

Comprehensive Examination – Algebra

Syllabus

1. Group Theory:

Subgroups, normal subgroups, factor groups, homomorphisms, simple groups, normal series, commutators, solvable groups, conjugate elements and the class equation, Sylow theorems with applications, finite abelian groups.

2. Ring Theory:

Ideals, idempotents, nilpotents, prime ideals, factor rings, homomorphisms, maximal ideals, integral domain, principal ideal domains, Euclidean domains, polynomial rings, irreducibility criteria.

3. Field Theory:

Vector spaces, linear independence, basis and dimension, extension fields, algebraic and transcendental elements, algebraic extensions, primitive elements and simple extensions, splitting fields, finite fields, normal extensions, separable extensions, automorphisms of fields, Galois theory, computation of Galois groups.

References

Herstein, *Topics in Algebra*, 2nd ed. (Chapters 2, 3, 4, 5)

Hungerford, *Algebra*, (Chapters 1, 2, 3, 5)

Fraleigh, *Abstract Algebra*, 5th ed.