8/29 - Thursday

Plot of $g(x,y) = \frac{x^2-y^2}{x^2+y^2}$

Let

$$g(x,y) = \frac{x^2-y^2}{x^2+y^2}.$$

What happens near (0,0)?

Note $g(0,0) = \frac{0}{0}$ (undefined)

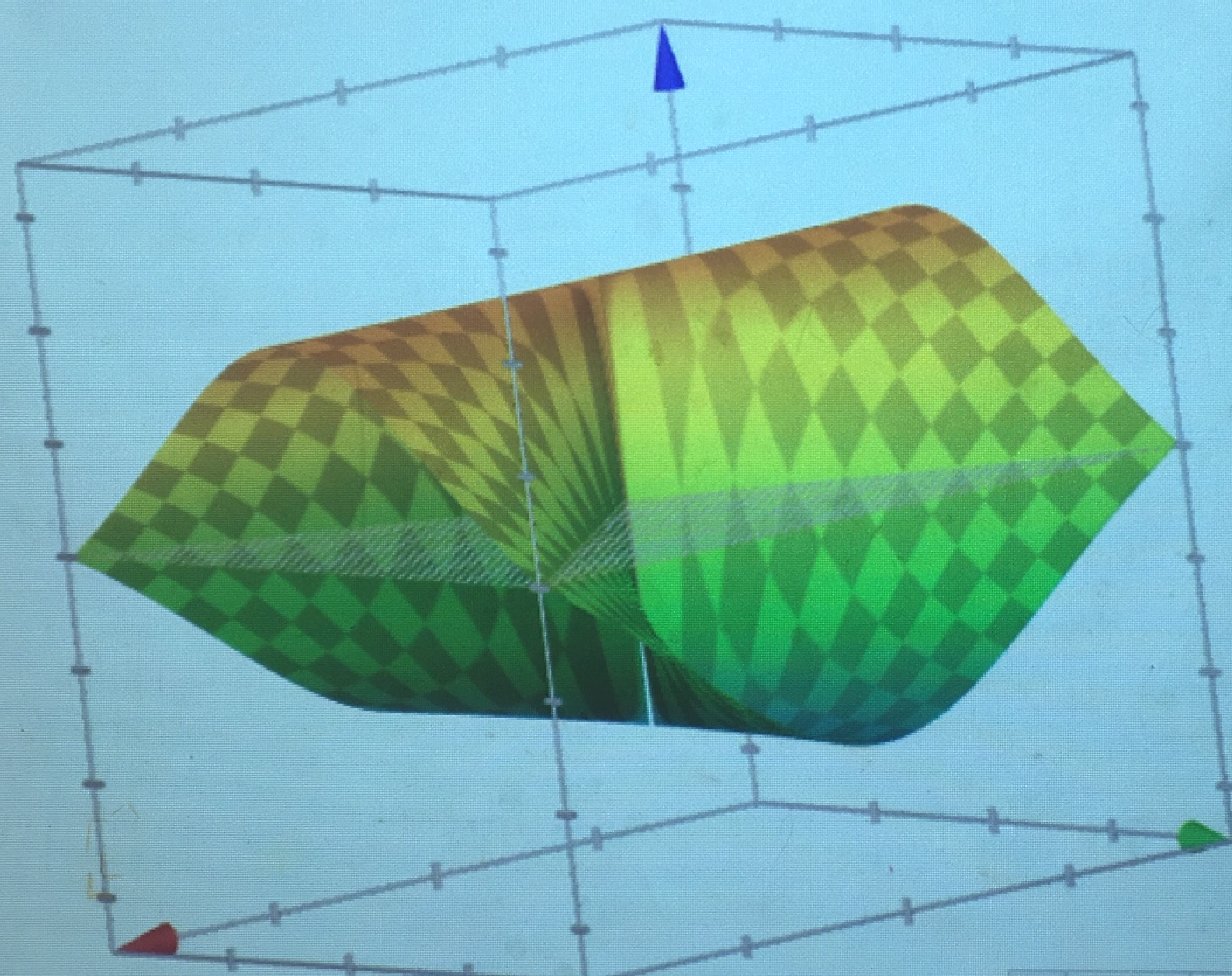
x \ y	-1	-0.5	-0.2	0	0.2	0.5	1.0
-1	0.000	0.600	0.923	1.000	0.923	0.600	0.000
-0.5	-0.600	0.000	0.724	1.000	0.724	0.000	-0.600
-0.2	-0.923	-0.724	0.000	1.000	0.000	-0.724	-0.923
0	-1.00	-1.000	-1.000		-1.000	-1.000	-1.000
0.2	-0.923	-0.724	0.000	1.000	0.000	-0.724	-0.923
0.5	-0.600	0.000	0.724	1.000	0.724	0.000	-0.600
1	0.000	0.600	0.923	1.000	0.923	0.600	0.000

