9/3/19
Tuesday

HW
12.3

36 Where is

$$f(x,y) = \frac{2xy}{x^2 - y^2}$$

continuous?

Since

$$\frac{2x}{x^2 - y^2}$$

is a rational function $\left(\frac{\text{polynomial}}{\text{polynomial}}\right)$

it's continuous everywhere in its domain.

To be in the domain we need $x^2 - y^2 \neq 0$.

When is $x^2 - y^2 = 0$?

When $y = \pm x$.

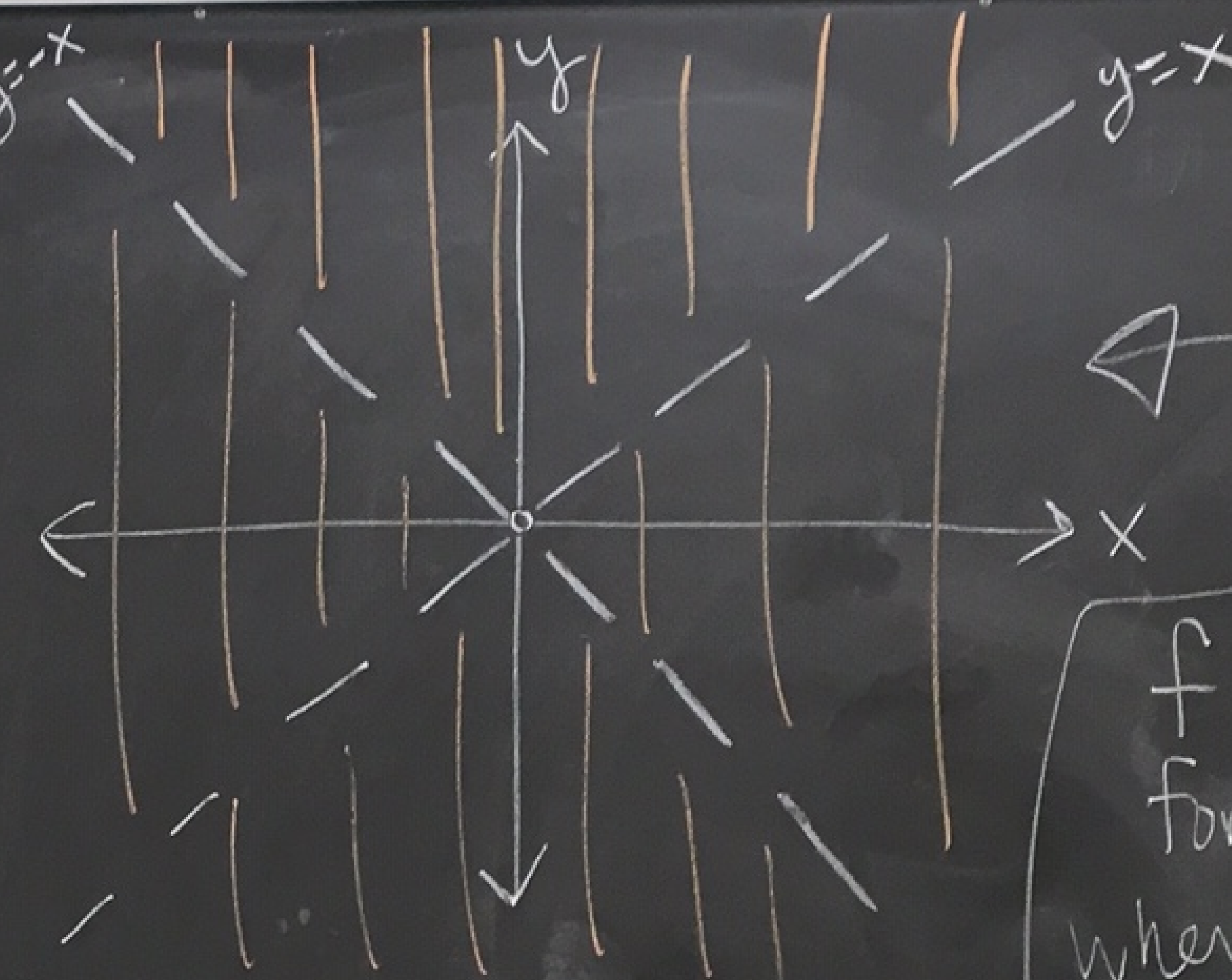
(x,y) is not in the domain when $y = \pm x$.

