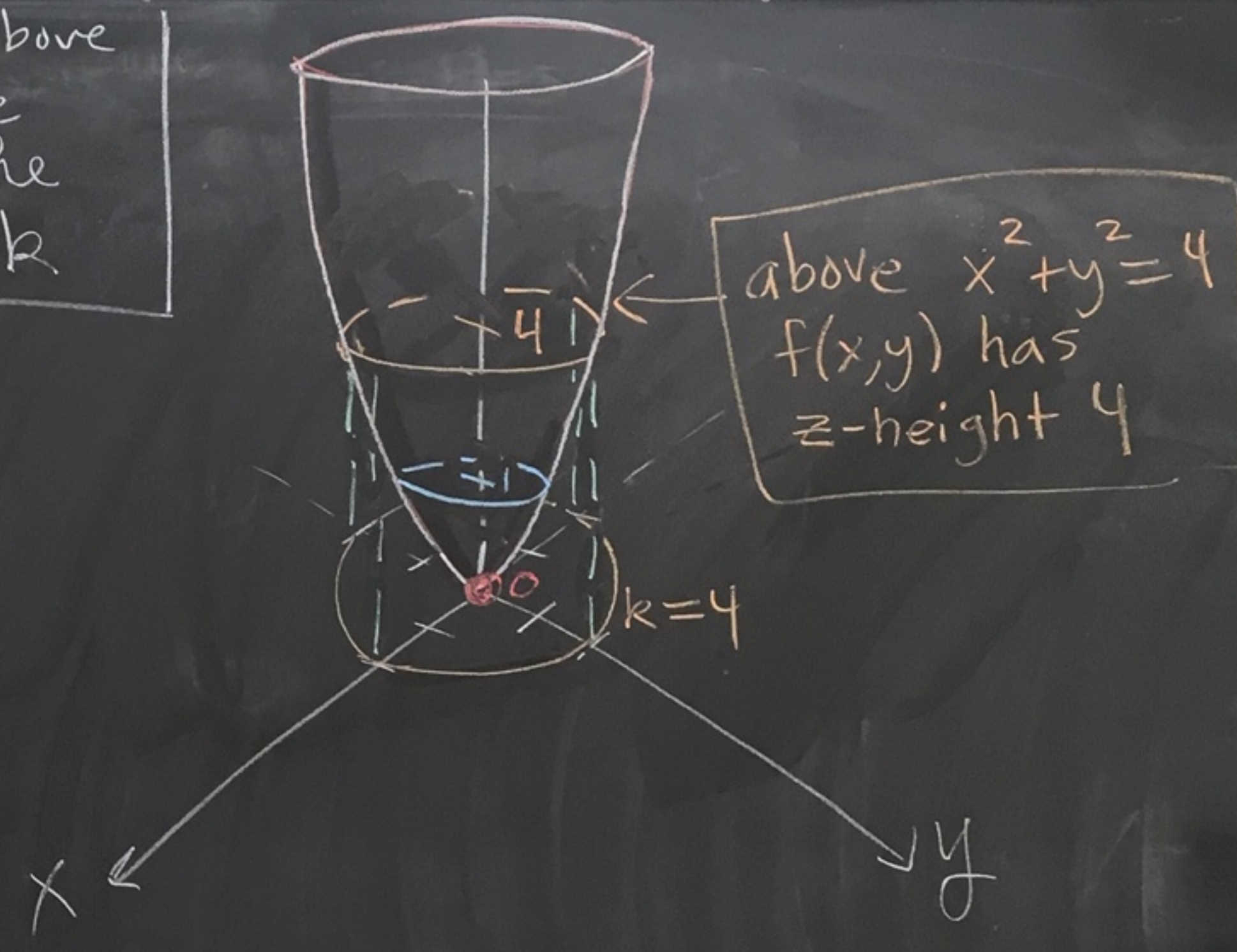