There's no way to make a limit as $(x,y) \to (0,0)$.

https://www.google.com/search?source=hp&ei=MztoXevnELXx9APAo6-wAg&c

Google

(x^2 - y^2) / (x^2 + y^2)



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Do more with Microsoft Edge – the fast, new browser built for Windows 10.

Change my default

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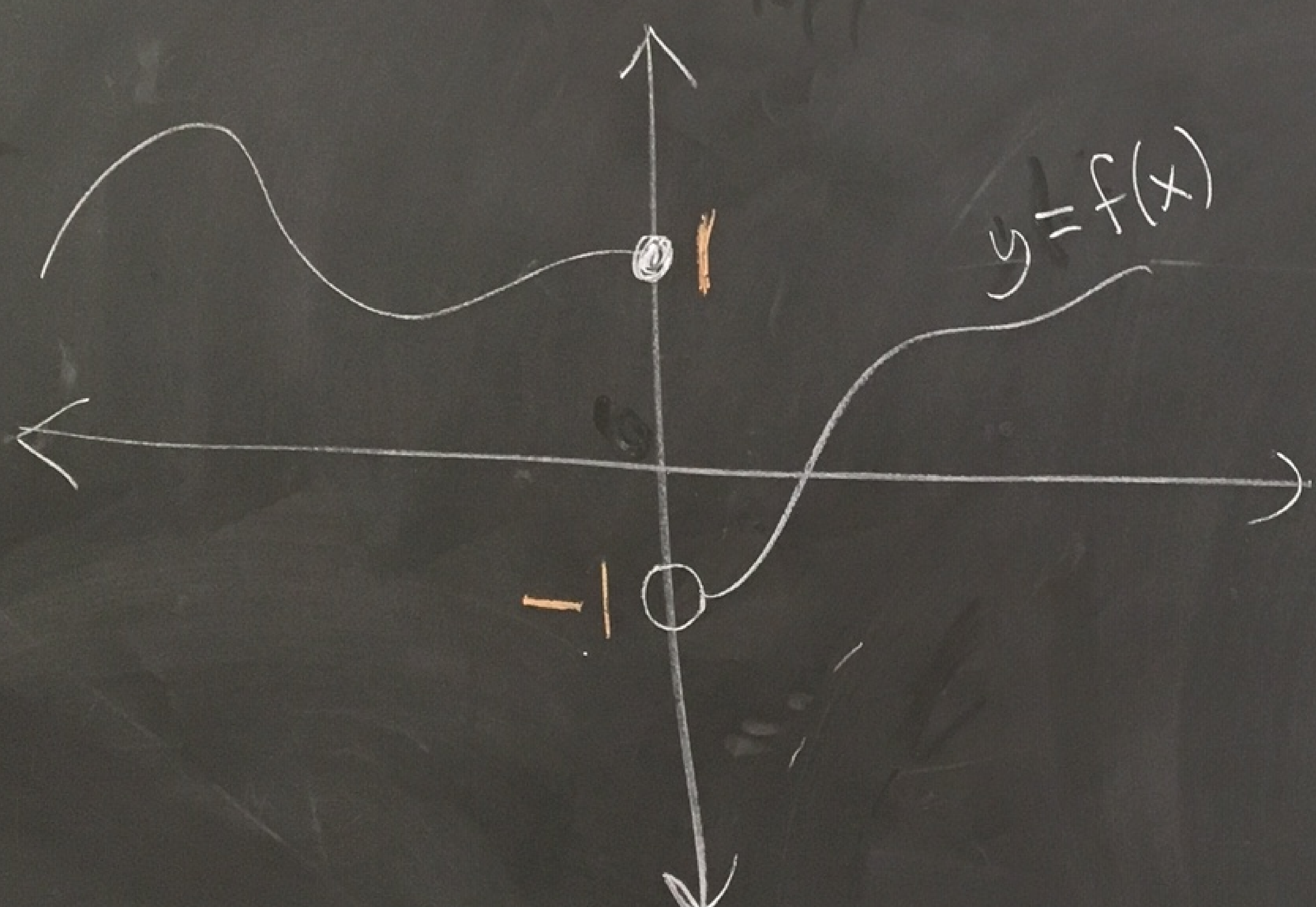
8/29/2019

