California State University Los Angeles, Department of Mathematics

Complex Analysis Comprehensive Examination Fall 2020

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Do five of the following seven problems.

1. Describe and sketch each of the following sets of complex numbers:

a. $A = \{z: 1 < |z^2| \leq 4\}$ b. $B = \left\{z: \text{Im}\left(\frac{1}{z}\right) > 1\right\}$

c. $C = \{z: \text{Re}(e^z) > 0\}$

2. Let γ be a simple closed curve in the plane, oriented counterclockwise. Suppose that f is analytic inside and on γ and that $f(z) = 2$ for all z on γ. Prove that $f(z) = 2$ for all z inside γ.

3. Evaluate the integral $\int_0^\infty \frac{dx}{1+x^6}$.

4. Evaluate the integral $\int_\gamma \frac{dz}{z(e^z-1)}$, where γ is the unit circle oriented counterclockwise.

5. Give two Laurent series expansions for the function

$$f(z) = \frac{1}{z^2(1-z)},$$

and specify the regions in which those expansions are valid.

6. Show that the polynomial $z^4 + z - 1$ has one root in the set $\left\{z: |z| < \frac{1}{3}\right\}$ and the remaining three roots in $\left\{z: \frac{1}{3} < |z| < 2\right\}$.

7. Consider the transformation $T(z) = \frac{z-a}{1-\bar{a}z}$, where a is a complex number of modulus less than 1.

a. Show that $T^{-1}(z) = \frac{z+a}{1+\bar{a}z}$ is the inverse of T in the domain of definition of T.

b. Show that T maps the circle $|z| = 1$ onto itself.

c. Show that T is a conformal map of $\mathbb{D} = \{z: |z| < 1\}$ onto itself.