$$x^2 = y^2$$
$$\pm\sqrt{x^2} = y$$
$$\pm|x| = y$$
$$\pm x = y$$

$$x^2 - y^2 = 0$$
$$(x-y)(x+y) = 0$$

$x - y = 0$ $\quad$ $x + y = 0$

$y = x$ $\quad$ $y = -x$



f is continuous for all (x,y) where $y \neq \pm x$

Theorem: If $f(x,y)$ is continuous at (a,b) and $g(t)$ is continuous at $f(a,b)$, then $(g \circ f)(x,y)$ is continuous at (a,b).

Ex: Consider $h(x,y) = e^{\frac{2xy}{x^2-y^2}}$

What's the domain of h?

$(x,y) = (1,3)$ is in the domain since $h(1,3) = e^{\frac{2(1)(3)}{1^2-3^2}} = e^{-3/4}$ is well-defined

You can

So the only

can't plug i

(x,y) can't

$\frac{2x}{x^2-y^2}$

same do

is domai

You can plug any t into e^t.
So the only (x,y) that you can't plug into h are where (x,y) can't be plugged into $\frac{2x}{x^2-y^2}$. So $h(x,y) = e^{\frac{2xy}{x^2-y^2}}$ has the same domain as $\frac{2xy}{x^2-y^2}$, that is domain of h is all (x,y) with $y \neq \pm x$

-defined

Note:

Set

$$f(x,y) = \frac{2xy}{x^2-y^2}$$

$$g(t) = e^t$$

then

$$(g \circ f)(x,y)$$

$$= g(f(x,y))$$

$$g\left(\frac{2xy}{x^2-y^2}\right) = e^{\frac{2xy}{x^2-y^2}}.$$

At $(a,b) = (1,3)$

f is continuous at $(1,3)$

$$f(1,3) = \frac{2(1)(3)}{1^2-3^2} = \frac{-3}{4}$$

$g(t) = e^t$ is continuous everywhere.

So its continuous at $t = -\frac{3}{4}$.

So, $h = g \circ f$ is continuous at $(1,3)$.

and

$$\lim_{(x,y)\to(1,3)} e^{\frac{2xy}{x^2-y^2}} = e^{\frac{2(1)(3)}{1^2-3^2}} = e^{-3/4}$$