This example is analogous to



Here $\lim_{x\to 0^-} f(x) = 1$

$\lim_{x\to 0^+} f(x) = -1$

So $\lim_{x\to 0} f(x)$ does not exist

Def: Let f be a function of two variables whose domain D includes points arbitrarily close to (a,b). We say that the <u>limit</u> of $f(x,y)$ as (x,y) approaches (a,b) is L and write $\boxed{\lim_{(x,y)\to(a,b)} f(x,y) = L}$ if

"We can make the values of $f(x,y)$ as close to L as we like by taking the point (x,y) to be sufficiently close to (a,b), but not equal to (a,b)."

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"We can make the values of $f(x,y)$ as close to L as we like by taking the point (x,y) to be sufficiently close to (a,b), but not equal to (a,b)."

Math version:

"for every $\varepsilon > 0$ there is a corresponding $\delta > 0$ such that

$$|f(x,y) - L| < \varepsilon$$

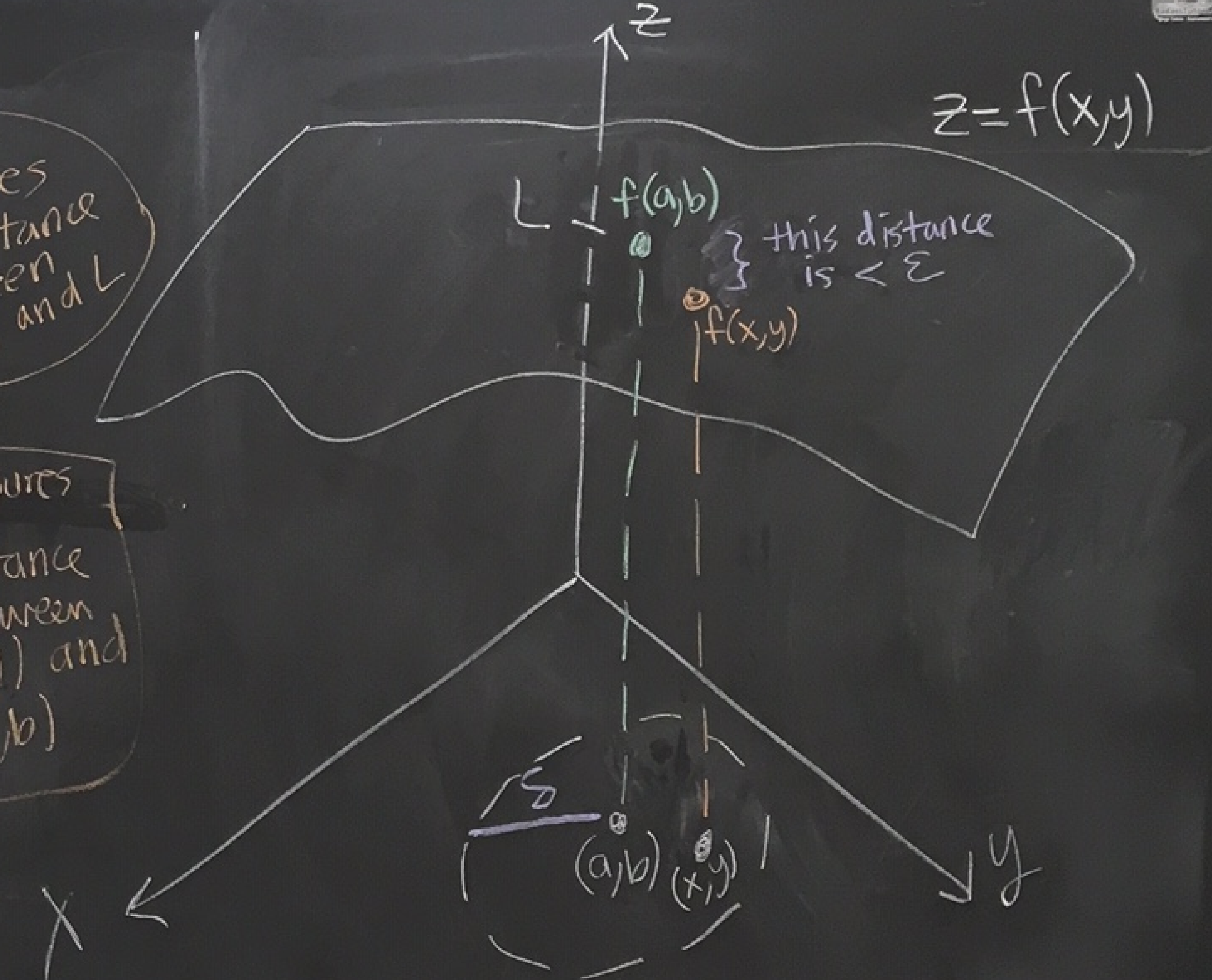
measures the distance between $f(x,y)$ and L