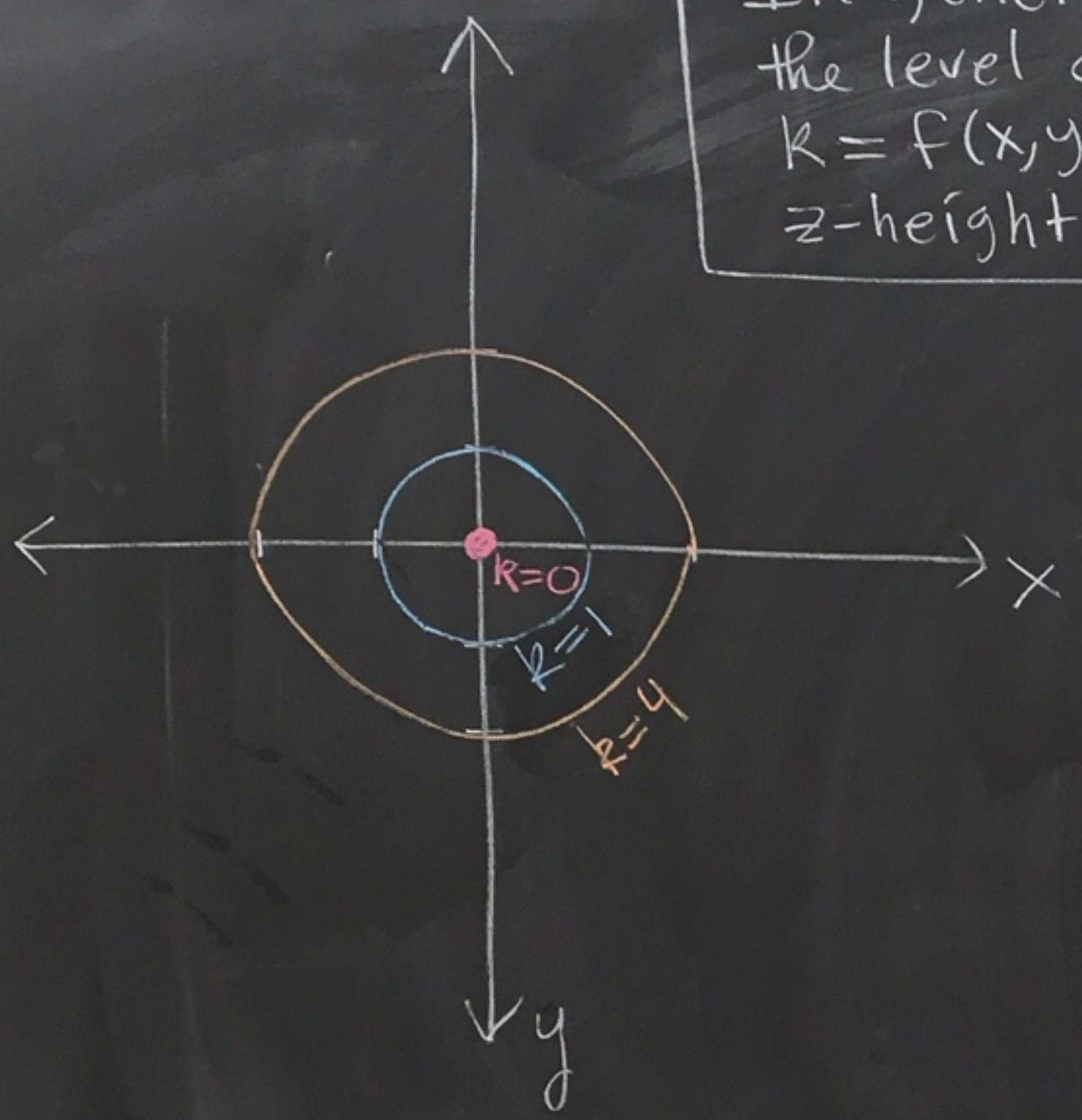
$$\frac{k=0}{0=x^2+y^2}$$

