California State University – Los Angeles Mathematics Masters Degree Comprehensive Examination

Complex Analysis   Fall 2017

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Do at least five of the following seven problems. All problems count equally. If you attempt more than five, the best five will be used.

Please

(1) Write in a fairly soft pencil (number 2) (or in ink if you wish) so that your work will duplicate well. There should be a supply available.
(2) Write on one side of the paper only
(3) Begin each problem on a new page
(4) Assemble the problems you hand in in numerical order

Exams are being graded anonymously, so put your name only where directed and follow any instructions concerning identification code numbers.

Notation: $\mathbb{C}$ denotes the set of complex numbers.
$\mathbb{R}$ denotes the set of real numbers.
$\operatorname{Re}(z)$ denotes the real part of the complex number z.
$\operatorname{Im}(z)$ denotes the imaginary part of the complex number z.
$\overline{z}$ denotes the complex conjugate of the complex number z.
$|z|$ denotes the absolute value of the complex number z.
$\operatorname{Log} z$ denotes the principal branch of $\log z$.
$\operatorname{Arg} z$ denotes the principal branch of $\arg z$.
$D(z;r)$ is the open disk with center z and radius r.
A *domain* is an open connected subset of $\mathbb{C}$.

MISCELLANEOUS FACTS

$$2\sin a\sin b = \cos(a-b) - \cos(a+b) \qquad 2\cos a\cos b = \cos(a-b) + \cos(a+b)$$

$$2\sin a\cos b = \sin(a+b) + \sin(a-b) \qquad 2\cos a\sin b = \sin(a+b) - \sin(a-b)$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b \qquad \cos(a+b) = \cos a\cos b - \sin a\sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$

$$\sin^2 a = \frac{1}{2} - \frac{1}{2}\cos(2a) \qquad \cos^2 a = \frac{1}{2} + \frac{1}{2}\cos(2a)$$

Fall 2016 # 1. Evaluate **TWO** of the following three integrals. Draw any contours and show any estimates needed to justify your methods.

a. $\displaystyle\int_{|z|=3} \frac{e^{3z}\,dz}{(z-1)^2(z-2)}$ **b.** $\displaystyle\int_{-\pi}^{\pi} \frac{d\theta}{2-\cos\theta}$ **c.** $\displaystyle\int_{-\infty}^{\infty} \frac{x\sin x}{x^2+1}\,dx$

Fall 2017 # 2. How many zeros (counting multiplicity) does the polynomial $z^5 + 3z^3 + 7$ have in the region $\{z \in \mathbb{C} : 1 < |z| < 2\}$?

Show your work and state carefully what theorems you are using.

Fall 2017 # 3. Consider the series $\sum_{n=1}^{\infty}(n+1)z^n$.

a. For which complex numbers z does this series converge?

b. For those z, let $f(z)$ be the sum of the series and Evaluate $f^{(5)}(0)$.

c. Find $f(z)$.

(Suggestion: Think about derivatives or integrals.)

Fall 2017 # 4. **a.** **[4 pts]** Find a nonconstant function $p : \mathbb{C} \to \mathbb{C}$ which is analytic on all of $\mathbb{C}$ with $p(1) = 0$ and $p(2) = 0$.

(This really is about as easy as it sounds. It is intended to get you started.)

b. **[9 pts]** Let $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disk in $\mathbb{C}$. Find a nonconstant function $f : D \to D$ which is analytic on all of D with $f(1/4) = 0$ and $f(1/2) = 0$. Justify your answer.

c.[7 pts] Can the function f in part **(b)** be a fractional linear (Möbius) transformation? (Why or why not)

Fall 2017 # 5. Suppose $f : \mathbb{C} \to \mathbb{C}$ is entire and that $|f(z)| > 1$ for all z in $\mathbb{C}$. Show that f must be constant on $\mathbb{C}$.

Fall 2017 # 6. If possible, give an example of an entire function $f : C \to C$ such that $f'(0) = 0$ and $f(z) = z$ for all z with $|z| > 1$. If not possible, explain fully stating any relevant theorems supporting your answer

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Fall 2017 # 7. **a.** State clearly and completely the Riemann Mapping Theorem

b. Which of the following sets are conformally equivalent to $\{z \in \mathbb{C} : |z| < 1\}$, the open unit disk in $\mathbb{C}$? Fully justify your answers (why or why not).

i. $\mathbb{C}$

ii. $\{z \in \mathbb{C} : \operatorname{Re} z < 0\}$

iii. $\{z \in \mathbb{C} : z \neq 0\}$

iv. $\{x + iy \in \mathbb{C} : 0 < x \text{ and } \sin(1/x) < y < 2\}$

v. $\mathbb{C} \setminus \{z \in \mathbb{C} : \operatorname{Re} z \leq 0 \text{ and } \operatorname{Im} z = 0\}$

End of Exam