whenever (x,y) is in D and

$$0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$$

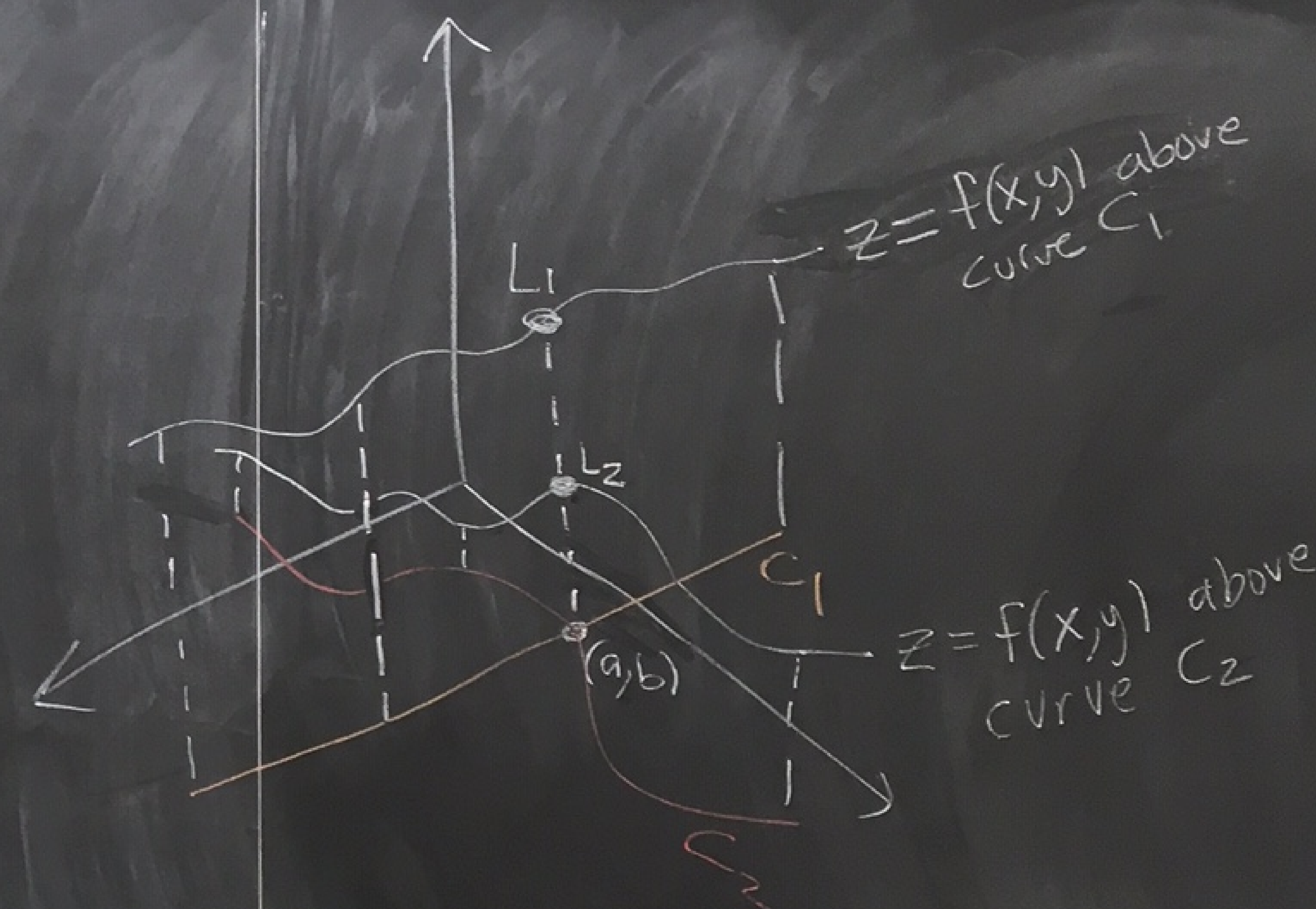
measures distance between (x,y) and (a,b)

$(x,y) \neq (a,b)$



Method to show a limit doesn't exist at (a,b)

If $f(x,y)$ approaches L_1 as (x,y) approaches (a,b) along a curve C_1 and $f(x,y)$ approaches L_2 as (x,y) approaches (a,b) along a curve C_2 and $L_1 \neq L_2$ then $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.