$$\frac{k=1}{1=x^2+y^2}$$

$$\frac{k=4}{4=x^2+y^2}$$

$$2^2=x^2+y^2$$

In general, above the level curve $k=f(x,y)$ the z-height is k



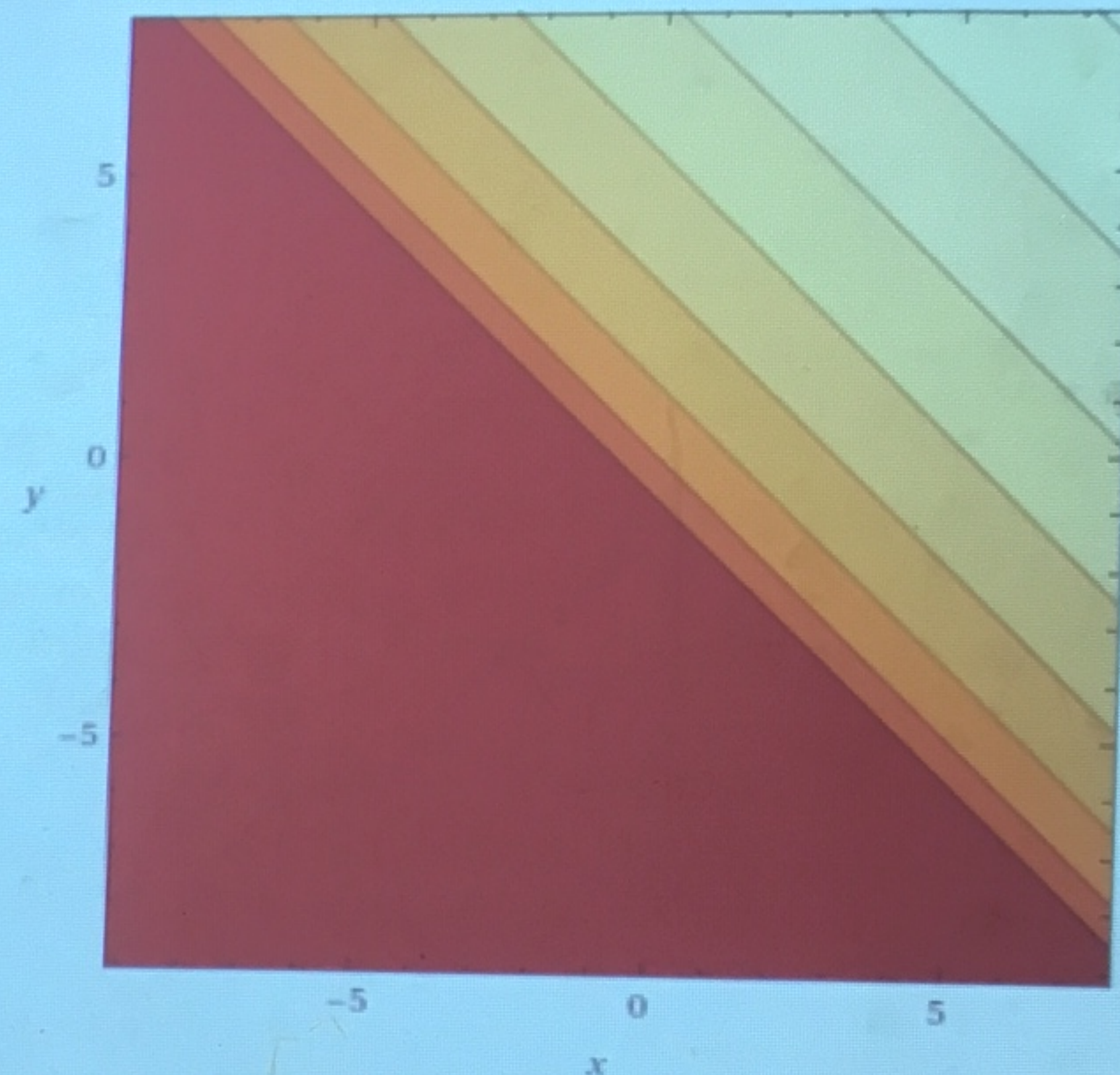
level
curves
for

$$f(x,y) = \sqrt{x+y+1}$$

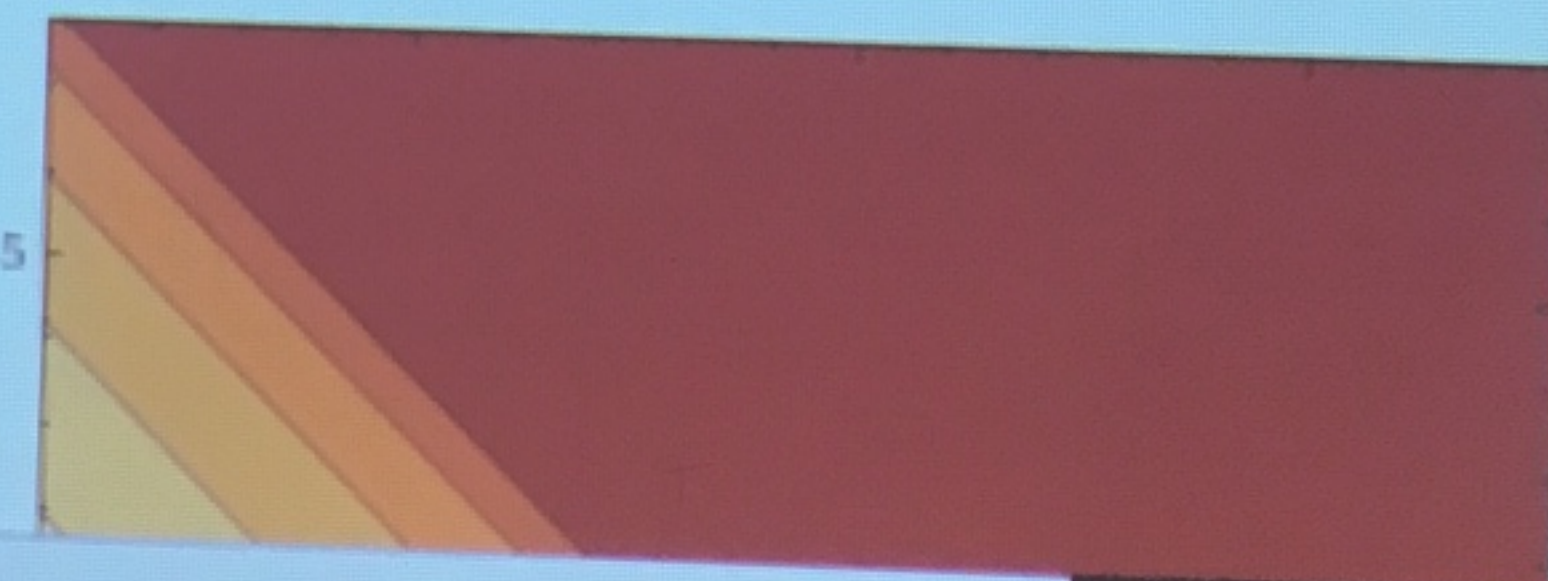


Contour plots:

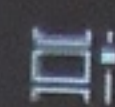
Real part



Imaginary part



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12.3 - Limits and Continuity

Ex: Let's look at the behavior of $f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$ near $(0,0)$.

Note: $f(0,0) = \frac{\sin(0)}{0}$ is undefined

Z-value plot

| X \ y | -1 | -0.5 | -0.2 | 0 | 0.2 | 0.5 | 1 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| -1 | 0.455 | 0.759 | 0.829 | 0.841 | 0.829 | 0.759 | 0.455 |
| -0.5 | 0.759 | 0.959 | 0.986 | 0.990 | 0.986 | 0.959 | 0.759 |
| -0.2 | 0.829 | 0.986 | 0.999 | 1.000 | 0.999 | 0.986 | 0.829 |
| 0 | 0.841 | 0.990 | 1.000 | | 1.000 | 0.990 | 0.841 |
| 0.2 | 0.829 | 0.986 | 0.999 | 1.000 | 0.999 | 0.986 | 0.829 |
| 0.5 | 0.759 | 0.959 | 0.986 | 0.990 | 0.986 | 0.959 | 0.759 |
| 1 | 0.455 | 0.759 | 0.829 | 0.841 | 0.829 | 0.759 | 0.455 |

