# California State University – Los Angeles Mathematics Masters Degree Comprehensive Examination

## Complex Analysis    Fall 2018

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**Do at least five of the following seven problems. All problems count equally. If you attempt more than five, the best five will be used.**

Please

(1) Write in a fairly soft pencil (number 2) (or in ink if you wish) so that your work will duplicate well. There should be a supply available.
(2) Write on one side of the paper only
(3) Begin each problem on a new page
(4) Assemble the problems you hand in in numerical order

**Exams are being graded anonymously, so put your name only where directed and follow any instructions concerning identification code numbers.**

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Notation: $\mathbb{C}$ denotes the set of complex numbers.
$\mathbb{R}$ denotes the set of real numbers.
$\mathrm{Re}(z)$ denotes the real part of the complex number $z$.
$\mathrm{Im}(z)$ denotes the imaginary part of the complex number $z$.
$\overline{z}$ denotes the complex conjugate of the complex number $z$.
$|z|$ denotes the absolute value of the complex number $z$.
$\mathrm{Log}\, z$ denotes the principal branch of $\log z$.
$\mathrm{Arg}\, z$ denotes the principal branch of $\arg z$.
$D(z; r)$ is the open disk with center $z$ and radius $r$.
A *domain* is an open connected subset of $\mathbb{C}$.

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**MISCELLANEOUS FACTS**

$$2\sin a \sin b = \cos(a-b) - \cos(a+b) \qquad 2\cos a \cos b = \cos(a-b) + \cos(a+b)$$

$$2\sin a \cos b = \sin(a+b) + \sin(a-b) \qquad 2\cos a \sin b = \sin(a+b) - \sin(a-b)$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \qquad \cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\sin^2 a = \frac{1}{2} - \frac{1}{2}\cos(2a) \qquad \cos^2 a = \frac{1}{2} + \frac{1}{2}\cos(2a)$$

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**Fall 2018 # 1.** Describe and sketch the set of all complex numbers $z$ satisfying each of the following.

**a.** $\operatorname{Im}(z^2) > 1$ **b.** $\left| e^{(z^2)} \right| < e$ **c.** $\left| \dfrac{z-4}{z-6} \right| = 1$

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**Fall 2018 # 2.** Suppose $u : A \to \mathbb{C}$ is harmonic on an open set $A$ and $v : A \to \mathbb{C}$ is a harmonic conjugate for $u$ on $A$. Let $h : A \to \mathbb{C}$ be given by $h(x,y) = (u(x,y))^2 - (v(x,y))^2$.

**a.** Show that $h$ is harmonic on $A$.
**b.** Find a harmonic conjugate $g(x,y)$ for $h$ on $A$.

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**Fall 2018 # 3.** Let $f(z) = \dfrac{e^{1/z}}{z+1}$

**a.** Find all singularities of $f$ in the complex plane $\mathbb{C}$ and classify each as removable, a pole, or essential. For poles, give the order.
**b.** Find the residue of $f$ at each of the singularities found in part **(a)**.

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**Fall 2018 # 4.** Let $f(z) = \dfrac{1}{z^3(z-1)(z-2)}$. Find the Laurent series for $f$ in each of the following regions.

**a.** $A = \{z \in \mathbb{C} : 0 < |z| < 1\}$
**b.** $B = \{z \in \mathbb{C} : 1 < |z| < 2\}$
**c.** $C = \{z \in \mathbb{C} : 2 < |z|\}$

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**Fall 2018 # 5.** Evaluate the integral $\displaystyle\int_0^{\pi} \frac{d\theta}{3 + \cos\theta}$. Show all work leading to your answer.

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**Fall 2018 # 6.** If possible, find an entire function $h : \mathbb{C} \to \mathbb{C}$ such that $h(z) = 0$ on the set $\{z \in \mathbb{C} : |z| < 1\}$ and $h(z) = z$ on the set $\{z \in \mathbb{C} : 2 < |z|\}$. If this is not possible, explain why such a function does not exist.

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**Fall 2018 # 7.** Let $A = \{z \in \mathbb{C} : 0 < \arg z < \pi/1000\}$. Determine whether there is a conformal map of $A$ onto the disk $B = \{z \in \mathbb{C} : |z| < \pi/1000\}$. State completely and clearly any theorems you are using.

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# End of Exam