Ex

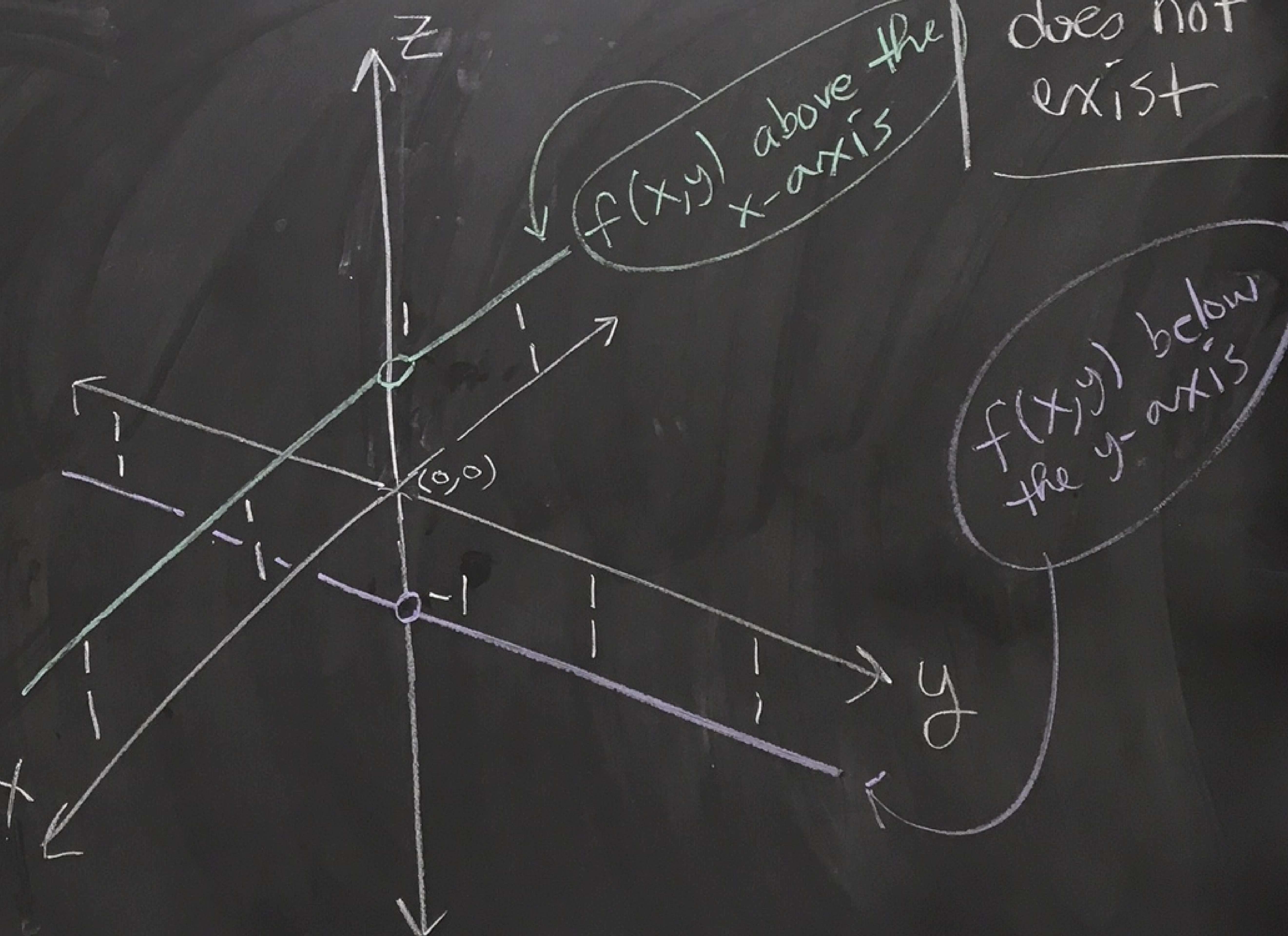
Ex: Show $\lim\limits_{(x,y)\to(0,0)} \left(\frac{x^2-y^2}{x^2+y^2}\right)$ does not exist.

Approach (0,0) on the y-axis (x=0)

If $x=0$ then
$$\frac{x^2-y^2}{x^2+y^2} = \frac{-y^2}{y^2} = -1$$

Approach on the x-axis

If $y=0$, then
$$\frac{x^2-y^2}{x^2+y^2} = \frac{x^2}{x^2} = 1$$



Since $1 \neq -1$ the limit does not exist

Ex: Does $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ exist?