$e^{\frac{2xy}{x^2-y^2}}$ is continuous at $(1,3)$

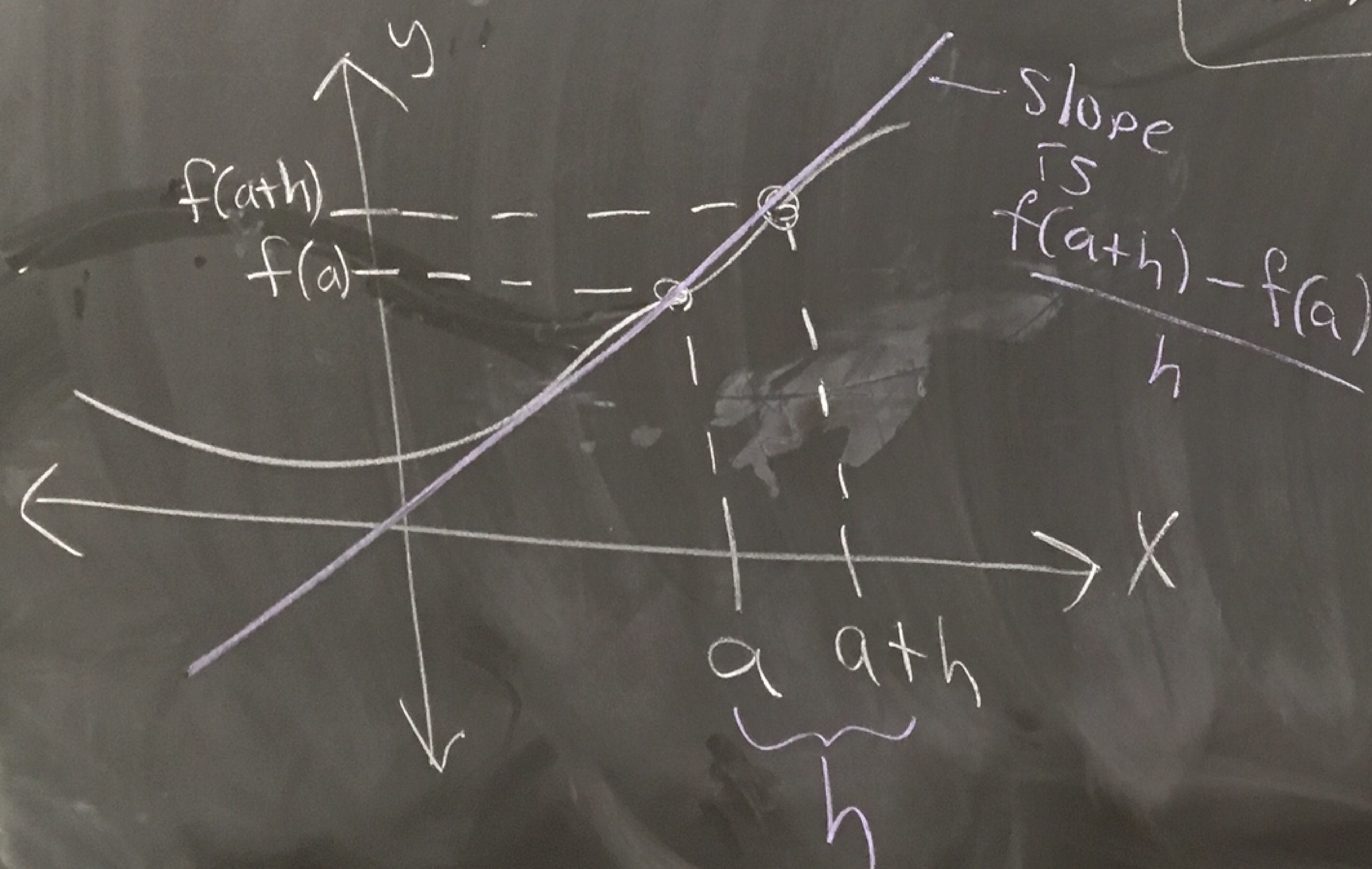
$e^{\frac{2xy}{x^2-y^2}}$ is continuous for all (x,y) with $y \neq \pm x$.

12.4 – Partial Derivatives

$\boxed{2d}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(if it exists)