No matter how you approach (0,0)

the function

$$f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$$

approaches 1.
After we define limit we will have

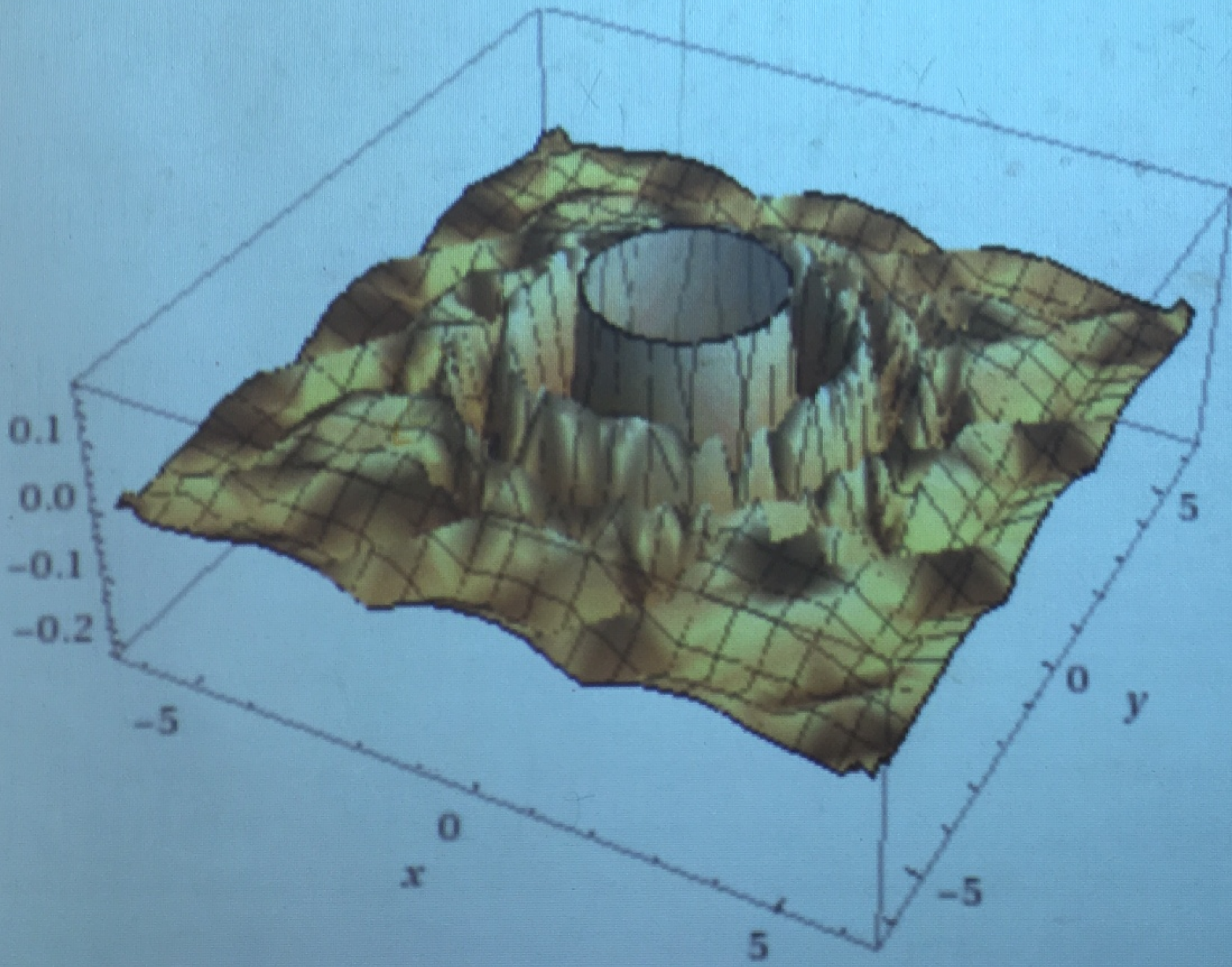
$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = 1$$

input interpretation.