Approach along x-axis

When $y=0$

$$\left(\frac{x\cdot y}{x^2+y^2}\right)=0$$

Approaching along y-axis

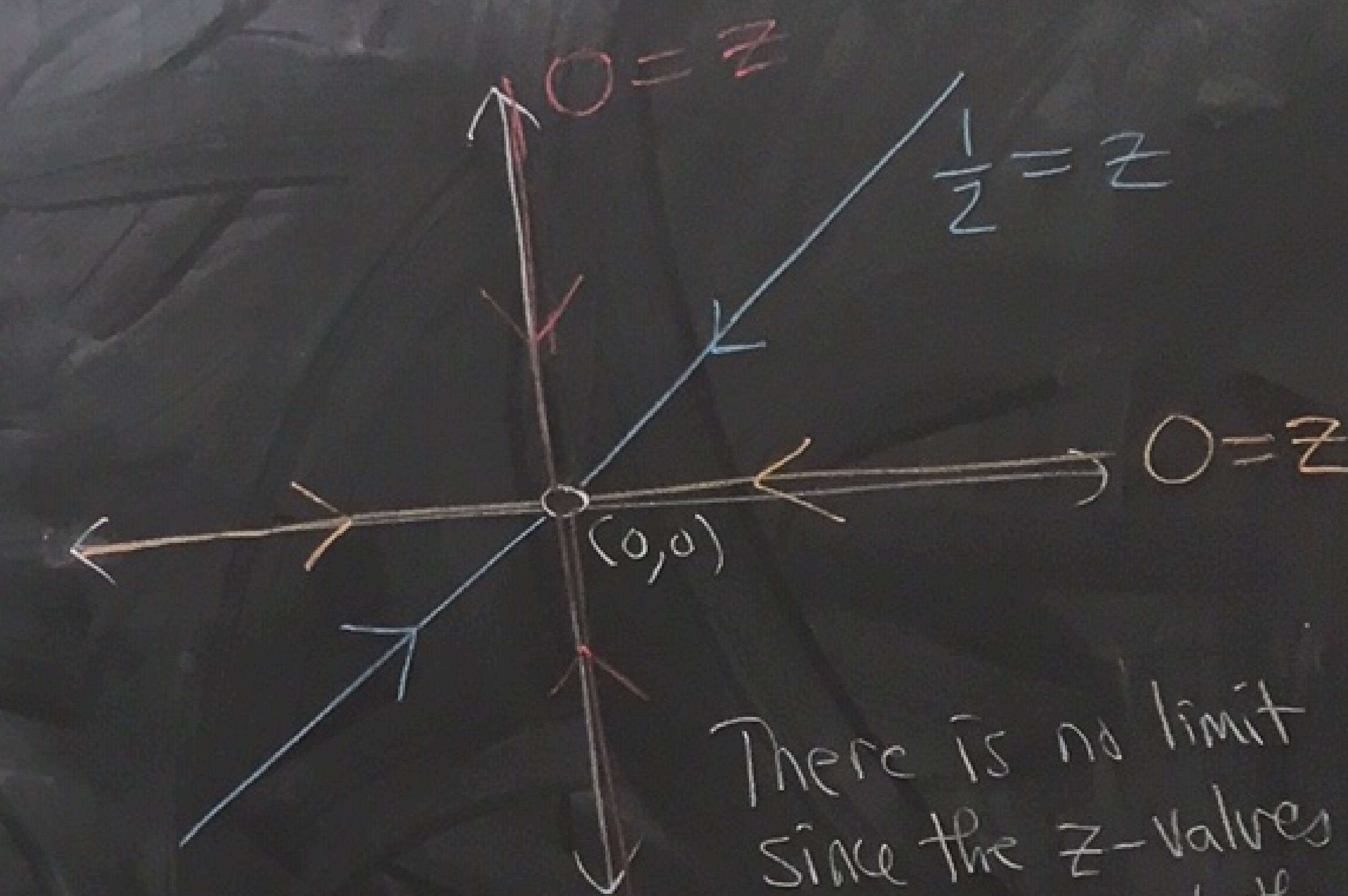
When $x=0$

$$\left(\frac{xy}{x^2+y^2}\right)=0$$

Approach along $y=x$ curve

When $y=x$

$$\frac{xy}{x^2+y^2}=\frac{x^2}{x^2+x^2}=\frac{x^2}{2x^2}=\frac{1}{2}$$



There is no limit since the z-values don't approach the same number at (0,0).

Def: A function f of two variables is called <u>continuous at (a,b)</u> if

1. $f(a,b)$ exists
2. $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ exists

and 3. $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

We say that f is continuous on a set A if f is continuous at every point in A.

Thm: A polynomial function of two variables is continuous everywhere.

A rational function (ratio of polynomials) of two variables is continuous everywhere in its domain.

Ex:

$$\lim_{(x,y)\to(2,1)} (xy+y^3) \overset{\uparrow}{=} (2)(1)+(1)^3 = 3$$

$xy+y^3$ is a polynomial so its continuous