Cal State Los Angeles Department of Mathematics
Complex Analysis Comprehensive Examination
Spring 2022
Committee: Akis, Gutarts, Shaheen*

Directions: Do five of the following seven problems. If you turn in more than five, the best five will be used.

Spring 2022 # 1.

(a) Where is the function $f(z) = \sqrt{z}$ analytic? Here the square root is defined using the principal branch of the logarithm.

(b) Where is the function $f(z) = \sqrt{z-2}$ analytic?

Spring 2022 # 2. Calculate $\displaystyle\int_\gamma \frac{e^z}{z^2(z-1)^3}\,dz$ where γ is the circle centered at 0 with radius 2, oriented counter-clockwise

---

Spring 2022 # 3. Find the Laurent series for $f(z) = \dfrac{z+1}{z^3(z^2+1)}$ on the region $A = \{z \mid 0 < |z| < 1\}$. Calculate the residue at $z_0 = 0$.

Spring 2022 # 4. Use Reside Theory to evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^4+16}$$

Spring 2022 # 5. Consider the domain $D = \{z \mid |z| < 1 \text{ or } 2 < |z|\}$. Let f be defined on D as follows

$$f(z) = \begin{cases} 1 & \text{if} \quad |z| < 1 \\ z & \text{if} \quad 2 < |z| \end{cases}$$

Find an entire function $g(z)$ such that $f(z) = g(z)$ for all $z \in D$, or show that such a function $g(z)$ does not exist.

Spring 2022 # 6. Let $f(z) = \dfrac{1}{\sin(\pi/z)}$. Find the singularities of $f(z)$. For each singularity of $f(z)$ characterize it as isolated or non-isolated. For each isolated singularity characterize it as essential, removable, or a pole.

Spring 2022 # 7. Suppose that $f(z)$ is analytic on

$$A_1 = \left\{ z \;\middle|\; \mathrm{Im}(z) < \frac{1}{2} \right\}$$

and that $g(z)$ is analytic on

$$A_2 = \{z \;\mid\; |z-3| > 1\}.$$

Let $h(z) = f(z) \cdot g(z)$.

Consider the Taylor series $\displaystyle\sum_{n=0}^{\infty} a_n (z-1)^n$ of $h(z)$ centered at $z_0 = 1$.

(a) Draw a picture of A_1 and a picture of A_2.

(b) Given the above information, what is the largest R such that we know for sure that $\displaystyle\sum_{n=0}^{\infty} a_n (z-1)^n$ converges on $A = \{z \;\mid\; |z-1| < R\}$?

(c) Give a formula for a_2 in terms of f and g.
[Hint: How is a_2 expressed in terms of $h(z)$?]