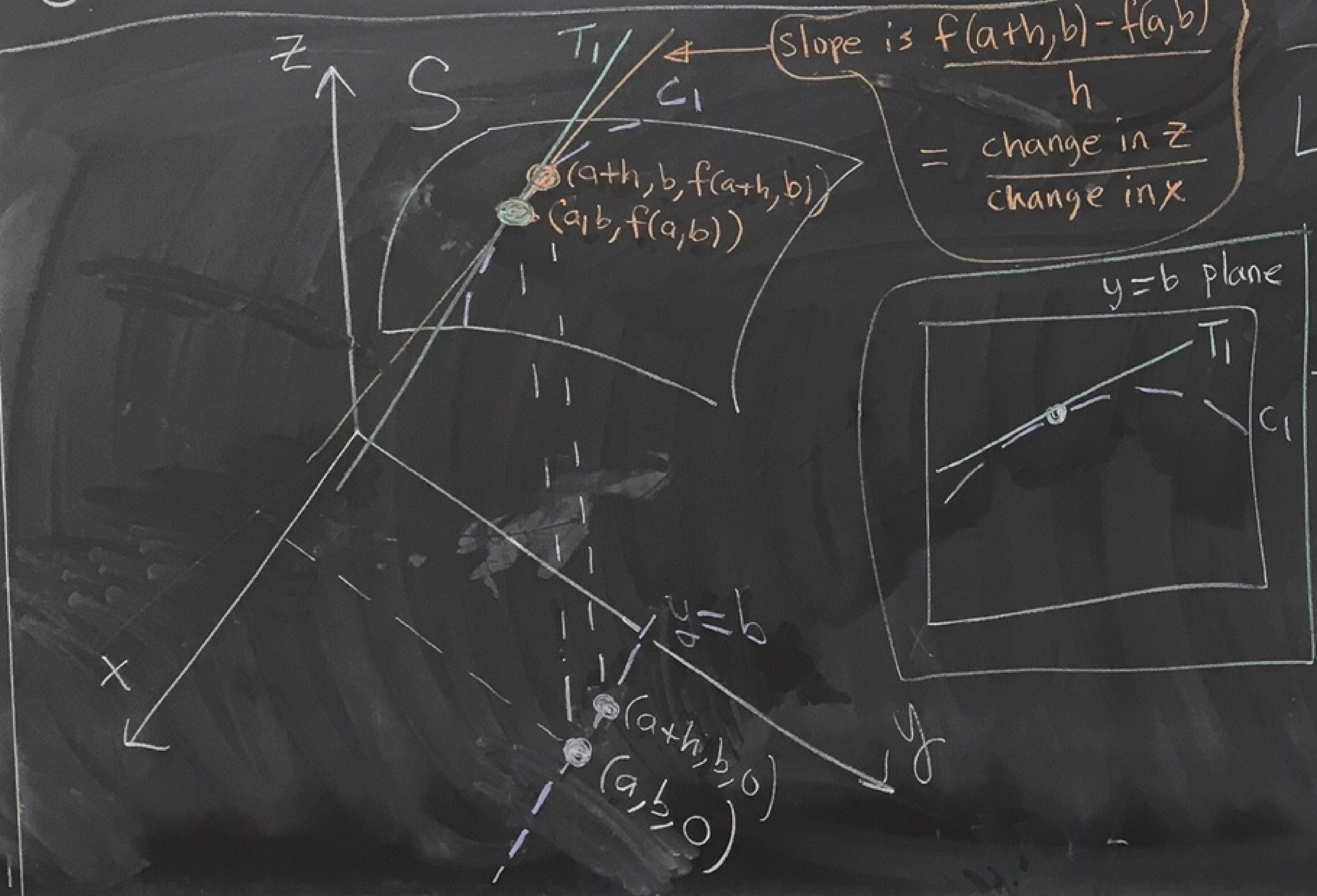
<u>Def:</u> Suppose we have a function $f(x,y)$ of two variables. Define the <u>partial derivatives</u> of f to be

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

and $$f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$$

if the limits exist.

Geometric Interpretation of f_x



What is $f_x(a,b)$? Let S be the surface given by $z=f(x,y)$.

Let C_1 be the curve on the surface S where the plane $y=b$ intersects S.

Then $f_x(a,b)$ is the slope of the tangent line T_1 that is tangent to C_1 at $(a,b,f(a,b))$.

How to calculate f_x and f_y

1. To calculate f_x, regard y as a constant and differentiate $f(x,y)$ with respect to x.

2. To calculate f_y, regard x as a constant and differentiate $f(x,y)$ with respect to y.

Ex: Let

$$f(x,y) = 4x^5y^6 - \frac{3}{7}x^3 + x^8y^{910} - 11y$$

Find f_x and f_y.

$$f_x(x,y) = 4y^6(5x^4) - \frac{3}{7}(3x^2) + y^{910}(8x^7) + 0$$

regard y as a fixed constant

$$= 20x^4y^6 - \frac{9}{7}x^2 + 8x^7y^{910}$$

$$f_y(x,y) = 4x^5(6y^5) + 0 + x^8(910y^{909}) - 11$$

regard x as a constant

$$= 24x^5y^5 + 910x^8y^{909} - 11$$

Notation: If $z = f(x,y)$ then
we have the following notations.

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) = \frac{\partial z}{\partial x} = D_x f$$

$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y) = \frac{\partial z}{\partial y} = D_y f$$