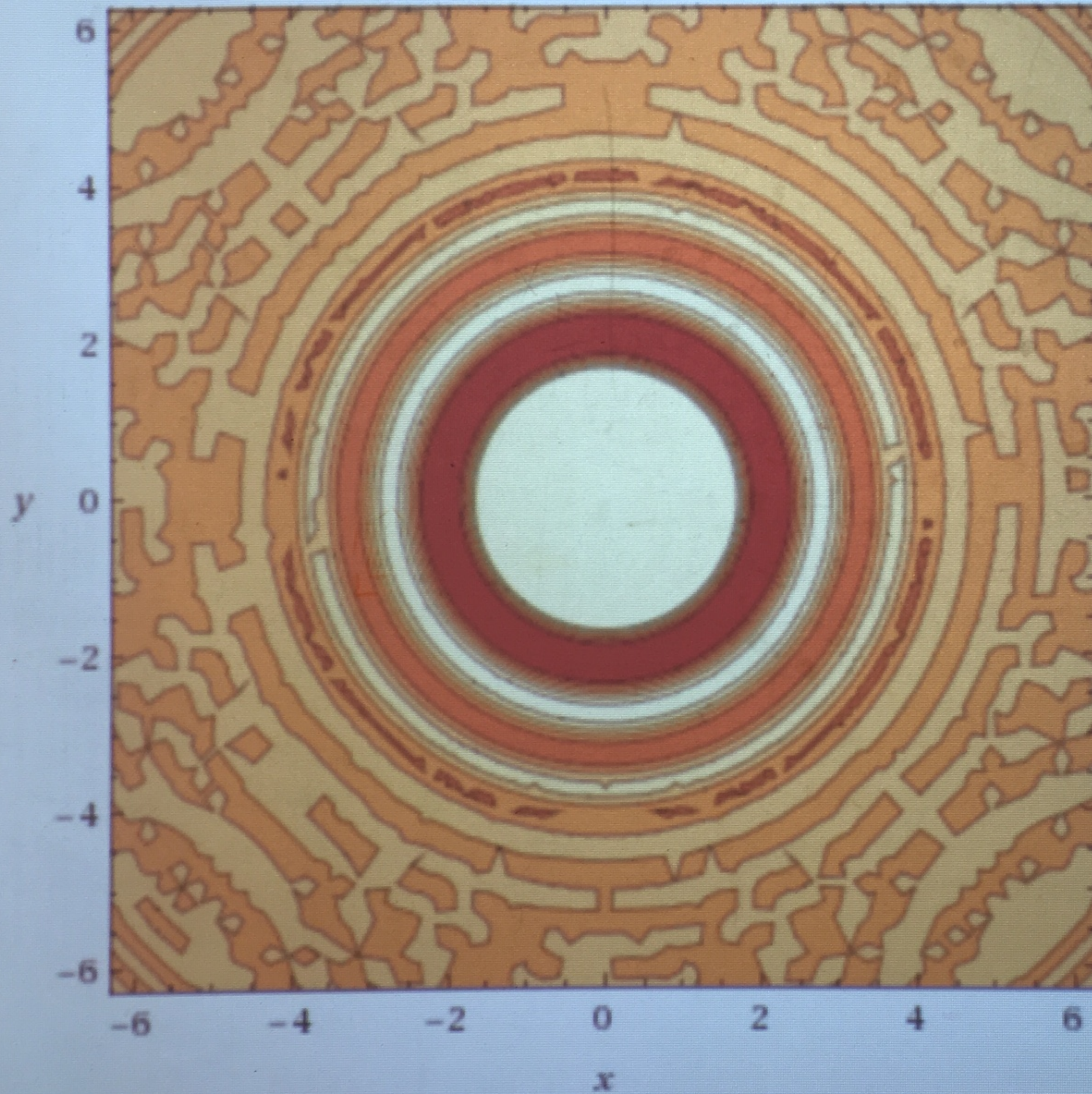
| | |
|------|---|
| plot | $z = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$ |
|------|---|

3D plot:

Show con



Contour plot:



Type here to search

